FILTER RULES AND STOCK-MARKET TRADING*

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I. INTRODUCTION

IN THE recent literature there has been a considerable interest in the theory of random walks in stock-market prices. The basic hypothesis of the theory is that successive price changes in individual securities are independent random variables. Independence implies, of course, that the past history of a series of changes cannot be used to predict future changes in any "meaningful" way.

What constitutes a "meaningful" prediction depends, of course, on the purpose for which the data are being examined. For example, the investor wants to know whether the history of prices can be used to increase expected gains. In a random-walk market, with either zero or positive drift, no mechanical trading rule applied to an individual security would consistently outperform a policy of simply buying and holding the security. Thus, the investor who must choose between the random-walk model and a more complicated model which assumes the existence of an excessive degree of either persistence (positive dependence) or reaction (negative dependence) in successive price changes, should accept the theory of random walks as the better model if the actual degree of dependence cannot be used to produce greater expected profits than a buy-and-hold policy.[1]

On the other hand, the statistician has different though equally pragmatic notions of what constitutes an important violation of the independence assumption of the random-walk model. He will

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[1] Although independence of successive price changes implies that the history of a price series cannot be used to increase expected gains, the reverse proposition does not hold. It is possible to construct models where successive price changes are dependent, yet the dependence is not of a form which can be used to increase expected profits. In fact, Mandelbrot [9] and Samuelson [12] show that, under fairly general conditions, in a market that fully "discounts" all available information prices will follow a "martingale" which may or may not have the independence property of a pure random walk. In particular, the martingale property implies only that the *expected values* of future prices will be independent of the *values* of past prices; the distributions of future prices, however, *may* very well depend on the values of past prices. In a martingale, though price changes may be dependent, the dependence cannot be used by the trader to increase his expected profits. A random walk is a martingale, but a martingale is not necessarily a random walk.

Unfortunately, however, most empirical work on the behavior of stock-market prices came about before the theoretical importance of the martingale model was established. Thus empirical work is usually concerned with the theory of random walks. In practice, this is not serious, since in most cases it is probably impossible to distinguish a series that follows a martingale with some dependence from a series that follows a random walk. In most cases the degree of dependence shown by a martingale will be so small that for practical purposes it will not do great violence to the independence assumption of the random-walk model.

The terminology used in this paper will be that of the more familiar theory of random walks rather than the more general (but perhaps simpler) theory of martingale processes. The reader will note, however, that the bulk of our discussions remain valid if the word "martingale" is substituted for "random walk" and the words "the martingale property" are substituted for "independence."

typically be interested in whether the degree of dependence in successive changes is sufficient to account for some particular property of the distribution of price changes or whether the dependence is sufficient to invalidate the results produced by statistical tools applied to the data. For example, price changes may be one variable in a regression analysis and the statistician will want to determine whether dependence in the series might produce serial dependence in the residuals. If the amount of dependence is low, he will probably conclude that it will not seriously damage his results. From the investor's point of view, however, the dependence may make the expected profits from some mechanical trading rule greater than those of a simple buy-and-hold policy.

It is important to note, however, that though a strict definition of "important dependence" is always specific to the case at hand, the ultimate criterion is always practical. In an encounter with a more complicated alternative, the theory of random walks is overthrown only if the alternative leads to a better action than the random-walk theory would have suggested.

Previously the independence assumption of the random-walk model has been tested primarily with standard statistical tools, and in most cases the results have tended to uphold the model. This is true, for example, of the serial correlation tests of Cootner [3], Fama [4], Kendall [8], and Moore [11]. In these studies the sample serial correlation coefficients computed for successive daily, weekly, and monthly price changes were extremely close to zero—evidence against "important" dependence in price changes. Similarly, Fama's [4] analysis of runs of successive price changes of the same sign and the spectral analysis techniques of Granger and Morgenstern [7], and Godfrey, Granger, and Morgenstern [6] also lend support to the independence assumption of the random-walk model.

Nevertheless, it is difficult to determine whether these results indicate that the random-walk model is adequate for the investor. For example, there is no obvious relationship between the magnitude of a serial correlation coefficient and the expected profits of a mechanical trading rule. Moreover, the market professional would probably object that common statistical tools cannot measure the types of dependence that he sees in the data. For example, the simple linear relationships that underlie the serial correlation model are much too unsophisticated to identify the complicated "patterns" that the "chartist" sees in stock prices. Similarly, runs tests are too rigid in determining the duration of upward and downward movements in prices. A run is considered terminated whenever there is a change in sign in the sequence of successive price changes, regardless of the magnitude of the price change that causes the reversal in sign. The market professional would require a more sophisticated method to identify movements—a method that does not always predict the termination of the movement simply because the price level has temporarily changed direction.

Not all the published empirical tests of independence have employed standard statistical models, however: Most notable, for example, is the work of Sidney S. Alexander [1, 2]. Professor Alexander's filter technique is a mechanical trading rule which attempts to apply more sophisticated criteria to identify movements in stock prices. An x per cent filter is defined as follows: If the daily closing price of a particular security moves up at least x per cent, buy and hold the security until its price moves down at least x per cent from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until the daily closing price rises at least x per cent above a subsequent low at which time one covers and buys. Moves less than x per cent in either direction are ignored.

Alexander formulated the filter technique to test the belief, widely held among market professionals, that prices adjust gradually to new information.

> The professional analysts operate in the belief that there exist certain trend generating facts, knowable today, that will guide a speculator to profit if only he can read them correctly. These facts are assumed to generate trends rather than instantaneous jumps because most of those trading in speculative markets have imperfect knowledge of these facts, and the future trend of price will result from a gradual spread of awareness of these facts throughout the market [1, p. 7].

For the filter technique, this means that for some values of x we would find that "if the stock market has moved up x per cent it is likely to move up more than x per cent further before it moves down by x per cent" [1, p. 26].

In his earlier article [1, Table 7] Alexander reported tests of the filter technique for filters ranging in size from 5 to 50 per cent. The tests covered different time periods from 1897 to 1959 and involved closing "prices" for two indexes, the Dow-Jones Industrials from 1897 to 1929 and Standard and Poor's Industrials from 1929 to 1959. In general, filters of all different sizes and for all the different time periods yielded substantial profits—indeed profits significantly greater than those of the simple buy-and-hold policy. This led Alexander to conclude that the independence assumption of the random-walk model was not upheld by his data.

