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Jennifer Conrad; Gautam Kaul

The Journal of Business, Vol. 61, No. 4. (Oct., 1988), pp. 409-425.

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Sat Sep 9 15:28:21 2006

Jennifer Conrad
University of North Carolina

Gautam Kaul
University of Michigan

Time-Variation in Expected Returns*

This article characterizes the stochastic behavior of expected returns on common stock. We assume market efficiency and postulate an autoregressive process for conditional expected returns. We use weekly returns of 10 size-based portfolios over the 1962–85 period and find that (1) the variation through time in expected returns is well characterized by a stationary first-order autoregressive process; (2) the extracted expected returns explain a substantial proportion (up to 26%) of the variance in realized returns, and the magnitude of this proportion has a monotonic (inverse) relation with size; (3) the degree of variation in expected returns also changes systematically over time; and (4) the forecasts subsume the information in other potential predictor variables.

I. Introduction

Theoretical models of asset pricing put few, if any, restrictions on the behavior of expected returns over time. However, in the implementation of most tests of market efficiency and/or a particular market equilibrium model, expected returns are assumed to remain constant over some period of time. Given market efficiency, such an assumption is unrealistic in light of recent evidence that security returns can (to some extent) be predicted (see, e.g., Fama and Schwert 1977; Fama 1981; Keim and Stambaugh 1986; Fama and French 1987, 1988; and Kaul 1987).

In this article, we attempt to characterize the stochastic nature of expected returns. Specifically, we assume market efficiency and test whether expected returns are constant relative to a particular alternative hypothesis. Based on the findings of recent empirical papers, we

* We are grateful to Victor Bernard, Michael Bradley, Mark Flannery, Mustafa Gultekin, Stanley Kon, Albert Madansky, Gregory Niehaus, Jay Ritter, Sarabjeet Seth, Nejat Seyhun, and especially Eugene Fama for helpful comments and suggestions. We would like to thank the finance workshop participants at the University of Michigan and the University of North Carolina. We also appreciate the invaluable research assistance of M. Nimalendran, the secretarial assistance of Dianne Haft and Patti Lamparter, and the editorial suggestions of Kristin Kaul. Partial financial support for this article was provided by the School of Business Administration, University of Michigan.

(*Journal of Business*, 1988, vol. 61, no. 4)
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0021-9398/88/6104-0003$01.50

hypothesize that expected returns follow an autoregressive process. We use weekly returns of 10 size-based portfolios over the 1962–85 period and attempt to eliminate market microstructure biases through careful sample selection. We use a Kalman filter technique proposed by Ansley (1980) to extract expected returns and find that constancy is strongly rejected for all portfolios. Movements in expected returns are well characterized by a parsimonious (stationary) first-order autoregressive model.

The most striking aspect of our results is that the variation through time in short-horizon expected returns is a relatively large fraction of return variances. Moreover, there is a monotonic relation between the size rankings of the portfolios and the relative time-variation in expected returns: variation in expected returns explains 26% of return variance for the smallest portfolio, and this proportion drops systematically to 1% for the largest portfolio. The significant time-variation, and its relation to size, is found during each of the 5-year subperiods, even when we separately allow for a January dummy. There is also strong evidence of systematic changes in the relative variation in expected returns across subperiods.

Finally, to gauge the economic content of the forecasts, we test whether they are informative with respect to other relevant ex ante information. Among others, we consider two predetermined variables—the treasury bill rate and the lagged return on an equal-weighted market portfolio. Although forecasts based on the autoregressive model rely solely on each portfolio's past returns, they subsume the information in such variables.

Section II describes the model; the empirical analysis is presented in Section III. Section IV contains a brief summary.

II. The Model

The choice of a particular model for the time-varying behavior of expected returns is, by nature, somewhat arbitrary. Ideally, an equilibrium model should specify both the stochastic process for, and the underlying economic determinants of, expected returns. However, existing asset-pricing theories do not specify any particular a priori restrictions on the variation through time in expected returns. In fact, more recent models of market equilibrium (e.g., Merton 1973; Lucas 1978; Breeden 1979; Cox, Ingersoll, and Ross 1985) do not rule out even negative expected returns.

Under these circumstances, it becomes essential to empirically characterize the stochastic nature of expected returns. Such an exercise will help us understand the behavior of expected returns over time and across different assets and perhaps also help identify the economic determinants of security returns. The basic approach in this article is one of stochastic-parameter estimation, which permits both the identification and estimation of the expected return process.[1]

We model movements in expected returns as a first-order autoregressive process. The choice of an autoregressive process is based largely on recent empirical evidence of the predictability of stock returns. A majority of the predetermined variables that have significant correlations with realized stock returns are themselves characterized by highly autocorrelated behavior. For example, all three of the (ex ante) predictive variables used by Keim and Stambaugh (1986) exhibit large and positive first-order autocorrelations, while higher order autocorrelations decay across longer lags. Similarly, Fama and French (1987) use dividend yield to forecast stock returns, and this variable also exhibits a similar autocorrelation structure (see also Campbell 1987; and Ferson, Kandel, and Stambaugh 1987).

