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Survival

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ABSTRACT

Empirical analysis of rates of return in finance implicitly condition on the security surviving into the sample. We investigate the implications of such conditioning on the time series of rates of return. In general this conditioning induces a spurious relationship between observed return and total risk for those securities that survive to be included in the sample. This result has immediate implications for the equity premium puzzle. We show how these results apply to other outstanding problems of empirical finance. Long-term autocorrelation studies focus on the statistical relation between successive holding period returns, where the holding period is of possibly extensive duration. If the equity market survives, then we find that average return in the beginning is higher than average return near the end of the time period. For this reason, statistical measures of long-term dependence are typically biased towards the rejection of a random walk. The result also has implications for event studies. There is a strong association between the magnitude of an earnings announcement and the postannouncement performance of the equity. This might be explained in part as an artefact of the stock price performance of firms in financial distress that survive an earnings announcement. The final example considers stock split studies. In this analysis we implicitly exclude securities whose price on announcement is less than the prior average stock price. We apply our results to this case, and find that the condition that the security forms part of our positive stock split sample suffices to explain the upward trend in event-related cumulated excess return in the preannouncement period.

LOOKING BACK OVER THE history of the London or the New York stock markets can be extraordinarily comforting to an investor—equities appear to have provided a substantial premium over bonds, and markets appear to have recovered nicely after huge crashes. The tendency of prices and yields to revert toward a mean appears suggestive of a long-term equilibrium in the financial markets. Less comforting is the past history of other major markets: Russia, China, Germany, and Japan. Each of these markets has had one or more major interruptions that prevent their inclusion in long-term studies. This observation suggests that it might be fruitful to consider the possible

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implications of the most pervasive ex post conditioning in empirical finance: the survival of the return history to be included in the sample.

We derive the distributional properties of stock prices that survive conditioning of this kind. As we would expect, expected returns are biased by this kind of conditioning. The magnitude of this bias is an increasing function of the volatility of returns. The result has immediate implications for the study of equity returns in emerging capital markets. Such markets are characterized by a significant ex ante probability of failure and are quite volatile. We should expect to see a significant equity premium in emerging capital market returns. There are implications for other kinds of studies as well.

Long-term autocorrelation studies focus on the statistical relation between successive holding period returns, where the holding period is of possibly extensive duration. If the equity market survives, then we find that average return in the beginning is higher than average return near the end of the time period. For this reason, statistical measures of long-term dependence are typically biased towards the rejection of a random walk. We find that the direction and magnitude of this bias is sensitive to the choice of return horizon, to the ex ante viability of the exchange in question, and to the criteria for survival. The issue of survival has been noted by researchers who use long-term financial data such as Shiller (1989), and researchers such as Harvey (1994) who use series that are subject to attrition. However the empirical implications of survival have yet to be specifically addressed. We provide preliminary numerical examples that show how statistics used to detect long-term market patterns are affected.[1]

Event studies typically look at the impact of corporate announcements on security prices after the announcement has been made, and then correlate this impact with the content of the announcement. The object is to discover the speed of market adjustment to this new information. There appears to be a strong association between the magnitude of an earnings announcement and the postannouncement performance of the equity. Firms in financial distress are in effect at-the-money call options, and we would expect the equity to have a higher return than a corresponding all-equity firm. In fact, the average return of firms that survive the announcement will be inversely related to the extent to which the equity represents an in-the-money call option. This will be true for both the announcement period and the postannouncement period. Provided there is cross-sectional dispersion in the extent to which the equity is in-the-money, there will be an induced cross-sectional relationship between average returns in the announcement and in the postannouncement period. Whether this effect is large enough to explain the observed postearnings drift phenomenon depends on a careful reexamination of the empirical evidence.

[1] Similar implications follow where the question of interest is the predictive properties of dividend yields for long-term returns (Goetzmann and Jorion (1995)).

Event studies sometimes find substantial price increases prior to the public announcement. Many would attribute apparent run-ups in price to market leakages or insider trading activity. In the analysis of positive stock splits, we implicitly exclude securities whose price on announcement is less than the prior average stock price. We apply our results to this case and find that this conditioning suffices to explain the upward trend in event-related security average return in the preannouncement period.

The article is organized as follows. Section I characterizes the properties of the price path, conditional upon surviving a sample selection criterion. Section II studies the implications of this result for the analysis of long-term autocorrelations. Section III analyzes the application of the results to the study of postevent performance subsequent to earnings announcements, whereas Section IV looks at the run-up in average prices prior to stock splits. Section IV concludes.

I. Properties of Surviving Return Histories

Virtually all empirical work in finance is conditioned upon the availability of data, but none more so than studies of long-term market behavior. Extending the history of the New York Stock Exchange back in time adds information to researchers about long-term mean, variance, and time-series behavior, but the cost of this information is the potential bias imparted by conditioning upon the survival of the market, or in less extreme cases, the unbroken continuity of transaction prices. As researchers seek to enhance the power of statistical tests by collecting longer and longer market price sequences, accounting for survival becomes a nontrivial problem. Does it comfort investors to know that the world's most successful stock market, a market that survived two world wars and a global depression over the last century, provided a six percent equity premium? How meaningful is it to show that markets which bounced back from great crashes in the 1930s and 1970s display ex post evidence of mean reversion?

A survey of the history of the world's equity markets shows that it is not uncommon to have an hiatus in trading that renders the index unsuitable for long-term econometric studies. Since the beginning of organized trading in shares in Holland in the 17th century, many stock markets have appeared, but only a few have survived continuously without a break for more than a few decades. For instance, when the New York Stock Exchange began in 1792, it was possible to speculate in shares in the financial markets of Britain, Holland, France, Germany, and Austria. Of these, only the United States and Britain yield continuous historical share price information.[2] Data on German and Austrian markets suffer from a major suspension during

[2] In fact, the New York and London exchanges were both shut down for a period of months during World War I, creating problems for researchers who use monthly data.