Mandelbrot [10, pp. 417–18] pointed out, however, that Alexander's computations incorporated biases which led to serious overstatement of the profitability of the filters. In each transaction Alexander assumed that his hypothetical trader could always buy at a price exactly equal to the low plus x per cent and sell at the high minus x per cent. In fact, because of the frequency of large price jumps,[2] the purchase price will often be somewhat higher than the low plus x per cent, while the sale price will often be below the high minus x per cent.

In his later paper [2, Table 1] Alexander reworked his earlier results to take account of this source of bias. In the corrected tests the profitability of the filter technique was drastically reduced.

However, though his later work takes account of discontinuities in the price series, Alexander's results are still very difficult to interpret. The difficulties arise because it is impossible to adjust the commonly used price indexes for the effects of dividends. This will later be shown to introduce serious biases into filter results.

II. THE FILTER RULE AND TRADING PROFITS

Alexander's filter technique has been applied to series of daily closing prices for each of the individual securities of the Dow-Jones Industrial Average. The initial dates of the samples vary from January, 1956, to April, 1958, but are usually about the end of 1957. The final date is always September 26, 1962. Thus there are thirty samples with 1,200 to 1,700 observations per sample.

Twenty-four different filters ranging from 0.5 per cent to 50 per cent have been simulated. Table 1 shows, for each

[2] The point is of central theoretical importance for the stable Paretian hypothesis. For additional discussion and empirical evidence, see Fama [4].

TABLE 1*

COMPARISON OF RATES OF RETURN, BEFORE COMMISSIONS, UNDER THE FILTER TECHNIQUE (F) AND UNDER A BUY-AND-HOLD POLICY (B)

Security	Filter Size															
	0.005		0.010		0.015		0.020		0.025		0.030		0.035		0.040	
	F	B	F	B	F	B	F	B	F	B	F	B	F	B	F	B
Allied Chemical....	.155	.068	.087	.069	.042	.063	−.030	.066	−.105	.069	.008	.066	−.002	.064	−.010	.051
Alcoa............	.401	.025	.308	.023	.318	.016	−.330	.021	.241	.022	.303	.025	.270	.008	.182	.006
American Can.....	.121	.085	−.065	.075	−.123	.075	−.088	.078	−.057	.074	−.129	.072	−.201	.071	−.226	.070
Amer. Tel. & Tel...	.150	.189	.146	.189	.158	.189	.133	.185	.135	.182	.131	.180	.143	.176	.076	.182
Amer. Tobacco....	.165	.170	.019	.168	.018	.172	.012	.168	−.057	.170	−.080	.168	.002	.163	.048	.162
Anaconda.........	.288	.047	.101	.049	−.012	.046	−.048	.042	−.038	.059	−.005	.057	−.030	.055	−.019	.055
Beth. Steel........	.082	.032	.051	.033	.030	.036	−.004	.038	−.038	.054	−.128	.052	−.250	.049	−.169	.044
Chrysler..........	.031	.004	−.090	−.002	−.090	.002	−.183	.016	−.234	.015	−.152	.015	−.082	.012	.029	.012
Dupont...........	.152	.107	.125	.106	.087	.108	.100	.105	.032	.097	.054	.097	.084	.098	.058	.103
Eastman Kodak....	.078	.194	.025	.195	.005	.189	.057	.185	.085	.183	.009	.183	.032	.178	.133	.175
General Elec.......	.080	.078	.046	.075	−.015	.075	−.016	.069	.013	.069	−.052	.069	.011	.072	−.010	.070
General Foods	.122	.257	.122	.256	.146	.257	.028	.251	.084	.250	.062	.246	.112	.250	.080	.250
General Motors....	.107	.088	.108	.091	.065	.091	.048	.094	−.063	.093	−.101	.098	−.151	.099	−.171	.095
Goodyear.........	−.229	.086	−.195	.083	−.151	.085	−.109	.076	−.092	.070	.048	.077	−.013	.077	.076	.112
Int. Harvester.....	−.088	.180	−.082	.177	−.206	.176	−.112	.174	−.142	.170	−.113	.178	−.036	.175	−.018	.178
Int. Nickel........	.218	.148	.170	.136	.118	.136	.077	.137	.005	.155	.088	.148	.105	.147	.041	.160
Int. Paper.........	.205	.010	.156	.007	.095	.005	.063	.003	.034	.010	.026	.011	.014	.011	−.013	.015
Johns Manville....	.021	.094	−.016	.093	−.162	.087	−.159	.085	−.070	.077	−.194	.072	−.204	.074	−.157	.074
Owens-Illinois.....	.008	.113	−.036	.116	−.043	.115	−.130	.119	−.120	.120	−.112	.120	−.091	.124	−.037	.106
Procter & Gamble..	.315	.210	.290	.212	.221	.206	.176	.208	.130	.212	.066	.212	.015	.219	.100	.222
Sears.............	.337	.258	.249	.256	.225	.252	.167	.252	.196	.251	.181	.255	.238	.247	.203	.241
Std. Oil (Calif.)....	.076	.093	.052	.090	−.079	.094	−.106	.099	−.124	.099	−.123	.094	−.117	.097	−.158	.098
Std. Oil (N.J.).....	.036	.077	−.072	.067	−.094	.067	−.093	.070	−.084	.068	−.083	.064	−.084	.057	−.086	.056
Swift & Co........	.010	.047	.002	.042	−.026	.037	.016	.035	−.044	.037	−.115	.037	−.052	.034	−.060	.031
Texaco...........	.172	.188	.165	.192	.105	.189	.095	.188	.109	.186	.166	.184	.144	.183	.115	.178
Union Carbide.....	.290	.052	.124	.052	.145	.049	.097	.050	.067	.049	.028	.047	.038	.038	.089	.037
United Aircraft....	−.025	.054	−.020	.052	−.023	.054	−.110	.059	−.134	.053	−.189	.048	−.025	.049	−.026	.046
U.S. Steel.........	.101	.014	−.039	.010	.036	.014	.049	.027	.077	.028	.072	.035	.027	.030	.032	.025
Westinghouse......	.008	.038	−.103	.040	−.047	.038	−.215	.054	−.216	.048	−.097	.049	−.083	.051	−.015	.047
Woolworth........	.068	.128	.012	.132	.088	.131	.029	.129	−.058	.131	−.076	.132	−.052	.141	−.061	.140
Average.........	.115	.104	.055	.103	.028	.102	.002	.103	−.016	.103	−.017	.103	−.008	.102	.001	.101

* See pp. 232–33 for Notes to Table 1.