Since movements in expected returns presumably reflect variation through time in such forecast variables, they themselves may be characterized by an autoregressive process. We, therefore, write our model as

$$R_t = E_{t-1}(R_t) + \epsilon_t, \tag{1}$$

and

$$E_{t-1}(R_t) = \phi E_{t-2}(R_{t-1}) + u_{t-1}, \tag{2}$$

where

R_t = realized return on a particular security over period $t - 1$ to t;
$E_{t-j}(R_{t-j+1})$ = expected return for a security over period $t - j$ to $t - j + 1$ as of period $t - j$;
$\epsilon_t \sim$ independently and identically distributed (i.i.d.) $N(0, \sigma_\epsilon^2)$;
$u_t \sim$ i.i.d. $N(0, \sigma_u^2)$; and
$\phi \leq 1$.

We use a Kalman filter technique proposed by Ansley (1980) to extract the expected returns. Before discussing the estimation procedure, we present a simple representation of the model.

If expected returns are represented by the process in equation (2), we can write realized returns as

$$R_t = \phi R_{t-1} + \epsilon_t - \phi\epsilon_{t-1} + u_{t-1}, \tag{3}$$

1. Ohlson and Rosenberg (1982) employ such an approach to estimate the stochastic behavior of the systematic risk of the equal-weighted common stock index.

which, in turn, implies that realized returns can be characterized by an ARMA (1,1) process of the form

$$R_t = \phi R_{t-1} + a_t - \theta a_{t-1}. \tag{4}$$

In the inverted form we can then write the conditional expected return as

$$\begin{aligned} E_{t-1}(R_t) = (\phi - \theta)R_{t-1} + \theta(\phi - \theta)R^2_{t-2} \\ + \theta(\phi - \theta)R_{t-3} + \dots . \end{aligned} \tag{5}$$

Invertibility of the process requires that the sum $\Sigma_{i=0}^{\infty} \theta^i(\phi - \theta)$ converge and, hence, that $|\theta| < 1$.

In the pure, moving average form, we can write

$$\begin{aligned} E_{t-1}(R_t) = (\phi - \theta)a_{t-1} + \phi(\phi - \theta)a_{t-2} \\ + \phi^2(\phi - \theta)a_{t-3} + \dots . \end{aligned} \tag{6}$$

For the process to be stationary, the sum of weights, $\Sigma_{i=0}^{\infty} \phi^i(\phi - \theta)$, must converge, and hence $|\phi| < 1$ (see Box and Jenkins 1970).

Therefore, conditional expected returns are an exponentially weighted sum of past returns, where the weights add up to less than 1.0 (eq. [5]). Conversely, conditional expected returns can be expressed as a weighted sum of all past shocks, where the weights given to past shocks decline exponentially (eq. [6]). In other words, if expected returns follow a stationary process, a shock at $t - 1$, a_{t-1}, has a progressively smaller effect on future expected returns.[2]

A special case of this model is one in which the autoregressive parameter, ϕ, is constrained to be equal to 1.0. Under this specification, expected returns follow a (nonstationary) random walk process, which leads to realized returns following a random walk-plus-noise process. This, in turn, implies that a shock at $t - 1$, a_{t-1}, has two parts, a permanent and a temporary one. We can interpret $(1 - \theta)a_{t-1}$ as the permanent contribution of a shock to realized returns in the sense that it affects *all* future expected returns by this amount. Correspondingly, θa_{t-1} can be viewed as the temporary contribution. We estimate both the constrained and unconstrained versions of the model.

2. Rosenberg (1973) first developed such a convergent parameter model, as opposed to the random-walk model, in which expected returns have no tendency to converge. Using his notation, we can rewrite our model in the following form:

$$R_t = E_{t-1}(R_t) + \epsilon_t,$$

$$E_{t-1}(R_t) = (1 - \delta)\bar{R} + \delta E_{t-2}(R_{t-1}) + u_{t-1},$$

where δ is the convergence parameter, and $\bar{R}$ is the population "norm" toward which the expected return process converges. We can then rewrite realized returns as

$$R_t = (1 - \delta)\bar{R} + \delta R_{t-1} + \epsilon_t - \delta\epsilon_{t-1} + u_{t-1},$$

which is an ARMA (1,1) process.

III. The Evidence

A. Data Description

We use the Center for Research in Security Prices (CRSP) daily master file to calculate weekly portfolio returns. The choice of a weekly sampling interval is largely a compromise between the relatively few monthly observations and the potential biases associated with nontrading, the bid-ask effect, and so on, in daily data. We use portfolio returns, rather than individual security returns, because it is much more difficult to extract the (expected return) signal from noisy weekly returns of a single security (see Lo and MacKinlay 1987).