World War II, and the hyperinflation in France during this century makes economic return calculation difficult.

Over a more recent horizon, there is historical evidence of at least thirty-six exchanges extant at the beginning of the century.[3] More than half of these suffered at least one major hiatus in trading.[4] Of the survivors, several are still considered emerging markets, suggesting that they have only recently experienced the level of growth that attracts international investors, and in turn induces researchers to gather price data. In fact the very term "emerging markets" admits the possibility that these markets might fail.

Despite historical evidence about the patterns of emergence and disappearance of stock exchanges, correlations across markets make it difficult to estimate the ex ante probability of survival for any given market. Most of the suspensions that prevent long-term econometric studies of investor return were the result of revolutions or major wars. The fact that most of the continuous markets are in former British colonies such as Australia, Canada, India, South Africa, and the United States is almost certainly an accident of political, and perhaps legal, history. Had the outcome of either world war been different, we might currently be studying the long-term behavior of continental European exchanges.

Just as it is difficult to estimate the ex ante probability of market survival, it is also difficult explicitly to model market breaks and suspensions. We have chosen to characterize a market as a stochastic variable, and a break or suspension of trading as an absorbing lower bound. In other words, we assume that market failure or appropriation by the state is likely to be anticipated by falling prices. Other processes are equally reasonable. A hyperinflation prelude to market closure may be characterized by an absorbing upper bound. Revolution may in fact be consequent on sustained excessive rates of return realized by domestic and foreign investors. In the analysis that follows we have taken a few simple rules as illustrative, but by no means definitive examples of market failure.

Suppose the researcher limits his or her analysis to a price series that has survived a specified period of study. The researcher has observed a time series of prices p_t which we are assuming is generated on $(0, T)$ by a simple absolute diffusion

$$dp = \mu dt + \sigma dz$$

[3] A survey of the *GT Guide to World Equity Markets* (O'Connor (1991)) suggests that of the cities that currently have significant exchanges, Amsterdam, Belgrade, Berlin, Bombay, Brussels, Budapest, Buenos Aires, Cairo, Caracas, Copenhagen, Dublin, Frankfort, Geneva, Helsinki, Hong Kong, Istanbul, Johannesburg, Lisbon, London, Madrid, Melbourne, Mexico City, Milan, Montreal, Moscow, New York, Oslo, Prague, Rio de Janeiro, Santiago, Seoul, Stockholm, Tokyo, Vienna, Warsaw, and Wellington all had exchanges in 1901.

[4] Amsterdam, Belgrade, Berlin, Brussels, Budapest, Buenos Aires, Cairo, Copenhagen, Frankfort, Hong Kong, Istanbul, Lisbon, Madrid, Mexico City, Moscow, Prague, Rio de Janeiro, Santiago, Seoul, Tokyo, Vienna, and Warsaw all suffered major suspensions in activity due to nationalizations or war.

where μ and σ are parameters and z is a Brownian path.[5] Usually we think of p as the log of asset price.

In a simple example of survival, assume there is a reservation price $\underline{p}$ below which the stock market ceases to function and securities cease to trade. For purposes of analysis, we start the price level at $p_0 + \underline{p}$. If the researcher studies only price paths that stay above $\underline{p}$ on the interval (0, T), what should he or she expect to see? The conditioning event that of interest is

$$A = \{\text{price path is greater than } \underline{p} \text{ on the interval } [0, T]\}$$

More generally, the set A defines the set of price paths that survive, where the ex ante probability of survival at date t depends on the level of prices at date t, p_t. Given that event A has occurred, we wish to find the distribution of the path of p_t. The paths in A will be diffusions, and the key to their analysis is the conditional probability that the path belongs to A given that $p_t = p$ at time t:

$$\pi(p, t) = \Pr(\mathrm{A} \mid p, t).$$

Ross (1987) introduces the following lemma to describe the properties of the transformed diffusion and shows why π is central to the analysis.[6] The relevant probability concepts may be found in Karlin and Taylor [1975, 1981].

LEMMA 1 (Karlin and Taylor): *Let p follow a diffusion with*

$$dp = \mu\, dt + \sigma\, dz$$

where μ and σ are constants. For any set A (with an interior) the process for p conditional on being in A, i.e., p/A, follows a diffusion

$$dp^* = dp \mid A = \mu^*\, dt + \sigma^*\, dz$$

with

$$\mu^* = \mu + \sigma^2 \frac{\pi_p}{\pi}$$

and

$$\sigma^* = \sigma$$

where π is the conditional density of A given (p, t).

[5] In this and what follows, the relevant material on diffusions and stochastic processes may be found in Karlin and Taylor (1975, 1981). For a discussion of the properties of conditional diffusions see Ross (1987).

[6] This result has recently been extended in an obvious way to vector-valued p diffusions by Shui (1995).

Proof: By definition

$$\mu^* = E(\Delta p \mid A, p, t)$$
$$= \int \Delta p f(\Delta p \mid A, p, t)\, d\Delta p$$

where $f(\Delta p \mid A, p, t)$ is the conditional density of Δp.

From Bayes' Theorem

$$f(\Delta p \mid A, p, t) = \frac{f(A, \Delta p \mid p, t)}{f(A \mid p, t)}$$
$$= \frac{1}{\pi} f(A, \Delta p \mid p, t).$$

Now, using Bayes' Theorem and the smoothness properties of diffusions,

$$f(A, \Delta p \mid p, t) = f(A \mid p + \Delta p, t + \Delta t) f(\Delta p \mid p, t)$$
$$= [\pi + \pi_p \Delta p + \pi_t \Delta t] f(\Delta p \mid p, t)$$

where we assume that (p, t) is in the interior of A.