TABLE 1—*Continued*

Security	0.045 F	0.045 B	0.050 F	0.050 B	0.060 F	0.060 B	0.070 F	0.070 B	0.080 F	0.080 B	0.090 F	0.090 B	0.100 F	0.100 B	0.120 F	0.120 B
Allied Chemical....	.026	.050	−.038	.052	−.073	.056	−.018	.066	−.163	.069	−.049	.069	.055	.056	.083	.126
Alcoa............	.221	.009	.037	.025	.108	.023	−.085	.020	−.083	.023	−.170	.019	−.157	.005	−.135	−.011
American Can.....	−.212	.069	−.125	.059	−.133	.062	−.047	.059	.048	.058	.019	.060	.090	.059	.100	.066
Amer. Tel. & Tel...	.133	.176	.135	.174	.169	.181	.118	.190	.087	.186	.119	.183	.083	.195	.157	.198
Amer. Tobacco....	−.016	.159	−.020	.157	.021	.150	.196	.161	.211	.171	.217	.170	.197	.164	.197	.148
Anaconda.........	.024	.052	−.027	.050	−.119	.050	−.167	.045	.153	.041	−.109	.034	−.045	.031	.134	.043
Beth. Steel........	−.113	.038	−.153	.038	.027	.034	−.037	.022	−.094	.025	−.039	.018	−.042	.010	−.006	.014
Chrysler..........	−.004	.018	−.060	.020	.054	.010	.064	.009	.069	.013	.017	.011	.090	−.025	.058	−.020
Dupont...........	.058	.099	.076	.096	.003	.079	.003	.079	.035	.071	.112	.074	.071	.074	.081	.072
Eastman Kodak....	.121	.171	.099	.170	.110	.168	.029	.169	.080	.168	.009	.170	.074	.164	−.004	.170
General Elec.......	.010	.070	.054	.060	.042	.061	.050	.057	−.001	.052	−.062	.051	−.116	.051	−.086	.044
General Foods.....	.058	.256	.061	.265	.070	.280	.073	.276	.217	.258	.249	.254	.226	.252	.326	.239
General Motors....	−.134	.099	−.117	.099	−.215	.095	−.098	.094	−.058	.097	−.017	.096	.025	.100	.055	.080
Goodyear.........	−.285	.120	−.341	.125	.062	.095	.036	.088	.072	.090	.050	.085	.018	.078	−.026	.088
Int. Harvester.....	−.036	.169	.054	.163	−.006	.163	−.070	.164	−.094	.163	−.176	.173	−.024	.169	.066	.165
Int. Nickel........	.024	.160	.047	.154	.019	.158	.040	.154	−.050	.148	−.025	.147	.019	.136	.061	.125
Int. Paper.........	−.055	.013	−.026	.014	.044	.013	−.000	.012	−.032	.027	−.030	.029	.001	.070	.068	.038
Johns Manville....	−.107	.060	−.065	.069	−.075	.069	.001	.062	−.023	.069	−.022	.094	−.074	.082	−.112	.087
Owens-Illinois.....	−.026	.095	.043	.096	.094	.088	.142	.092	.130	.090	.057	.090	.062	.089	.068	.084
Procter & Gamble..	.151	.222	.133	.226	.163	.222	.176	.215	.121	.222	.190	.223	.156	.226	.169	.217
Sears............	.220	.246	.200	.247	.192	.231	.101	.234	.078	.232	.191	.229	.163	.229	.136	.233
Std. Oil (Calif.)....	−.128	.096	−.134	.089	−.055	.083	−.088	.082	−.052	.073	−.061	.078	−.113	.081	.019	.079
Std. Oil (N.J.).....	−.115	.057	−.164	.061	−.124	.057	−.008	.052	−.002	.048	−.051	.048	−.077	.043	−.102	.048
Swift & Co........	−.053	.035	−.049	.038	.039	.031	−.083	.027	−.079	.038	−.153	.044	−.121	.048	−.063	.070
Texaco...........	.070	.161	.005	.162	.140	.156	.048	.154	.069	.159	.023	.162	.061	.158	.057	.154
Union Carbide.....	.049	.034	.017	.035	−.005	.029	−.036	.040	.122	.049	.142	.048	.098	.042	.169	.039
United Aircraft....	−.102	.051	−.050	.047	−.076	.052	−.128	.013	−.121	.018	.045	.016	−.017	.017	−.098	.011
U.S. Steel.........	.018	.014	−.023	.013	.039	.014	.058	.024	.086	.031	.025	.029	.084	.014	.090	.010
Westinghouse......	−.060	.048	−.083	.048	−.084	.046	−.033	.057	−.016	.052	.014	.039	−.025	.036	.068	.042
Woolworth........	−.092	.132	−.049	.145	.030	.153	.011	.153	.094	.153	.062	.151	.091	.150	.055	.153
Average.........	−.012	.099	−.019	.100	.013	.097	.008	.096	.017	.096	.019	.096	.030	.093	.053	.094