At the end of each year, stocks are sorted into 10 portfolios based on market value (number of shares outstanding times price per share). For each week (Wednesday close–Wednesday close) of the following year, 1-week simple returns are calculated for securities that actually *traded* on both Wednesdays. For the July 1962–December 1962 period we form portfolios based on market values as of December 1962. Weekly holding-period security returns in each portfolio are equally weighted to form 10 series of portfolio returns, which are then continuously compounded. We also construct an equal-weighted "market portfolio" return using all our sample securities. Hence, we have a total of 1,226 weekly, continuously compounded returns of 10 size-based portfolios and one market portfolio from July 1962 to December 1985.

B. Autocorrelations

1. *The Weekly Evidence.* Table 1 shows the summary statistics for the weekly portfolio returns for the 1962–85 period. The first-order autocorrelations are large and significant, and the higher-order autocorrelations (though significant) decay across longer lags. The returns on the equal-weighted market portfolio (EWMR) exhibit similar persistence in autocorrelations. Fama (1965) and Lo and MacKinlay (1987) also find positive autocorrelations in short-horizon returns. (We replicate all our results using excess returns, i.e., returns in excess of the weekly risk-free return. The results are virtually identical.)

The autocorrelation structure displays a consistent pattern as we go from the smallest portfolio ($R1$) to the largest ($R10$): the magnitude and persistence of the autocorrelations decline monotonically. However, higher-order autocorrelations remain significant for all but the largest portfolio (which exhibits only first-order autocorrelation).

Finally, the first-order autocorrelations of weekly *changes* in returns, ΔR, are all significantly negative, while higher-order autocorrelations are close to zero. This behavior of sample autocorrelations is consistent with slowly moving expected returns.

2. *Market Microstructure Biases.* The positive autocorrelations in portfolio returns may also be consistent with the presence of market

TABLE 1 **Summary Statistics of Weekly Returns of 10 Equal-weighted Portfolios of New York and American Stock Exchange Common Stocks, Formed by Decile Rankings of Market Value of Equity Outstanding at the End of the Previous Year, 1962–85 (1,226 weeks)**

Variable (x)	$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$	$\hat{\rho}_6$	$\bar{x}$*	$s(x)$*
$R1$	.41	.24	.16	.10	.01	.04	.7231	3.158
$R2$	.35	.20	.14	.10	.01	.04	.4650	2.736
$R3$	.31	.17	.12	.08	.00	.03	.3949	2.638
$R4$	.30	.15	.10	.05	.00	.01	.3515	2.528
$R5$	.29	.13	.08	.05	.00	.00	.2896	2.481
$R6$	.27	.12	.07	.04	−.01	−.00	.2921	2.395
$R7$	.24	.11	.06	.04	−.00	−.01	.2747	2.256
$R8$	.22	.09	.07	.03	−.00	−.01	.2466	2.153
$R9$	.18	.07	.07	.02	−.02	−.03	.2370	2.076
$R10$	.09	−.00	.04	.00	−.02	−.03	.1919	1.978
EWMR†	.28	.14	.10	.06	−.00	.01	.3358	2.295
$\Delta R1$‡	−.35	−.08	−.01	.02	−.10	.07	.0031	3.435
$\Delta R2$	−.38	−.07	−.01	.04	−.10	.07	−.0009	3.110
$\Delta R3$	−.40	−.07	−.01	.03	−.08	.06	−.0026	3.098
$\Delta R4$	−.39	−.07	−.00	−.00	−.04	.02	−.0012	2.996
$\Delta R5$	−.39	−.07	−.01	.01	−.04	.00	−.0025	2.958
$\Delta R6$	−.40	−.07	−.01	.01	−.04	−.00	−.0019	2.893
$\Delta R7$	−.41	−.06	−.01	.01	−.02	−.01	−.0016	2.777
$\Delta R8$	−.42	−.06	.01	−.00	−.01	−.02	−.0007	2.692
$\Delta R9$	−.43	−.07	.03	−.00	−.02	−.03	−.0007	2.658
$\Delta R10$	−.45	−.08	.05	−.01	−.00	−.05	−.0002	2.673
ΔEWMR	−.40	−.07	.00	.01	−.05	.01	−.0009	2.758

NOTE.—$R1$–$R10$ are the continuously compounded weekly returns on 10 size-based portfolios in ascending order from smallest to largest. Values $\bar{x}$ and $s(x)$ are the sample mean and standard deviation of the variable, and $\hat{\rho}_t$ is the sample autocorrelation at lag t. Under the hypothesis that the true autocorrelations are zero, SEs of the estimated autocorrelations are about .03. $R1$ is the smallest portfolio; $R10$ the largest.