Hence

$$\mu^* = E(\Delta p \mid A, p, t)$$
$$= \int \Delta p f(\Delta p \mid A, p, t)\, d\Delta p$$
$$= \int \Delta p \frac{1}{\pi} [\pi + \pi_p \Delta p + \pi_t \Delta t] f(\Delta p \mid p, t)\, d\Delta p$$
$$= \mu + \sigma^2 \frac{\pi_p}{\pi}.$$

A similar analysis verifies that $\sigma^* = \sigma$.

In our case, A is the set of all paths that survive to period $T > t$, i.e., those that do not get absorbed at $\underline{p}$. Define $m(t)$ as

$$m(t) = \inf_{s \in (t, T)} p(s).$$

We use the sets

$$A = \{\text{paths such that } p(t) = p, m(t) > \underline{p}\}.$$

Clearly,

$$\pi(p, t) = \Pr(A(p, t) \mid p(t) = p)$$
$$= \Pr\big(m(t) > \underline{p}\big).$$

We will conduct the analysis assuming that the unconditional process has zero drift, i.e., $\mu = 0$, although the analysis can be easily extended to cases allowing for a drift term.[7] We use the result from Ingersoll [1987], p. 352, for a Brownian motion with absorbing state at zero

$$\pi(p,t) = \Phi\left[\frac{p - \underline{p}}{\sigma\sqrt{T-t}}\right] - \Phi\left[\frac{-(p - \underline{p})}{\sigma\sqrt{T-t}}\right]$$
$$= 2\Phi\left[\frac{p - \underline{p}}{\sigma\sqrt{T-t}}\right] - 1$$

where $\Phi[\cdot]$ denotes the standard Normal distribution function. Then it follows that

$$\mu^* = \mu + \sigma^2 \frac{\pi_p}{\pi}$$
$$= \frac{2\sigma\varphi[w]}{\sqrt{T-t}\,(2\Phi[w] - 1)}$$

where $\varphi[\cdot]$ is the standard Normal density function and

$$w = \frac{p - \underline{p}}{\sigma\sqrt{T-t}}.$$

With zero drift, the conditional diffusion is then

$$dp^* = \frac{2\sigma\varphi[w]}{\sqrt{T-t}\,(2\Phi[w] - 1)}\, dt + \sigma\, dz.$$

It follows that even when the underlying process has zero drift, the expected return is positive for all $t < T$. Furthermore, it is easy to show that expected return is an increasing function of the standard deviation parameter σ for all $t < T$.

What does this path look like? The mean Brownian path is represented by the differential equation

$$\frac{dp}{dt} = \frac{2\sigma\varphi[w]}{\sqrt{T-t}\,(2\Phi[w] - 1)}.$$

We can characterize some important properties of this equation, although we cannot solve it directly. Applying L'Hopital's rule and letting T approach infinity, we obtain a simpler expression for the conditional diffusion:

$$dp^* = \frac{\sigma^2}{p - \underline{p}}\, dt + \sigma\, dz$$

[7] A trivial extension would be to consider a case where the lower bound K increases with the expected increase in stock price. In this case we would interpret *pt* as the price in excess of the lower bound. The zero drift results apply. More general results can be obtained using the result cited in Ingersoll (1987; p. 352) for the case of a Brownian motion with drift.

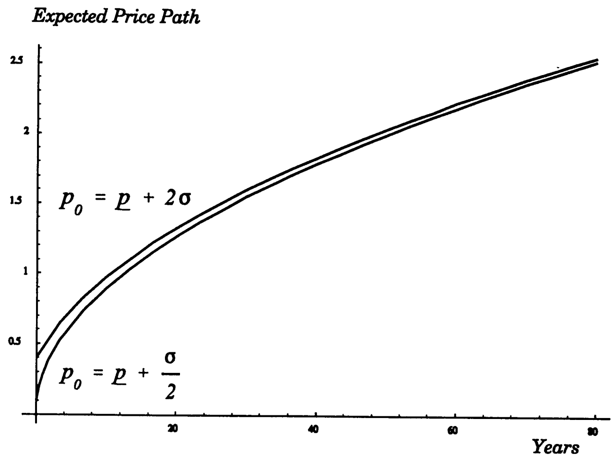

Figure 1. The mean Brownian path (in excess of the minimum price $\underline{p}$) for annual standard deviation σ = 0.2 for initial prices 0.5, and 2 standard deviations above $\underline{p}$.

and we can solve for the mean Brownian path directly:

$$\bar{p}_t = \sigma\sqrt{\frac{(p_0 + \underline{p})^2}{\sigma^2} + 2t}$$

which is evidently an increasing concave function of time, with the degree of concavity an increasing function of the volatility parameter σ. This function is given in Figure 1 for annual standard deviation 0.2.

This analysis assumes a zero drift process. If we consider the unconditional process to represent excess returns, then the mean Brownian path would correspond to the cumulative average return measure popular in event studies (see Brown and Warner (1980)). In other applications, it is appropriate to consider positive drift processes.