TABLE 1—*Continued*

Security	Filter Size															
	0.140		0.160		0.180		0.200		0.250		0.300		0.400		0.500	
	F	B	F	B	F	B	F	B	F	B	F	B	F	B	F	B
Allied Chemical	−.017	.128	−.075	.119	−.047	.123	.036	.101	.067	.082	.037	.037	−.015	−.015	......	......
Alcoa	−.082	−.022	−.169	−.036	.050	−.039	−.015	−.038	.001	−.050	−.112	−.047	−.040	−.053	−.129	−.129
American Can	.080	.035	.072	.031	.026	−.012	.001	−.015	−.065	−.013	−.160	−.004	......	......	......	......
Amer. Tel. & Tel.	.192	.183	.197	.197	.185	.185	.159	.159	.154	.153	......	......	......	......	......	......
Amer. Tobacco	.221	.125	.229	.229	.223	.223	.218	.218	.193	.193	.174	.174	.134	.134	......	......
Anaconda	.052	.099	−.044	.081	.007	.073	.102	.059	.029	.036	−.060	.004	−.251	.134	......	......
Beth. Steel	−.039	.033	.072	.045	.036	.043	.021	.032	−.070	.001	−.158	−.008	−.034	−.034	......	......
Chrysler	−.065	−.011	−.049	−.009	−.113	−.004	−.091	−.002	−.079	.011	−.142	−.032	−.376	−.021	......	......
Dupont	.105	.067	.058	.070	.032	.071	.008	.078	−.093	.094	−.118	.111	......	......	......	......
Eastman Kodak	.062	.162	.061	.162	.108	.168	.105	.152	.262	.262	.125	.125	......	......	......	......
General Elec.	.045	.045	−.001	.043	−.062	.046	−.135	.057	−.052	.050	−.017	.072	−.015	−.015	......	......
General Foods	.321	.240	.309	.236	.291	.235	.283	.240	.270	.270	.250	.250	.200	.200	......	......
General Motors	−.282	.097	.011	.081	−.058	.089	−.080	.064	−.094	.168	......	......	......	......	......	......
Goodyear	−.063	.088	−.052	.081	−.048	.069	.083	.100	.007	.086	−.090	.072	.007	.007	......	......
Int. Harvester	.097	.153	.152	.146	.123	.135	.094	.131	.041	.219	......	......	......	......	......	......
Int. Nickel	.091	.109	.128	.093	.149	.135	.137	.141	.128	.128	.113	.113	......	......	......	......
Int. Paper	.015	.029	−.006	.065	.032	.062	.002	.061	−.041	.004	−.116	.015	−.050	−.050	......	......
Johns Manville	−.134	−.159	.032	.154	−.023	.144	.033	.131	.100	.100	.068	.068	.017	.017	......	......
Owens-Illinois	.083	.080	.034	.078	−.038	.084	.030	.077	−.051	.069	.055	.055	......	......	......	......
Procter & Gamble	.237	.234	.223	.233	.294	.230	.279	.228	.242	.242	.220	.220	.180	.180	......	......
Sears	.116	.243	.150	.226	.245	.230	.224	.226	.166	.245	.204	.204	......	......	......	......
Std. Oil (Calif.)	−.009	.079	−.024	.019	−.067	.021	−.130	.028	−.146	.037	−.095	.034	......	......	......	......
Std. Oil (N.J.)	.004	.044	−.026	.045	−.051	−.030	−.076	−.035	−.161	−.019	−.285	.007	......	......	......	......
Swift & Co.	−.092	.096	−.011	.084	−.056	.095	−.111	.106	.024	.095	−.036	.111	......	......	......	......
Texaco	−.050	.159	−.049	.162	−.114	.167	−.124	.174	......	......	......	......	......	......	......	......
Union Carbide	.108	.087	.071	.080	.022	.070	−.002	.055	−.090	.037	.004	.004	−.056	−.056	......	......
United Aircraft	−.062	.016	−.099	.002	−.173	.010	−.174	.003	−.166	.008	−.127	−.175	.209	.209	−.670	.670
U.S. Steel	.091	.119	.036	.112	−.135	.009	.072	.083	.025	.047	−.107	.053	.011	.011	−.043	−.043
Westinghouse	.038	.034	−.031	.039	.076	.200	.175	.175	.128	.128	.111	.111	.061	.061	−.014	−.014
Woolworth	.111	.177	.063	.220	.166	.166	.162	.162	.137	.137	.121	.121	......	......	......	......
Average	.039	.103	.042	.103	.036	.100	.043	.098	.027	.097	−.005	.064	−.027	.044	−.214	.121

* See pp. 232–33 for Notes to Table 1.

NOTES TO TABLE 1

In applying the filter technique, the data determine whether the first position taken will be long or short. With an x per cent filter, an initial position is taken as soon as there is an up-move or a down-move (whichever comes first) where the total price change is equal to or greater than x per cent. The position is assumed to be taken on the first day for which the price change equals or exceeds the x per cent limit. Any positions open at the end of the sampling period are disregarded. Thus only completed transactions are included in the calculations.

The closing price on the day a position is opened defines a reference price: a peak in the case of a long transaction and a trough in the case of a short transaction. On each subsequent day it is necessary to check whether the position should be closed, i.e., whether the current price is x per cent below the reference (peak) price in a long position or x per cent above the reference (trough) price if the open position is short. If the current position is not to be closed, it is then necessary to check whether the reference price must be changed. In a long position this will be necessary when the current price exceeds the reference price so that a new peak has been attained, whereas in a short position a new trough will be defined when the current price is below the reference price. Of course, when the reference price changes all subsequent testing uses the new value as base.

On ex-dividend days the reference price is adjusted by adding back the amount of the dividend. Such an adjustment is necessary in order to insure that the filter will not be triggered simply because the stock's price typically falls on an ex-dividend date. In addition, if a split occurs when a position is open, the price of the security subsequent to the split is adjusted upward by the appropriate factor until the position is closed.

With this background discussion we shall now consider the rate-of-return calculations summarized in Table 1. The following are the basic variables in the calculations:

$P_{ti}^{(j)}$ = the closing price of security j for the day on which transaction t for filter i was initiated.

$I_{ti}^{(j)}$ = the total dollar profit on transaction t of filter i when applied to security j. The profits are capital gains plus dividends, which are positive for long transactions and negative for short transactions.

$n_{ti}^{(j)}$ = the duration in terms of total trading days of transaction t for filter i when applied to security j.

$N_i^{(j)}$ = the total number of trading days during which positions were open under filter i when applied to security j. Thus

$$N_i^{(j)} = \sum_{t=1}^{T_i^{(j)}} n_{ti}^{(j)} ,$$

where $T_i^{(j)}$ is the total number of transactions initiated by filter i for security j.

$r_{ti}^{(j)}$ = the rate of return with daily compounding on transaction t of filter i when applied to security j. It is computed as

$$P_{ti}^{(j)}[1 + r_{ti}^{(j)}]^{n_{ti}^{(j)}} = P_{ti}^{(j)} + I_{ti}^{(j)} .$$

$r_i^{(j)}$ = the over-all rate of return with daily compounding provided by filter i when applied to security j. It is computed as

$$r_i^{(j)} = \left\{ \prod_{t=1}^{T_i^{(j)}} [1 + r_{ti}^{(j)}]^{n_{ti}^{(j)}} \right\}^{1/N_i^{(j)}} - 1 .$$

$R_i^{(j)}$ = the nominal annual rate of return for filter i when applied to company j. It is computed as

$$R_i^{(j)} = 260 r_i^{(j)} .$$

$R_i^{(j)}$ are the returns shown for the filter technique (F) in Table 1.