* The returns are rates of return per week in decimal fraction units $\times 10^2$.

† EWMR is the equal-weighted market portfolio return.

‡ The operator Δ denotes first differences.

microstructure biases caused by infrequent or nonsynchronous trading and/or bid-ask effects (see, e.g., Fisher 1966; Scholes and Williams 1977; and Cohen et al. 1983).

The basic intuition for positive autocorrelation in portfolio returns is as follows: an infrequently traded (or small firm) security's observed return at time t may contain a component of the price adjustment to news released at $t - 1$, which will tend to be positively related to the observed returns of frequently traded (large firm) securities at $t - 1$. In other words, good (bad) news is incorporated immediately in frequently traded firms' returns and with a lag in the returns of infrequently traded firms, thus inducing artificial serial correlation in the returns on a *portfolio* of securities.

However, our sampling procedure minimizes the possibility of such biases, and the evidence indicates that the autocorrelation structure of returns is unlikely to have been caused by infrequent trading in small-firm stocks. First, we use weekly portfolio returns and exclude all securities that do not trade on the adjacent Wednesdays of a particular week. Hence, any remaining nonsynchronous trading bias would be restricted to differences in trading intervals of less than one day, and, relative to weekly returns, such a bias should be small. Moreover, we construct weekly portfolio returns by first calculating weekly security returns and then forming equally weighted portfolios. This method minimizes the bias in some earlier studies that use arithmetic averages of returns within the review period. See Blume and Stambaugh (1983) and Roll (1983) for a discussion of this issue.

Second, the magnitude of the first-order autocorrelations (see table 1) is unlikely to have been caused by microstructure biases. Lo and MacKinlay (1987) model the nontrading phenomenon as a binomial process and show that even if (on average) *50%* of stocks on the New York and American Stock Exchanges do not trade each day, the theoretical first-order weekly autocorrelation of portfolio returns would be about 17%. Furthermore, Working (1960) shows that averaging temporally ordered data can induce first-order autocorrelation in the average/index, but the magnitude of this autocorrelation approaches a maximum of 25% as the number of items in the average/index approaches infinity. Our evidence indicates that most of the portfolio returns have first-order serial correlations greater than such extreme theoretical values. Hence, the high first-order autocorrelations are not peculiar to just infrequently traded (small) firms.

Third, table 1 also shows persistence in the serial correlation at longer lags for a majority of the portfolios. Such a pattern in *weekly* returns, which should exist if our model for expected returns is a robust representation of the true process, is again unlikely to have been caused by market microstructure biases. Finally, our analysis (see Sec. III) of the properties of the extracted expected returns suggests that such biases are unlikely causes of the observed autocorrelation structure in weekly returns.

C. The Model Estimates

We use the Kalman filter and a Marquardt maximum-likelihood procedure to estimate the parameters of our model. The first-order autoregressive parameter, ϕ, is allowed to vary across portfolios. The results are reported in table 2.

A stationary autoregressive process for expected returns appears to be well specified. The estimates of ϕ are significantly different from both 0.0 and 1.0 for all portfolios, and the residuals behave like white noise. Moreover, there is a monotonic relation between the estimates of ϕ and size: the magnitude of ϕ declines systematically from the smallest- to the largest-sized portfolios. We also estimate the model with a dummy variable for the first week of January; the results are

TABLE 2 **Weekly Estimates of the Parameters of the Model in Which Expected Returns Follow a Stationary AR(1) Process**

$$R_t = E_{t-1}(R_t) + \epsilon_t, \quad (1)$$

$$E_{t-1}(R_t) = \phi E_{t-2}(R_{t-1}) + u_{t-1}, \quad (2)$$

where $\phi \leq 1$

	Overall Period: July 1962–December 1985 (N = 1,226)						
	Without January Dummy			With January Dummy (D)			
Portfolio	$\hat{\phi}$*	$s(\epsilon)$†	$\hat{\rho}_1$‡	$\hat{\phi}$	$\hat{\delta}$§	$s(\epsilon)$	$\hat{\rho}_1$
1	.589	.02872	−.001	.649	.068	.02707	.004
	(.054)			(.037)	(.005)		
2	.584	.02549	−.002	.617	.047	.02462	.003
	(.061)			(.046)	(.005)		
3	.555	.02501	−.003	.601	.035	.02452	.003
	(.071)			(.056)	(.005)		
4	.489	.02409	−.004	.566	.029	.02376	.004
	(.080)			(.064)	(.005)		
5	.430	.02373	−.006	.523	.023	.02352	.002
	(.086)			(.072)	(.005)		
6	.403	.02304	−.006	.498	.017	.02292	.001
	(.094)			(.081)	(.005)		
7	.394	.02187	−.006	.489	.014	.02179	−.000
	(.104)			(.091)	(.004)		
8	.217	.02101	−.010	.463	.010	.02095	−.001
	(.028)			(.105)	(.004)		
9	.180	.02042	−.007	.409	.007	.02039	−.003
	(.028)			(.132)	(.004)		
10	.087	.01971	.001	.086	.001	.01971	.001
	(.028)			(.028)	(.004)		

NOTE.—Portfolio size ranges from smallest size (1) to largest size (10). Numbers in parentheses below estimated coefficients are SEs. R_t = continuously compounded realized return for week t; $E_{t-j}(R_{t-j+1})$ = expected return over week $t-j$ to $t-j+1$ as of week $t-j$; $\epsilon_t \sim N(0, \sigma_\epsilon^2)$; $u_t \sim N(0, \sigma_u^2)$.