For a nonzero drift μ, it is more difficult to derive the marginal probability

$$\pi(p,t) = \Pr(A(p,t) \mid p(t) = p).$$

We can solve this problem by going directly to $t = \infty$ and searching for a stationary solution. Notice that for $\mu = 0$

$$\pi(p) \equiv \Pr\left(p(s) \geq \underline{p}, \text{all } s\right) = 0,$$

since the price path is certain to hit $\underline{p}$. For $\mu > 0$ the problem is more interesting. The conditioning set A is equivalent to the condition

$$p(t) = \mu t + \sigma z(t) + p_0 > \underline{p}$$

or $z(t) > \frac{\underline{p} - p_0}{\sigma} - \frac{\mu}{\sigma}t$ for all t. For the process with local drift μ and speed σ we can solve for the marginal probability $\pi(p)$ by employing the (backward) diffusion equation. We want to solve

$$\frac{1}{2}\sigma^2\pi_{pp} + \mu\pi_p = 0$$

subject to a marginal probability of zero at the lower bound $\pi(\underline{p}) = 0$, and a probability approaching unity for an infinite price $\pi(\infty) = 1$. This is easy to solve by conversion to a first order equation. The solution is

$$\pi(p) = 1 - e^{-\frac{2\mu}{\sigma^2}(p-\underline{p})}.$$

Hence, the conditional process is

$$\begin{aligned} dp^* &= \left[\mu + \sigma^2\frac{\pi_p}{\pi}\right]dt + \sigma\, dz \\ &= \left[\mu + \frac{2\mu e^{-\frac{2\mu}{\sigma^2}(p-\underline{p})}}{1 - e^{-\frac{2\mu}{\sigma^2}(p-\underline{p})}}\right]dt + \sigma\, dz. \end{aligned}$$

Note that as μ goes to zero we approach

$$dp^* = \frac{\sigma^2}{p - \underline{p}}\, dt + \sigma\, dz$$

as before.

This result suggests that survival will induce a substantial spurious equity premium. Note that the mean return conditional on survival is

$$\mu^* = \mu + \frac{2\mu(1 - \pi(p))}{\pi(p)}$$

where we interpret $\pi(p)$ as the probability that the stock market will survive over the very long term given the current level of prices p.

To take an extreme case, suppose that the true equity premium is in fact zero, with μ equal to the (positive) return on a risk-free security. The observed equity premium is then $\mu^* - \mu$. If the return on a risk free security is 4 percent, an observed equity premium of 8 percent[8] is consistent with $\pi(p)$ equal to 50 percent. This is perhaps not unreasonable, given the number of stock markets that have survived the past 100 years. If we take a

[8] In Ibbotson (1991) the average risk-free rate for the period 1926 to 1990 is 3.7 percent, and the average equity premium (stock return minus U.S. Treasury bill return) is 8.4 percent.

less extreme case, a true equity premium of 4 percent would imply a much higher probability of long-term survival, with $\pi(p)$ equal to 80 percent. This may go at least part of the way to explaining the equity premium puzzle posed by Mehra and Prescott [1985]. They observe that the ex post historical premium provided by stocks over bonds is much larger than that can be justified by reasonable specifications of investor risk aversion. We find that, for any positive probability of series survival, the *unconditional* equity premium is lower than would be observed by averaging the differences in *conditional* return series when those series do not have the same variance.[9] Emerging markets provide an example where volatility appears to be associated with substantial equity premia (Harvey [1994]).[10]

II. Long-Term Dependence

Given that an empirical investigator has observed a price series up to date T that has survived an absorbing barrier and that obeys the conditional diffusion given above, what can we say about long-term dependence? Concavity of the mean Brownian path suggests that long-term holding period returns will be negatively autocorrelated, as average holding period returns in the initial period will be higher than average returns measured over the entire period, and average holding period returns in the latter part of the data will be less than the overall average return.

Analysis of the variance of holding period returns as the holding period grows longer provides a popular measure of return persistence. Positive autocorrelation in returns implies that annualized variances increase with the holding period for which returns are computed. A decrease in annualized variance suggests that stock prices exhibit mean reversion properties. In fact, our results show that annualized variance measures for long horizon returns will fall to 43 percent of variances estimated using short-horizon data, regardless of the volatility of returns and distance from the critical reservation price $\underline{p}$.

[9] Rietz (1988) suggests that a small probability of a large "crash" in consumption can justify a large equity premium. The Mehra and Prescott (1988) reply challenges Rietz to identify such catastrophic events, and estimate their probabilities. Our analysis suggests that, when such events are correlated with market closure, they will appear much less frequently in surviving economic histories than their ex ante probabilities would indicate.

[10] Harvey (1994) argues that this association is due to an asymmetric response of volatility to price level in emerging markets. The present analysis provides an alternative explanation for this phenomenon. To the extent that investments in emerging markets are like at-the-money call options, we would expect that return increases in the volatility of those markets that survive, and decreases in the extent to which the option is in-the-money $(p_t - \underline{p})$. Notice that the expected price path shown in Figure 1 suggests that extending data series back further in time does not necessarily reduce the bias induced by survival. In fact, for fixed lower bounds the bias in average returns is exacerbated.