${}_bR_i^{(j)}$ = the nominal annual rate of return from buy-and-hold during the time period for which filter i had open positions in security j.

$${}_bR_i^{(j)} = 260\, {}_br_i^{(j)} ,$$

NOTES TO TABLE 1—*Continued*

where ${}_b r_i^{(j)}$ is defined as

$$ {}_b r_i^{(j)} = \left\{ \prod_{t=1}^{T_i^{(j)}} [1 + {}_b r_{ti}^{(j)}]^{n_{ti}^{(j)}} \right\}^{1/N_i^{(j)}} - 1 , $$

where ${}_b r_{ti}^{(j)} = r_{ti}^{(j)}$ if the corresponding filter transaction is long, and ${}_b r_{ti}^{(j)} = -r_{ti}^{(j)}$ if the corresponding filter transaction is short. ${}_b R_i^{(j)}$ are the returns reported for the buy-and-hold policy (B) in Table 1.

This roundabout procedure for computing buy-and-hold returns is necessary to insure that the buy-and-hold returns cover exactly the same time periods and are computed on exactly the same basis as the returns under the filter technique.

Finally, it should also be noted that the results for the largest filters are probably not reliable since for these filters the number of transactions is very small. Cf. Table 3.

security and filter size, the annual returns, adjusted for dividends but not for brokerage fees, under both the filter technique and a simple buy-and-hold policy. For each security and filter size, buy-and-hold returns are computed only for the period during which active positions are open under the filter rule, which requires that multiple buy-and-hold figures be reported for each security. The exact procedure used to compute the returns is discussed in the note to Table 1.

Table 1 presents only a small fraction of the results of this study. For example, returns under the filter technique have been computed in many different ways: gross and net of brokerage fees, with and without dividends, etc. Since presenting all the empirical work would require a small book of tables, we shall be constrained to concentrate on summary versions of the results—summarized by security and by filter size. Table 1 presents the most important of the basic results in full detail, however, and permits the reader to verify conclusions that will be drawn from the summary statistics.

A. ANALYSIS OF RESULTS BY SECURITY

Table 2 summarizes the filter results by security. For each security the table shows average returns per filter under both the filter rule and the buy-and-hold policy. The reported returns are variously adjusted for dividends and for commissions.

When commissions are taken into account the largest profits under the filter technique are those of the broker. Only four securities (American Tel. and Tel., General Foods, Procter & Gamble, and Sears) have positive average returns per filter when commissions are included (col. [2]). When commissions are omitted, the returns from the filter technique (col. [1]) are, of course, greatly improved but are still not as large as the returns from simply buying and holding. Comparison of the profits before commissions under the filter technique (col. [1]) and under a buy-and-hold policy (col. [6]) indicates that, even ignoring transactions costs, the filter technique is inferior to buy-and-hold for all but two securities: Alcoa and Union Carbide.

This last result is inconsistent with some of Alexander's latest empirical work [2, Tables 1 and 2]. When commissions are omitted, Alexander finds that the filter technique is typically superior to a buy-and-hold policy, at least for the period 1928–61. A bias in Alexander's computations, however, tends to overstate the actual profitability of the filter technique relative to buy-and-hold. This bias arises because using common price in-

TABLE 2*

NOMINAL ANNUAL RATES OF RETURN BY COMPANY: AVERAGED OVER ALL FILTERS

SECURITY	AVERAGE RETURNS ADJ. FOR DIVDS.		BREAKDOWN OF COL. (1)		AVERAGE RETURNS	BUY AND HOLD RETURNS NOT ADJ. FOR COMM.		PROFITABLE FILTERS/TOTAL FILTERS: ADJ. FOR DIVDS.	
	Not Adj. for Comm. (1)	Adj. for Comm. (2)	Long (3)	Short (4)	Not Adj. for Divds. or Comm. (5)	Adj. for Divds. (6)	Not Adj. for Divds. (7)	Not Adj. for Comm. (8)	Adj. for Comm. (9)
Allied Chemical	−.0079	−.2371	.0486	−.1453	−.0221	.0712	.0384	9/23	4/23
Alcoa	.0664	−.1388	.0744	.0627	.0643	−.0064	−.0224	13/24	4/24
American Can	−.0489	−.3022	.0052	−.1347	−.0639	.0507	.0061	9/22	6/22
Amer Tel. & Tel.	.1410	.0581	.2156	−.0727	.1221	.1824	.1484	21/21	17/21
Amer. Tobacco	.1095	−.0491	.1706	−.0724	.0814	.1704	.1307	18/23	12/23
Anaconda	−.0170	−.3091	.0398	−.1069	−.0255	.0540	.0125	8/23	4/23
Beth. Steel	−.0459	−.3214	−.0100	−.1282	−.0733	.0283	−.0266	7/23	3/23
Chrysler	−.0609	−.3695	−.0598	−.0643	−.0645	.0017	−.0311	8/23	2/23
Dupont	.0512	−.0164	.1135	−.0605	.0431	.0889	.0348	20/22	12/22
Eastman Kodak	.0757	−.0649	.1786	−.1761	.0653	.1756	.1555	21/22	12/22
General Elec.	−.0125	−.1963	.0394	−.1079	−.0237	.0576	.0285	9/23	1/23
General Foods	.1740	.0103	.2780	−.0621	.1607	.2509	.2283	23/23	12/23
General Motors	−.0581	−.3420	.0337	−.1868	−.0708	.0956	.0500	7/21	1/21
Goodyear	−.0538	−.3501	.0179	−.1942	−.0731	.0843	.0467	10/23	2/23
Int. Harvester	−.0274	−.3474	.1020	−.2624	−.0410	.1677	.1192	7/21	6/21
Int. Nickel	.0776	−.0843	.1517	−.0895	.0632	.1395	.1104	20/22	7/22
Int. Paper	.0167	−.1654	.0346	−.0178	.0026	.0193	−.0238	13/23	2/23
Johns Manville	−.0576	−.3577	.0157	−.2302	−.0707	.0878	.0497	7/23	5/23
Owens-Illinois	.0056	−.1584	.0763	−.1401	−.0010	.0958	.0679	12/22	10/22
Procter & Gamble	.1847	.0480	.2736	−.0459	.1720	.2193	.1966	23/23	15/23
Sears	.1903	.0069	.2772	−.2014	.1735	.2396	.2154	22/22	16/22
Std. Oil (Calif.)	−.0756	−.3405	.0018	−.1911	−.0915	.0748	.0302	3/22	0/22
Std Oil (N.J.)	−.0818	−.3020	−.0314	−.1670	−.0963	.0432	−.0033	2/22	0/22
Swift & Co.	−.0542	−.3793	−.0028	−.2098	−.0623	.0553	.0095	4/22	1/22
Texaco	.0605	−.1516	.1828	−.3054	.0410	.1710	.1349	16/20	4/20
Union Carbide	.0649	−.0335	.0909	−.0031	.0533	.0421	.0133	18/23	9/23
United Aircraft	−.1117	−.4478	−.0459	−.1500	−.1166	.0578	.0066	1/24	0/24
U.S. Steel	.0264	−.1622	.0467	−.0433	.0135	.0303	−.0087	18/24	7/24
Westinghouse	−.0186	−.2804	.0177	−.1164	−.0305	.0610	.0338	9/24	7/24
Woolworth	.0414	−.1491	.1296	−.2158	.0267	.1472	.1080	16/22	10/22
Average	.0185	−.1978	.0822	−.1279	.0032	.0986	.0620	12.5/22.5	6.4/22.5