* $\hat{\phi}$ = the estimated autoregressive parameter in eq. (2).

† $s(\epsilon)$ = residual standard error.

‡ $\hat{\rho}_1$ = residual autocorrelation at lag 1. Under the hypothesis that the true autocorrelations are zero, the SEs of the residual autocorrelations are about .03.

§ $\hat{\delta}$ = estimated coefficient on the dummy variable (D), where D = 1 for first week in January, and $D = 0 \ \forall$ other weeks.

shown in table 2. The dummy variable is significant for the smaller eight market-value portfolios, which indicates that seasonality is not just a small-firm phenomenon. Moreover, the estimates of ϕ maintain their systematic relation with size.

Evidence of a stationary expected-return process is consistent with the results of Fama and French (1987), who find positive autocorrelations in expected returns, which are documented in their regressions of long-horizon returns on dividend/price ratios. The pattern of the coefficients in these regressions for different holding periods is suggestive of a mean-reverting expected-return process. However, the time-variation in short-horizon expected returns is quite different from the slowly decaying variation in long-horizon expected returns found by Fama and French. Specifically, our estimates of ϕ (which are always less than 0.65) indicate rapidly decaying time-variation in expected returns: the effects of an expected-return shock are largely dissipated after a month.

The conclusion that expected returns follow a stationary autoregressive process is supported when we estimate the model in which ϕ is restricted to be equal to 1.0 (or, equivalently, expected returns follow a random walk). The results, not reported, show strong evidence of movements in expected returns across all portfolios and a systematic relation between the magnitude of such variation and the size rankings of the portfolios. However, the constrained specification is clearly misspecified. First, the residuals from the model exhibit significant first-order autocorrelations for all portfolios. Second, although the extracted expected returns from the constrained model contain significant information about subsequent realized returns, they have no marginal explanatory power in regressions of realized returns on forecasts obtained from both the constrained and unconstrained models.

Brown, Kleidon, and Marsh (1983), hereafter BKM, also use a Kalman filter technique to analyze the variation through time in the size effect. Specifically, they test for nonstationarity in the *excess* returns of 10 size-based portfolios, where excess returns are defined as deviations from returns predicted by the Sharpe-Lintner version of the capital asset-pricing model (CAPM). Their results indicate that the assumption of constancy of excess returns is most seriously violated for the smallest and largest portfolios.

Our objective is quite different from the BKM study since we are concerned more generally in the stochastic process for time-varying expected returns. Unlike BKM, our approach has the advantage of not imposing, and therefore not requiring the estimation of, any particular asset-pricing model with its attendant assumptions. We only require the assumption of market efficiency, which enables us to extract expected returns by a linear projection of realized returns on *past* information (in particular, past portfolio returns). Consequently, we do not entertain the notion of excess returns.

D. *Statistical Properties of Weekly Expected-Return Forecasts*

Table 3 shows regressions of weekly portfolio returns, R_t, on the extracted expected returns (with the January dummy) from the stationary autoregressive model, ERAR_{t-1}. The criteria for a good extracted expected-return series are: (1) conditional unbiasedness, that is, an intercept close to zero and a slope coefficient close to 1.0, and (2) serially uncorrelated residuals. The extracted expected returns are condition-

TABLE 3 Estimates of Regressions of Weekly Realized Portfolio Returns on Extracted Expected Returns, July 1962–December 1985

$$R_t = \alpha + \beta \mathrm{ERAR}_{t-1} + \eta_t$$

Portfolio	$\hat{\alpha}$	$\hat{\beta}$	$\bar{R}^2$*	$\hat{\rho}_1$†
1	−.000003	1.000	.265	.004
	(.00088)	(.086)		
2	−.000001	.998	.191	.004
	(.00083)	(.099)		
3	.000009	.994	.136	.005
	(.00084)	(.112)		
4	.000001	.997	.117	.005
	(.00084)	(.116)		
5	.000005	.995	.101	.003
	(.00080)	(.115)		
6	.000004	.995	.084	.002
	(.00081)	(.125)		
7	.000007	.994	.067	.001
	(.00081)	(.140)		
8	.000004	.995	.053	.000
	(.00077)	(.152)		
9	.000010	.992	.035	−.002
	(.00078)	(.184)		
10	−.000000	1.000	.007	.001
	(.00104)	(.421)		

NOTE.—The numbers in parentheses below the estimated regression coefficients are SEs based on White's (1980) consistent heteroskedasticity correction. Portfolios are listed from smallest size (1) to largest size (10).