To analyze the variance of long horizon returns we consider the distribution of the logarithm of prices p_T at the conclusion of a T-year holding period, assuming that the return series survives to that point. We again use the result for a Brownian motion with absorbing state at zero to establish that for price paths that survive a reservation price of $\underline{p}$, the probability that prices will exceed a level $p_T > \underline{p}$ will be given by

$$\begin{aligned}\Pr(p > p_T) &\equiv P(p_T)\\ &= 1 - \Phi\left[\frac{-(p_T - \underline{p}) + (p_0 - \underline{p})}{\sigma\sqrt{T}}\right] - \Phi\left[\frac{-(p_T - \underline{p}) - (p_0 - \underline{p})}{\sigma\sqrt{T}}\right]\\ &= \Phi\left[\frac{p_0 - p_T}{\sigma\sqrt{T}}\right] - \Phi\left[\frac{-(p_T + p_0 - 2\underline{p})}{\sigma\sqrt{T}}\right]\end{aligned}$$

where $\Phi[\cdot]$ denotes the standard Normal distribution function. Then it follows that the probability density function of p_T given that it survives is

$$f(p_T) \equiv \frac{\dfrac{-\partial P(p_T)}{\partial p_T}}{P(\underline{p})} = \frac{\dfrac{1}{\sigma\sqrt{T}}\varphi\left[\dfrac{p_T - p_0}{\sigma\sqrt{T}}\right] - \dfrac{1}{\sigma\sqrt{T}}\varphi\left[\dfrac{p_T + p_0 - 2\underline{p}}{\sigma\sqrt{T}}\right]}{2\Phi\left[\dfrac{p_0 - \underline{p}}{\sigma\sqrt{T}}\right] - 1}.$$

The moment generating function $\psi_c(\theta)$ of this conditional distribution is

$$\begin{aligned}&\psi_c(\theta)\\ &\quad\equiv E[e^{\theta x}]\\ &\quad= \int_{\underline{p}}^{\infty} e^{\theta x} f(x)\,dx\\ &\quad= \frac{\displaystyle\int_{\underline{p}}^{\infty}\frac{1}{\sigma\sqrt{T}}e^{\theta x - \frac{(x-p_0)^2}{2\sigma^2 T}}\,dx - \int_{\underline{p}}^{\infty}\frac{1}{\sigma\sqrt{T}}e^{\theta x - \frac{(x+p_0-2\underline{p})^2}{\sigma^2 T}}\,dx}{2\Phi[w] - 1}\\ &\quad= \frac{e^{\theta p_0 + \frac{\sigma^2 T}{2}\theta^2}(1 - \Phi[w - \sigma\sqrt{T}\theta]) - e^{\theta(2\underline{p} - p_0) + \frac{\sigma^2 T}{2}\theta^2}(1 - \Phi[-w - \sigma\sqrt{T}\theta])}{2\Phi[w] - 1}\end{aligned}$$

for $w = \dfrac{p_0 - \underline{p}}{\sigma\sqrt{T}}$. By contrast, with no conditioning the terminal price will be distributed as Normal with mean p_0, and the associated moment generating function will be

$$\psi_u(\theta) = e^{\theta p_0 + \frac{\sigma^2 T}{2}\theta^2}$$

with mean $E_u[p_T] = \psi_u'(0) = p_0$ and variance $\mathrm{Var}_u[p_T] = \psi_u''(0) - \psi_u'(0)^2 = \sigma^2 T$. After some simplification, the moments of the conditional distribution are given as

$$E_c[p_T] = \psi_c'(0) = \frac{2\underline{p}\Phi[w] - (2\underline{p} - p_0)}{2\Phi[w] - 1}$$

and

$$\mathrm{Var}_c[p_T] = \psi_c''(0) - \psi_c'(0)^2 = \sigma^2 T + (p_0 - \underline{p})^2 \left(1 - \frac{1}{(2\Phi[w] - 1)^2} + \frac{2\varphi[w]}{w(2\Phi[w] - 1)}\right).$$

Since it can shown that the second term in the above expression is negative for all T and for $w > 0$, the conditional variance is always less than the unconditional variance. To determine whether the conditional price displays general mean reverting behavior, or alternately, a general positive autocorrelation property, we need to know the behavior of the annualized variance

$$\omega[T] \equiv \frac{1}{T} \mathrm{Var}_c[p_T].$$

If $\omega[T]$ is falling, this is an indication of mean reversion. A rising $\omega[T]$ indicates positive autocorrelation since the long-run variance exceeds the sum of the short-run variances. This function is given in Figure 2 for two assumptions about the initial price. In the first case, we assume the initial price is half a per period standard deviation above the reservation price. In the second case, we assume the initial price is two standard deviations above the reservation price. In each case, the conditional variance is less than the unconditional variance, and appears to approach an asymptote that is independent of the initial price.

The limit of $\omega[T]$ as T approaches zero is the unconditional variance σ^2. It can be shown that $\omega[T]$ is everywhere a decreasing function of T and approaches an asymptote as T grows large

$$\lim_{T \Rightarrow \infty} \omega[T] = \frac{4 - \pi}{2} \sigma^2$$

which does not depend on how far the initial price p_0 is from the reservation price $\underline{p}$. This result suggests that substantial mean reversion will be evident for any stock return history that has survived, so long as the investigator studies a sufficiently long holding period. This formula indicates that the variance ratio will approach $(4 - \pi)/2$ or 0.429204 regardless of how volatile the market has been or how secure it is from failure.

Of course, empirical studies of long-period return variance ratios (for example, Lo and MacKinlay (1988) and Poterba and Summers (1988) do not find numbers this low. The precise value of the variance ratio conditional on

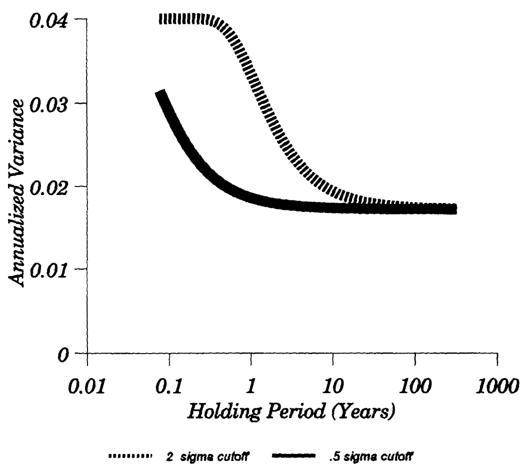

Figure 2. Annualized holding period variance ($\omega[T]$ in text) plotted as a function of holding period T given annual standard deviation $\sigma = 0.2$ and initial prices 0.5, and 2 standard deviations above the reservation price $\underline{p}$.

survival depends crucially on assumptions that are made about the conditions that define whether or not a particular price path survives. The above results are derived for the special case where returns exhibit zero drift. They can be trivially extended to cases where the lower absorbing barrier rises through time at a rate equal to the positive drift in the sequence of returns. It is difficult to obtain analytic results for the case where the absorbing barrier rises at a rate that differs from the expected return, or where survival depends on whether prices are greater than a certain fixed value relative to the previous maximum price.