* See Notes to Table 2.

NOTES TO TABLE 2

The numbers in columns (1), (2), and (5) are average returns per filter under different assumptions concerning what is included in computing dollar profits on individual transactions. The returns in column (2) are adjusted for both dividends and brokerage fees; those in column (1) are adjusted only for dividends; while those in column (5) are not adjusted for either dividends or commissions. The general formula for computing the average return per filter is

$$R^{(j)} = \frac{\sum_{i=1}^{24} R_i^{(j)}}{S^{(j)}},$$

where $S^{(j)}$ is the number of filters that resulted in completed transactions in security j and $R_i^{(j)}$ is the return from filter i when applied to security j. $R_i^{(j)} = 0$ for security j if the ith filter resulted in no completed transactions. The general procedure used in computing the $R_i^{(j)}$ is discussed in the Notes to Table 1.

Columns (6) and (7) of Table 2 show the average returns per filter from buy-and-hold. The returns in column (7) do not include either dividends or brokerage fees, while those in column (6) include only dividends. The general formula used in computing average returns per filter from buy-and-hold is

$${}_bR^{(j)} = \frac{\sum_{i=1}^{24} {}_bR_i^{(j)}}{S^{(j)}},$$

where ${}_bR_i^{(j)}$ is the rate of return from buy-and-hold during the time period for which filter i resulted in open positions in security j. ${}_bR_i^{(j)} = 0$ for security j if the ith filter resulted in no completed transactions. The procedure for computing the ${}_bR_i^{(j)}$ is discussed in the Notes to Table 1.

Columns (3) and (4) of Table 2 show the average returns per filter separately for long and short transactions. Although the returns in columns (3) and (4) are computed in the same way as those in column (1), it is important to note that the returns in column (1) are not a simple average of the returns on long and short positions. In order to use columns (3) and (4) to compute the returns in column (1), it is necessary to know the number of days that long and short positions are open.

dices makes it impossible to adjust properly for dividends. Under a buy-and-hold policy the total profit is the price change for the time period plus any dividends that have been paid. Dividends simply increase the profitability of holding shares. Under the filter technique, however, the investor alternates between long and short positions. In a short sale the borrower of the securities typically reimburses the lender for any dividends that are paid while the short position is outstanding. Thus adjusting for dividends will reduce the profitability of short sales and thereby reduce the profitability of the filter technique relative to buy-and-hold.

The size of the bias introduced by omitting dividends from the calculations can be seen by comparing returns before commissions under the filter technique and under the buy-and-hold policy, first for the case where the calculations are properly adjusted for dividends and second for the case where they are not. In our results adjusted for dividends (cols. [1] and [6] of Table 2) the filter technique only surpasses the buy-and-hold policy for two securities: The difference between the average return for all securities under the filter technique (.0185) and the average return from buy-and-hold (.0986) is 8.01 percentage points. On the other hand, when dividends are excluded (col. [5] and [7] of Table 2), average returns per filter for five securities are greater than the corresponding returns provided by buy-and-hold: The difference between the over-all average return under the filter rule (.0032) and the average return from buy-and-hold (.0620) is 5.9 percentage points. Thus adjustment for dividends increases the average advantage of buy-and-hold over the filter technique by at least 2 percentage points. If such an adjustment were applied to Alexander's data, it would probably account for much of the favorable showing of the filter rule.[3]

[3] Another possible explanation of the differences between Alexander's results and ours is that there may be "dependence" in successive changes in a price index, even though successive price changes in the individual securities of the index are independent. This spurious dependence in index changes arises from lack of synchronization in the trading of individual securities in the index. The reasoning is as follows: Suppose there is a market factor which affects the behavior of all securities. When there is a change in the market factor, the prices of individu-

The breakdown of returns before commissions for long and short transactions adds further evidence that the simple filter rule probably cannot be used to increase expected profits. Column (4) of Table 2 makes it clear that the short positions initiated by the filter rule are usually disastrous for the investor. Only one security, Alcoa, has positive average returns per filter on short transactions. For all securities, the average return on short transactions is $-.1279$, while the average return from buy-and-hold is .0986.

On long positions thirteen securities have greater average returns per filter (col. [3]) than the corresponding returns from buy-and-hold. Averaging over-all securities, the return on long transactions under the filter technique is .0822 while that from buy-and-hold is .0986. Thus even if the filter technique were restricted to long positions, it would not consistently outperform the buy-and-hold policy.[4]

al securities have also implicitly changed. All securities will not trade at precisely the time of the change in the market factor; thus for some securities the effect of the change on reported prices will only be recognized with some lag.

If successive changes in the market factor are independent, this lag in the adjustment of reported prices will not in itself produce positive dependence in successive price changes for individual securities. This is not true, however, for an average of, say, daily "closing" prices of a sample of individual securities. If the "closing" prices are really the prices on the last trade of the day, yesterday's "closing" price for some securities will not fully reflect all of yesterday's movement in the market factor since some securities will not have traded at exactly the end of yesterday's trading period. This means, of course, that the price changes today for such securities will be affected by yesterday's changes in the market factor, which in turn will tend to introduce persistence in successive changes in the average.