R_t = continuously compounded realized return for week t; ERAR_{t-1} = expected return for week t as of week $t-1$ extracted from the model, in which expected returns follow a (stationary) AR(1) process (with January dummy); η_t = random disturbance term.

* $\bar{R}^2$ = (adjusted) coefficient of determination.

† $\hat{\rho}_1$ = residual autocorrelation at lag 1. Under the hypothesis that the true autocorrelations are zero, the SEs of the residual autocorrelations are about .03.

ally unbiased for all portfolios: the slope coefficients are all close to 1.0, while the intercepts are close to zero.[3] Moreover, the regression residuals behave like white noise.

The most striking aspect of the regressions in table 2 is the large proportion of variance of short-horizon returns explained by variation through time in expected returns. The (adjusted) R^2 values of up to 26% are much larger than those typically found in tests that use monthly data. Moreover, there is a monotonic relation between the

3. The heteroskedasticity test of White (1980) produces chi-square statistics well above conventional significance levels for most portfolio regressions. We therefore report the more conservative standard errors based on the heteroskedasticity-consistent method. We also have measurement errors in the extracted expected returns, ERAR_{t-1}, since they are estimates. However, most methods for the computation of corrected standard errors (for the errors-in-variables problem) assume homoskedastic errors (see, e.g., Murphy and Topel 1985). Since heteroskedasticity is potentially a more serious problem in stock return regressions, we choose to report the already conservative heteroskedasticity-consistent standard errors.

TABLE 4 **Estimates of Variances of Realized Returns, Expected Returns, and Unexpected Returns on 10 Equal-weighted Portfolios**

Portfolio	R_t*	$ERAR_{t-1}$	$URAR_t$
1	.099751	.026500	.073291
2	.074866	.014374	.060592
3	.069596	.009615	.060136
4	.063921	.007562	.056435
5	.061530	.006300	.055320
6	.057340	.004887	.052519
7	.050903	.003498	.047464
8	.046336	.002502	.043871
9	.043099	.001574	.041560
10	.039138	.000298	.038847

NOTE.—R_t = continuously compounded realized return for week t; $ERAR_{t-1}$ = estimated expected return for week t as of week $t - 1$ extracted from the model in which expected returns follow a (stationary) AR(1) process (with January dummy); $URAR_t$ = unexpected return.

* The estimated variances are reported in decimal fraction units $\times 10^2$.

size rankings of the portfolios and the relative time-variation in expected returns: variation in expected returns explains 26% of return variance for the smallest portfolio, and this proportion drops systematically to about 1% for the largest portfolio. Hence, not only is the variance of *realized* returns for small firms systematically higher than for large firms (see table 1), but the relative variance of *expected* returns is higher as well. This, in turn, enables the signal extraction technique to detect the significant variation in the expected returns of (especially) the smaller firms.

The variability of movements in expected returns across portfolios can perhaps be better gauged by comparing the *absolute* variances of the expected-return components of different portfolio returns. In table 4 we present the estimated variances of realized, expected, and unexpected returns for all 10 portfolios. The absolute variation in the expected returns of the smallest portfolio is about 90 times the corresponding variation of the largest portfolio. Moreover, the estimated variance of expected returns declines systematically as the size of the portfolio increases.

Finally, to analyze the extent of variation in expected returns over different time periods, we reestimate the model over five subperiods (results not reported). The significant aspect of the results is the evidence of systematic changes in the relative variation in expected returns over time, across portfolios. Typically, the proportion of variance in realized returns explained by the extracted forecasts during the seventies is about twice the proportion explained in the sixties and eighties. This result can serve as a useful guide in identifying the important economic determinants of security returns.

TABLE 5 **Estimates of Regressions of Weekly Realized Portfolio Returns on Predetermined Variables and Extracted Expected Returns, July 1962–December 1985**

$$R_t = \alpha + \beta_1 \text{RF}_{t-1} + \beta_2 \text{EWMR}_{t-1} + \beta_3 \text{ERAR}_{t-1} + \eta_t$$