To address these issues, we consider a simple simulation experiment. Weekly returns are generated from a zero mean Normal distribution with annualized standard deviation equal to 0.20. Holding period returns are computed for 4-week, 26-week, 1-, 10-, 20-, 40-, 80-, 160-, and 320-year holding periods. The variance of terminal wealth is computed across 60,000 replications of this experiment. As illustrated in Figure 3, the variance of terminal wealth is 0.04 for each holding period. We then consider two cases. In the first case, we assume the initial price is half an annualized standard deviation above the reservation price. In the second case, we assume the initial price is two standard deviations above the reservation price. The results closely correspond in numerical value to the theoretical results presented in Figure 2.

We then change the experiment to consider a 10 percent annualized expected return. We then assume that the reservation price is constant in real terms, with a 2.3 percent assumed annual rate of inflation. We then consider two examples that correspond to the cases considered above. The variance of terminal wealth for short holding periods corresponds to the zero-drift case. In addition, long holding period variances appear to converge

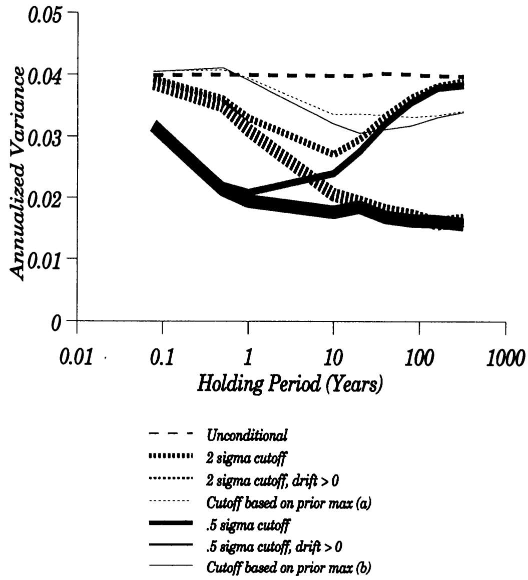

Figure 3. Effect of different survival criteria on annualized holding period variance. Simulation results based on 60,000 replications with annual standard deviation $\sigma = 0.2$ and initial prices 0.5 (*solid lines*), and 2 standard deviations (*dotted lines*) above the reservation price $\underline{p}$. The *dashed line* gives the unconditional variance. We consider 3 cases. In the first we assume zero drift (corresponding to Figure 2). In the second case, we allow drift to be 10 percent per annum, with reservation price constant in real terms (2.3 percent inflation). In the third case, the cutoff is based on price greater than a fixed value below the prior maximum. In case (a) the test excludes as many price paths as the zero-drift case with $p_0 = \underline{p} + 2\sigma$. Case (b) excludes as many price paths as zero drift with $p_0 = \underline{p} + \sigma/2$.

to an asymptote independent of the initial price. This asymptote is the unconditional variance. However, for holding periods less than 100 years, the variance of terminal wealth is significantly below the unconditional variance, implying variance ratios less than one.

In the third experiment, we consider the case where survival depends on the price level being above a certain fixed value relative to the previous maximum. This corresponds to the case where a market crash precipitates the closure of the market. We again consider two cases. In the first case, we consider the fixed value to be set in such a way as to exclude as many price paths as are excluded with zero drift and an initial price half an annual standard deviation above the reservation price. In the second case, we consider a test as stringent, as where before we assume the initial price is two standard deviations above the reservation price. With an expected return of 10 percent, we obtain results intermediate between those of the first and second experiments reported above. The variance of terminal wealth approaches an asymptote independent of how stringent we make the test for survival. The variance of terminal wealth is everywhere below the unconditional variance.

Each of the three experiments provide results consistent with the view that survival implies that the variance of terminal wealth will be less than the unconditional variance. In other words, the variance ratio will be less than one. The variance ratio appears to converge to a value independent of how stringent we make the test. At least part of the apparent mean reversion in long-period asset returns may be an artifact of survival. To determine how big or small this effect might be, we need additional information relating to the structure of these asset markets and the conditions for their survival.

Another important caveat to these findings is that they apply to the variance of terminal wealth for a potentially long holding period. Empirical work typically examines the time sequence of N period returns, where N is strictly less than the number of time-series observations available to the investigator. In Figure 3 each simulated price path starts at the same point, and has an identical a priori probability of survival. When we examine the time sequence of N period returns, the unconditional variance ratio will be greater than $(4 - \pi)/2$, the difference reflecting the variance of the conditional mean return.[11] Simulation evidence (not reported here) confirms that the variance ratio, calculated as in Lo and MacKinlay (1988) with appropriate overlap correction is less than one but greater than $(4 - \pi)/2$. The same simulation experiment shows that other measures of long-term dependence such as the rescaled range (Mandelbrot (1972)) and long-term autocorrelation measures of the kind employed by Fama and French (1988) are also affected by survival considerations, to a greater or lesser extent. In addition, Goetzmann and Jorion (1995) show how survival similarly affects the distribution of the widely used Dickey-Fuller test for unit roots in time series.