This line of reasoning was first suggested by Professor Lawrence Fisher. A more complete discussion, along with some empirical results, is provided in Fama [5, pp. 296–98].

[4] Even on extremely close scrutiny Table 2 does not yield evidence of dependence. For example, the average returns before commissions under the filter technique (col. [1]) are positive for fifteen securities and negative for fifteen. Col. (8) shows, for each security, the ratio of number of profitable filters to active filters. For fifteen securities, over half of all active filters for each security show positive returns, while for the other fifteen securities less than half of all the active filters are profitable. The average number of profitable filters for all securities is 12.5 while the average number of active filters is 22.5. Thus the ratio of profitable to active filters is slightly greater than one-half. But this discrepancy is not surprising since the securities do not in general follow driftless random walks.

B. ANALYSIS OF RESULTS BY FILTER

Although analysis of the filter results by security has not produced evidence of important dependence, this may not be conclusive. For example, even though the filter technique in general does not do better than a simple buy-and-hold policy, some filters may be consistently better than others and indeed better than buy-and-hold. This along with other possibilities will now be examined.

Table 3 shows the average returns per security provided by each of the different size filters. From column (2) it is evident that when brokerage fees are included none of the filters consistently produce large returns. All filters below 12 per cent and above 25 per cent produce negative average returns per security after commissions. Although filters between 12 per cent and 25 per cent yield positive returns, they are small when compared to .0986, the average return for all securities from a buy-and-hold policy. These results support the conclusion that the filter technique cannot be used to increase the expected profits of the investor who must pay the usual brokerage commissions.

Although the random-walk model is adequate for the average investor, close scrutiny of Table 3 indicates that there are very slight amounts of both positive and negative dependence in the price changes. Note that if successive price changes conformed strictly to the random-walk model, the average returns per security on long positions should be approximately equal to the average returns from buy-and-hold while the average returns on short positions should be approximately equal to the negative of the average returns from buy-and-hold.[5] In Table 3, however, for three filter sizes, 0.5, 1.0, and 1.5 per cent, the average returns per security on long positions (col. [3]) are greater than the average return from buy-and-hold, .0986. For the same filter sizes the losses on short positions are smaller than the gains from buy-and-hold. The returns on both long and short positions, however, fall dramatically as the filter size is increased.

This behavior of the returns on the smallest filters is evidence of persistence or positive dependence in very small movements of stock prices. The results

[5] In a random walk with positive drift, long positions will be open for longer periods than short positions. Thus, although the expected rate of return from short positions is just the negative of the expected return on long positions, the net expected return from the filter will be positive.

TABLE 3*

NOMINAL ANNUAL RATES OF RETURN BY FILTER: AVERAGED OVER ALL COMPANIES

Filter	Average Return per Security		Breakdown of Average Return per Security before Commissions		No. of Profitable Securities per Filter	Total Transactions
	Before Commissions (1)	After Commissions (2)	Long (3)	Short (4)	(5)	(6)
0.005	.1152	−1.0359	.2089	.0097	27/30	12,514
.010	.0547	− .7494	.1444	−.0518	20/30	8,660
.015	.0277	− .5614	.1143	−.0813	17/30	6,270
.020	.0023	− .4515	.0872	−.1131	16/30	4,784
.025	−.0156	− .3732	.0702	−.1378	13/30	3,750
.030	−.0169	− .3049	.0683	−.1413	14/30	2,994
.035	−.0081	− .2438	.0734	−.1317	13/30	2,438
.040	.0008	−..1950	.0779	−.1330	14/30	2,013
.045	−.0117	− .1813	.0635	−.1484	14/30	1,720
.050	−.0188	− .1662	.0567	−.1600	13/30	1,484
.060	.0128	− .0939	.0800	−.1189	18/30	1,071
.070	.0083	− .0744	.0706	−.1338	16/30	828
.080	.0167	− .0495	.0758	−.1267	15/30	653
.090	.0193	− .0358	.0765	−.1155	17/30	539
.100	.0298	− .0143	.0818	−.1002	19/30	435
.120	.0528	.0231	.0958	−.0881	21/30	289
.140	.0391	.0142	.0853	−.1108	19/30	224
.160	.0421	.0230	.0835	−.1709	17/30	172
.180	.0360	.0196	.0725	−.1620	17/30	139
.200	.0428	.0298	.0718	−.1583	20/30	110
.250	.0269	.0171	.0609	−.1955	15/29	73
.300	−.0054	− .0142	.0182	−.2264	12/26	51
.400	−.0273	− .0347	−.0095	−.0965	7/16	21
0.500	−.2142	− .2295	−.0466	−.1676	0/4	4

* Cols. (1) and (2) show the average returns per security provided by each of the different filters. The figures in col. (2) are adjusted for both dividends and commissions, while those in col. (1) are adjusted only for dividends. The general formula, in the notation of the Notes to Table 1, is

$$R_i = \sum_{j=1}^{30} R_i^{(j)}/F_i ,$$

where $R_i^{(j)}$ is the return from filter i when applied to security j, and F_i is the number of securities that had at least one complete transaction under filter i. $R^{(j)}$ is considered zero for security j if the ith filter resulted in no computed transactions.

indicate that the conditional probability of a positive (negative) change tomorrow, given a positive (negative) change today, is greater than the unconditional probability, but the effect of today's change on subsequent changes decreases very rapidly as one predicts further into the future. In this model the best way to utilize the dependence in the changes is to transact frequently, which is in effect what happens with the smallest filters.

On the other hand, there is also evidence in Table 3 of negative dependence in intermediate size-price movements. No filter larger than 1.5 per cent produces an average return per security on long positions greater than the average return from buy-and-hold, and the returns on long positions fall fairly steadily up to a filter size of 5 per cent. Similarly, for filter sizes greater than 1.5 and less than 12 per cent the average losses on short positions are greater absolutely than the average return from buy-and-hold.[6] These results suggest that for values of x greater than 1.5, when the price level of a security has moved down (up) x per cent, the conditional probability that it will move down (up) x per cent further before it moves up (down) x per cent is lower than the unconditional probability. Or in other words, the average duration of intermediate size-price movements is shorter than would be predicted under a pure random walk.