Portfolio	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\bar{R}^{2*}$	$\hat{\rho}_1$†
1:						
(i)	.0107	−3.263	...	...	.002	.41
	(.0018)	(1.478)				
(ii)	.0054	...	.550	...	.159	.02
	(.0008)		(.052)			
(iii)	.0012	−.666	.166	.849	.273	−.06
	(.0016)	(1.309)	(.059)	(.108)		
2:						
(i)	.0086	−3.708	...	...	.004	.35
	(.0016)	(1.319)				
(ii)	.0032	...	.434	...	.132	−.02
	(.0008)		(.045)			
(iii)	.0019	−1.382	.121	.825	.195	−.05
	(.0015)	(1.222)	(.060)	(.139)		
3:						
(i)	.0075	−3.335	...	...	.003	.31
	(.0015)	(1.298)				
(ii)	.0026	...	.378	...	.108	−.04
	(.0007)		(.044)			
(iii)	.0020	−1.371	.119	.769	.140	−.04
	(.0016)	(1.234)	(.067)	(.176)		
4:						
(i)	.0071	−3.362	...	...	.004	.29
	(.0015)	(1.294)				
(ii)	.0023	...	.342	...	.096	−.03
	(.0007)		(.042)			
(iii)	.0021	−1.553	.097	.775	.119	−.02
	(.0016)	(1.250)	(.066)	(.190)		
5:						
(i)	.0062	−3.148	...	...	.003	.28
	(.0014)	(1.287)				
(ii)	.0018	...	.310	...	.082	−.01
	(.0007)		(.041)			
(iii)	.0019	−1.551	.040	.883	.101	−.00
	(.0015)	(1.257)	(.075)	(.217)		
6:						
(i)	.0063	−3.178	...	...	.004	.26
	(.0014)	(1.270)				
(ii)	.0020	...	.270	...	.067	.00
	(.0007)		(.039)			
(iii)	.0017	−1.640	−.027	1.063	.084	.01
	(.0015)	(1.247)	(.085)	(.272)		
7:						
(i)	.0056	−2.631	...	...	.003	.24
	(.0013)	(1.192)				
(ii)	.0019	...	.231	...	.055	.00
	(.0006)		(.037)			
(iii)	.0015	−1.440	−.007	1.005	.067	.01
	(.0014)	(1.164)	(.075)	(.287)		
8:						
(i)	.0050	−2.370	...	...	.002	.21
	(.0013)	(1.150)				

TABLE 5 (*Continued*)

Portfolio	$\hat{\alpha}$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\bar{R}^2$*	$\hat{\rho}_1$†
(ii)	.0018	...	.191	...	.041	.01
	(.0006)		(.035)			
(iii)	.0010	−1.331	−.060	1.235	.053	.01
	(.0014)	(1.131)	(.082)	(.360)		
9:						
(i)	.0046	−2.086	...	...	.002	.18
	(.0012)	(1.122)				
(ii)	.0019	...	.143	...	.024	.03
	(.0006)		(.033)			
(iii)	.0005	−1.291	−.098	1.498	.036	.01
	(.0014)	(1.107)	(.074)	(.412)		
10:						
(i)	.0039	−1.836	...	...	.001	.08
	(.0012)	(1.131)				
(ii)	.0017	...	.067	...	.005	.02
	(.0005)		(.031)			
(iii)	.0018	−1.583	.002	.947	.007	.00
	(.0018)	(1.123)	(.060)	(.825)		

NOTE.—The numbers in parentheses below the estimated regression coefficients are SEs based on White's (1980) consistent heteroskedasticity correction. Portfolios are listed from smallest size (1) to largest size (10). R_t = continuously compounded realized return for week t; RF_{t-1} = the risk-free rate calculated as the continuously compounded return on a 1-week treasury bill for week t, known at week $t - 1$; $EWMR_{t-1}$ = continuously compounded return on the equal-weighted market portfolio for week $t - 1$; $ERAR_{t-1}$ = estimated expected return for week t as of week $t - 1$ extracted from the model in which expected returns follow a (stationary) AR(1) process (with January dummy); η_t = random disturbance term.

* $\bar{R}^2$ = (adjusted) coefficient of determination.

† $\hat{\rho}_1$ = residual autocorrelation at lag 1. Under the hypothesis that the true autocorrelations are zero, the SEs of the residual autocorrelations are about .03.

E. The Information Content of the Extracted Expected Returns

We now consider the information content of the expected returns extracted using the stationary model, $ERAR_{t-1}$, with respect to other relevant ex ante information. It is by no means necessary that the expected returns, conditioned solely on each portfolio's past returns, should also incorporate other information. However, we can get an idea of the economic content of the forecasts by testing whether the information in other relevant ex ante variables is already impounded in them.

The choice of the predetermined variables is largely dictated by the findings in other papers and data availability considerations. Among other variables, we consider the nominal risk-free rate (Fama and Schwert 1977; and Shanken 1987)[4] and the lagged return on the equal-

4. Since new treasury bills are introduced every Thursday in the *Wall Street Journal* quotations, the estimated risk-free rate series is for an 8-day instrument. We assume skip-day settlement and continuously compound the returns. The price used to calculate the return is an average of the bid-and-ask prices derived from the quoted bid/ask discount rates.

weighted market portfolio (Gibbons and Ferson 1985). Moreover, since the January effect is statistically significant, we use the extracted expected returns which include the January dummy.