III. Postearnings Drift

These results find interesting application in the study of cross-sectional cumulated excess return measures (CARs) that are commonly used in the context of event studies (see Brown and Warner (1980)). Ball and Brown (1968) note an upward drift in cumulated excess returns subsequent to a positive earnings announcement surprise. Subsequent work by Foster (1977) and Foster, Olsen, and Shevlin (1984) among others has documented that this drift is related to size of the firm in question. The current state of this literature is summarized in Ball (1992).

The dynamics of the conditional price path have a direct bearing on these results. Firms that are otherwise in financial distress are more likely to survive on a favorable earnings surprise than an unfavorable earnings sur-

[11] Note that

$$\frac{1}{N}E_0\{r_{\tau+N} - \mu\}^2 = \frac{1}{N}E_0\{r_{\tau+N} - \mu_{\tau+N} \mid p_\tau\}^2 + \frac{1}{N}E_0\{\mu_{\tau+N} \mid p_\tau - \mu\}^2$$

$$\geq \frac{1}{N}E_0\{r_{\tau+N} - \mu_{\tau+N} \mid p_\tau\}^2 = \frac{4 - \pi}{2}\sigma^2 \text{ for large } N.$$

prise. In fact the ex ante probability of survival will be an increasing function of the magnitude of this surprise for any given level of financial distress. In a surviving sample, firms working their way out of financial distress will typically have higher earnings announcements and subsequent returns than will other firms whose ex ante probability of survival does not so nearly depend on favorable announcements.[12]

In other words, firms in financial distress are at-the-money call options, and we would expect the equity to have a higher return than a corresponding all-equity firm (Stapleton (1982)).[13] In fact, the average return of firms that survive the announcement will be inversely related to the extent to which the equity represents an in-the-money call option. This will be true for both the announcement period and the postannouncement period.[14] Provided there is cross-sectional dispersion in the extent to which the equity is in-the-money, there will be an induced cross-sectional relationship between average returns in the announcement and in the postannouncement period. This effect works at the level of the individual security. To the extent that portfolios are formed according to the size of the earnings surprise, this effect will be magnified at the portfolio level (cf. Lo and MacKinlay (1990)).

Event studies typically look at the impact of the earnings announcement on security prices after the announcement has been made, and then correlate this impact with the content of the earnings announcement. These studies typically assume that before the event, the expected change in security prices is zero. Knowing that a quarterly earnings announcement is to be made at some date τ in the future simply adds volatility to the ex ante distribution of stock prices. This increase in volatility reflects the likely magnitude of the earnings announcement, good or bad.

It can be shown (Brown and Ross (1995)) that we can write the security price process as

$$
\begin{aligned}
dp &= \frac{\sigma^2}{p - \underline{p}}\, dt + \sigma\, dz, \quad \text{for } \tau \notin \{\alpha l \mid l = 1, \ldots\} \\
&= \frac{\sigma^2}{p - \underline{p}}\, dt + \sigma\, dz + \delta_\tau(p), \quad \text{for } \tau \in \{\alpha l \mid l = 1, \ldots\},
\end{aligned}
$$

[12] Note that this skewness argument derived from the option-like characteristics of firms in financial distress (c.f. Ball, Kothari, and Shanken (1995)) does not require that any firms actually fail ex post in a finite sample. Therefore, the argument does not really depend on the inclusiveness (or lack thereof) of COMPUSTAT (Chan, Jegadeesh, and Lakonishok (1994)).

[13] This assumes that the equity in the corresponding all-equity firm has an expected return greater than the risk-free rate (Cox and Rubinstein (1985), p. 189).

[14] Brown and Pope (1995) find that skewness in size-adjusted returns is indeed associated with the postearnings drift phenomenon, and that skewness persists from the announcement to the postannouncement period.

where the event-related change in price $\delta_\tau(p)$ is a random variable capturing the event-related change in price and $\{\alpha l \mid l = 1, \ldots\}$ represents the set of earnings dates separated by a time of α (three months).

We can use this result to determine the relevant distribution of price changes conditional on earnings announcements. Brown and Ross (1995) show that an increase in Δp_τ implies a first-order stochastic increase in $\Delta p_{\tau'}$ for $\tau < \tau'$ in the surviving sample of returns. This result is consistent with empirical results found by Bernard and Thomas (1989, 1990) among others.

IV. Stock Splits

We have chosen to condition upon the price exceeding the reservation price $\underline{p}$; it may be of interest to condition upon other features of the price path. For instance, Ross (1987) derives the conditional diffusion for the case where the conditioning set A is defined as the set of all price paths on $[0, T]$ that achieve a maximum at point t^*, $0 < t^* < T$. Suppose the empirical investigator observes that the price attains a maximum of m at point t^*, and considers the time series of data up to that point. Ross derives a closed form for the diffusion prior to t^* and shows that the mean Brownian path is increasing at an increasing rate up to t^*. The implication is that the perils of data snooping extend even to preliminary curiosity!

Another interesting example is the case of stock splits studied by Fama *et al.* (1969). Very rarely does a positive stock split come upon a *decrease* in security prices. If we consider the conditioning set A as the set of all price paths where the price on the event date is greater than the average price measured over the prior period, the cumulative average return statistic measured as the cross-section average of excess returns is in fact the mean Brownian path introduced earlier.