Regressions (i) and (ii) in table 5 show estimates of regressions of realized portfolio returns on predetermined variables for the overall period. The risk-free rate, RF_{t-1}, is significantly negatively related to the returns of all portfolios (see regressions [i]). There is residual autocorrelation in all the regressions, which implies that the standard errors are biased. However, if the information set incorporated in expected returns contains elements (other than RF_{t-1}) that are autocorrelated, then we would expect such autocorrelated residuals.

The lagged return on the market, $EWMR_{t-1}$, is also significantly (and positively) related to realized returns of all the portfolios (see regressions [ii]). These regressions have lower standard errors than regressions (i), and the residuals behave like white noise.

Estimates of regressions, in which all of the predictor variables *and* the extracted expected returns from the stationary model, $ERAR_{t-1}$, are included simultaneously, are also shown in table 5 (see regressions [iii]). The expected return forecasts are informative with respect to the two ex ante variables. Specifically, the forecasts contain all the information in the risk-free rate that is pertinent to expected returns; RF_{t-1} has no marginal explanatory power in any of the portfolio regressions.

The lagged return on the market, $EWMR_{t-1}$, again has no marginal explanatory power in the larger portfolio (4–10) regressions and has only some explanatory power in the regressions of portfolios 1–3. The fact that the explanatory power of the lagged market return drops substantially due to the inclusion of the extracted expected returns suggests that the autoregressive model cannot merely be capturing autocorrelation in portfolio returns due to infrequent trading in small stocks. In fact, the remaining explanatory power of $EWMR_{t-1}$ in the small-sized portfolio regressions may be a reasonable estimate of the infrequent trading effect (see simulation results of Lo and MacKinlay [1987]). More important, the lagged market return does not significantly alter the explanatory power of the extracted forecasts.

We also use other ex ante information to predict returns (results not reported). Based on the findings of Campbell (1987), we use various term-structure variables—for example, lagged weekly returns on 1-, 3-, 6-, and 12-month treasury bills (in excess of the weekly risk-free rate). Following Shanken (1987), we use a measure of interest rate volatility in an attempt to capture shifts in the investment opportunity set (see Merton 1973). Finally, we use the lagged return on the smallest portfolio. However, these variables either do not have systematically significant relations with portfolio returns or are rendered insignificant in regressions that also include $ERAR_{t-1}$.

Hence, forecasts extracted from the stationary autoregressive model (which rely solely on each portfolio's past returns) tend to subsume the information in other potential predictor variables. This result is important because it supports the robustness of our parsimonious, autoregressive model for expected returns. Regressions (iii) in table 5 also indicate that the predictor variables do contain some information about expected returns because the standard errors of the coefficients of the extracted expected returns are larger than in table 3 (especially for the large-firm portfolios).

IV. Summary and Conclusions

In this article, we attempt to characterize the stochastic behavior of expected returns on common stock. We assume market efficiency, and, based on recent empirical evidence, we postulate an autoregressive process for conditional expected returns.

We use weekly returns of 10 size-based portfolios over the 1962–85 period and extract expected returns for all portfolios. In implementing our signal-extraction methodology, we attempt to eliminate market microstructure biases through careful sample selection. Our major findings are: (1) the time-variation in expected returns is well characterized by a parsimonious (stationary) first-order autoregressive model; (2) the variation through time in short-horizon expected returns is a relatively large fraction of return variance, and this fraction has a monotonic (inverse) relation with the size rankings of the portfolios; (3) there is strong evidence of systematic changes in the relative variation in expected returns across subperiods; and (4) although the forecasts based on a stationary model rely solely on each portfolio's past returns, they subsume the information in other potential predictor variables.

This article, therefore, explicitly characterizes the nature of expected return movements. Our findings reveal significant variation in weekly expected returns, and this variation has systematic patterns both over time and across portfolios. Furthermore, the fact that the expected returns tend to incorporate other relevant economic information (including the effects of the lagged market return) suggests that the signal extracted from past returns is not due to market microstructure biases.

However, there are several unexplored issues. First, future research is needed to determine the underlying economic determinants of the time-variation in expected returns. This will help us understand the nature of the systematic differences in the degree of relative variation in expected returns both across assets and over time.[5] Second, the rapidly decaying time-variation in short-horizon expected returns ap-

5. Our results indicate that the expected risk premium on the market follows a stationary process. Preliminary results also show that systematic risk measures (i.e., betas computed relative to the equal-weighted market portfolio) change slowly over time (see also Ohlson and Rosenberg 1982). Hence, it appears that expected returns on common stock vary due to changes in both risk premiums and risk measures.

pears to be quite different from (and unrelated to) the slowly moving long-horizon expected returns. An understanding of the basic determinants of expected returns may also help explain these differences.

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