If we assume as before that we observe a time series of prices p_t which is generated on $(0, T)$ according to

$$dp = \mu\, dt + \sigma\, dz$$

we only observe a positive stock split on date T when the current price is at least equal to the geometric average of past prices. For observations prior to T, the conditioning set is then

$$A = \left\{ \text{price path such that } p_T > \frac{\sum_{t=0}^{T} p_t}{T} \right\}$$

we sample the price path on infinitesimal increments, the summation is approximated by the integrated Brownian process

$$w(T) = \int_0 p_t\, dt$$

and we can represent the conditioning set as

$$A = \left\{ \text{paths such that } p(t) = p, p(T) > \frac{w(T)}{T} \right\}$$

It is a well-known result (e.g., Ross (1983), (p. 196) and $p(T)$ and $w(T)$ are bivariate Normal conditional on $p(t)$ and $w(t)$. The quantity

$$e(T) = p(T) - \frac{w(T)}{T}$$

has a conditional Normal distribution with mean

$$E[e(T) \mid p(t), w(t)] = \frac{tp(t) - w(t)}{T} + \mu \frac{T^2 - t^2}{2T} \equiv m(t, p, w)$$

and standard deviation

$$\sqrt{\operatorname{Var}[e(T) \mid p(t), w(t)]} = \sigma \sqrt{\frac{t^3 - T^3}{2T}} \equiv s(t).$$

Hence the marginal probability of set A is given as

$$\pi(p, t) = 1 - \Phi[z],$$
$$z = \frac{-m(t, p, w)}{s(t)}$$

and the mean of the conditional diffusion is given as

$$\begin{aligned} \mu^* &= \mu + \sigma^2 \frac{\pi_p}{\pi} \\ &= \mu + \sigma^2 \frac{t\varphi[z]}{Ts(t)(1 - \Phi[z])}. \end{aligned}$$

The mean Brownian path given as the solution to the differential equations

$$\begin{aligned} \frac{dp}{dt} &= \mu + \sigma^2 \frac{t\varphi[z]}{Ts(t)(1 - \Phi[z])}, \quad t < T, \\ &= \mu, \quad t \geq T, \\ \frac{dw}{dt} &= p(t). \end{aligned}$$

with initial conditions $p(0) = 0$ and $w(0) = 0$ is particularly interesting, as it represents the cumulative average return statistic (where excess returns are cross sectionally uncorrelated—as, for example, where events occur at different points in calendar time). For market model R^2 of 0.15, market return mean of 0.10, and standard deviation of 0.20 (corresponding to Ibbotson (1991)), the mean Brownian path corresponds closely to that reported in Fama *et al.* (1969) for $T = 2$ Years, as shown in Figure 4.

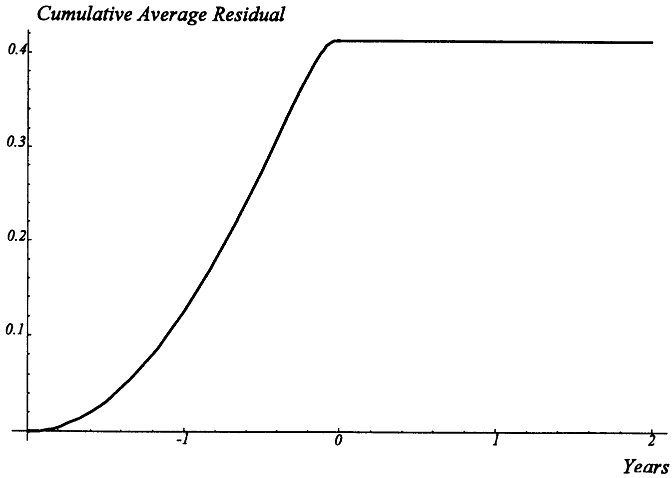

Figure 4. Mean Brownian path for stock split example, for market model R^2 of 0.15, market return mean of 0.10 and standard deviation of 0.20.

We should be careful how we interpret this figure. The fact that the cumulative performance measure is positive on the event date is not an artifact of survival. After all, the experimental design conditions on the price at zero being greater than the average prior price, which is ex post good news. Rather, we should be careful how we interpret the apparent price pattern prior to the event. This point is actually made in the Fama *et al.* (1969) article, and is also explicit in the work of Mandelkar (1974) who writes in reference to the apparent price run-up prior to merger announcements that the decision to announce a merger at a particular time may not be independent of the pattern of price changes in the period prior to the announcement. Suppose that bad news in the days prior to a planned announcement is sufficient to cause a delay of several days in announcing the acquisition. The present analysis demonstrates that the absence of material bad news prior to the announcement may explain part of the apparent run-up of prices immediately prior to a merger announcement.

VI. Conclusion

Survival is an issue to some extent whenever a researcher chooses to use historical data. We have provided some analysis of the consequences of survival for studies of temporal dependency in long-term stock market returns, event studies, and other applications of empirical finance. The importance of these results is not limited to economics. Mandelbrot and Wallis (1969) identify very long-term dependencies in geophysical records such as river levels and tree rings using the Hurst statistic. They conclude that "the span of statistical interdependence of geophysical data is infinite." They failed, however, to take into account the survival bias in their data.[15] This oversight leads to a tempting conjecture. Given that a series is subject to some form of survival bias, does the probability of false rejection of temporal independence approach one as the period of survival grows to infinity?

These results are not intended to discourage the analysis of historical data. Rather they are intended to describe what researchers should expect to find due to survival alone. Our hope is that this analysis and further extensions of it will help disentangle survival effects from meaningful economic phenomena. Given that all econometricians are, to some extent, prisoners of history, perhaps we should seek to further our understanding of its constraints.

[15] Survival bias in this case may be due to disappearances of lakes and rivers, to changing climatic conditions leading to the disappearance of forests, or it could be due to the disappearance of the human observer, and/or the erasure of the geological record.

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