The question that now arises, of course, is whether either the positive dependence in extremely small price moves or the negative dependence in larger moves can be used to increase expected profits. The nature of the positive dependence in the price series suggests two possible trading procedures that would seem to produce greater expected profits than a simple buy-and-hold policy, at least for the floor trader who does not pay the usual brokerage fees. First, one could operate a 0.5 per cent filter, opening and closing long and short positions whenever such actions were signaled by the filter rule. Second, one could operate only the long positions triggered by the 0.5 per cent rule. With such a small filter, signals to open new long positions in some securities will usually occur very soon after receiving signals to close positions in others. Thus, if one follows the policy of investing all available funds in the security which triggers the next open position, capital should not be idle for a very large proportion of the time.

Yet because of out-of-pocket transactions costs which even the floor trader cannot avoid, neither of these policies can outperform buy-and-hold by any significant margin. The most important of these transactions costs is the clearinghouse fee which varies according to the price of the stock but averages approximately 0.1 per cent on each complete transaction (i.e., purchase plus sale or sale plus purchase). For our thirty securities and across a time period of approximately five years the 0.5 per cent filter initiated 12,514 transactions. This is an average of eighty-four transactions per security per year. The clearinghouse fees alone from this many transactions will reduce the average annual return per security from the 0.5 per cent filter by about 8.4 percentage points, which is more than sufficient to push the returns from the simple filter rule below those of a buy-and-hold policy.[7]

[6] The results for the largest filters are probably not reliable since the number of transactions per security is very small.

[7] Because it neglects the effects of discounting, however, this rough and ready adjustment for clearinghouse fees slightly overstates the effects of such fees on filter returns. For example, the clearinghouse fee incurred when a position is closed should be dis-

Let us now consider the policy of operating only the long positions initiated by the 0.5 per cent filter. If one succeeded in remaining fully invested nearly all of the time, the clearinghouse fees would be about as large as under the simple filter technique.[8] Thus taking only clearinghouse fees into account causes the average return per security from this modified filter rule to fall from .209 to .125, which is still about 2.5 percentage points in excess of the average return from the buy-and-hold policy, .0986.

There are other factors, however, which indicate that even the modified filter rule would not in practice be better than a simple buy-and-hold policy. First, the .209 annual average rate of return per security on long positions is computed under the implicit assumption that funds are never idle. In restricting oneself to long positions, however, even with a 0.5 per cent filter some funds will be idle part of the time, and this will reduce the return under the filter rule.[9] Second, since the filter rule is more complicated than a buy-and-hold policy, it will be more expensive to operate (e.g., costs of search, etc.). Finally, if the filter is allowed to trigger only long positions, in order to minimize the amount of time that funds are idle it will be necessary to follow closely the price movements of many securities. In practice this will probably mean that to better the chances of getting in and out at the proper times, it will sometimes be necessary to place orders with specialists. Since the floor brokerage fees of the specialist are almost twice as large as clearinghouse charges, this alone will probably be sufficient to erase any remaining advantage of the filter rule over buy-and-hold.

We now wish to determine whether the negative dependence that is evident in the results for the intermediate and larger size filters can be used to increase expected profits. As noted earlier, for filter sizes larger than 1.5 per cent, average returns per security on long positions are less than the average return from buy-and-hold; thus the long signals for these filters should be ignored. On the other hand, for filter sizes larger than 1.5 per cent average losses on short positions exceed in absolute value the average returns from buy-and-hold; and up to a filter size of 5 per cent, the losses on short positions rise as the filter size is increased. This suggests that we pick a filter, say 5 per cent, and watch only for short signals, operating them in reverse (i.e., go long when the filter signals a short position). To go long when a short signal is received has the effect of reversing the signs of the returns from short positions. Thus the negative annual average return

counted back to the point in time when the position was opened. With the smallest filters, however, positions are open for such short periods of time (an average of about three days per transaction for the 0.5 per cent filter) that proper discounting of the fees would have little effect.

[8] In this refinement of the filter technique, when a long position is closed in one security, the proceeds from the sale of the stock are used to increase the investment base for the next long position that the filter signals for some other security. Thus, although only half as many transactions are triggered as under the simple filter rule, the average investment per transaction is twice as large, so that the clearing house fees under the two policies would be almost equal.

[9] For a given security and filter size, the "annual" rate of return on long positions is computed by first finding the rate of return with daily compounding on long positions and then multiplying this daily rate by the number of trading days in the year. For a given filter size the average "annual" rate of return per security on long positions is just a simple average of the "annual" returns for each security. Since long positions, of course, are not continuously open in a single security, this procedure implicitly assumes that when a position is closed in some security the funds can be *immediately* reinvested at the average return on long positions for all securities. In fact, however, this will not be the case since immediate reinvestment will not always be possible.

of $-.160$ on the short positions of the 5 per cent filters becomes a positive return of the same magnitude. This compares with the average return on buy-and-hold of .0986.

In practice, however, it is unlikely that this modified reverse filter rule would have any advantage over the buy-and-hold policy. First, clearinghouse fees would reduce the annual returns from the filter rule by about 1 percentage point. Second, the .160 average return per security that comes from operating the short signals of the 5 per cent filter as long signals implicitly assumes that funds are never idle. With a 5 per cent filter, however, funds will probably be idle a substantial fraction of the time even if we apply the filter rule to many securities and follow the policy of investing all available funds in the next security for which a position is signaled. Finally, if in order to minimize the time during which funds are idle the filter rule is applied to many securities, it will probably be necessary to place many orders with specialists. Specialists' commissions, of course, will further reduce the returns from the reverse filter rule.[10]

Thus if the costs of operating different versions of the filter rule are considered, it seems that even the floor trader cannot use it to increase his expected gains appreciably. Since the marginal transaction costs of the floor trader are the minimum trading costs under present institutional arrangements, our results also indicate that the market is working rather efficiently from an economic viewpoint. In conclusion, there appears to be both positive and negative dependence in price changes. The order of magnitude of the dependence is so small, however, that our results add further to the evidence that for practical purposes the random-walk model is an adequate description of price behavior.

This concludes our discussion of the practical economic implications of the filter tests. The next and final section of the paper will be concerned with the more esoteric statistical implications of the empirical results.

III. THE FILTER RULE AND THE SERIAL CORRELATION MODEL: A COMPARISON

A major reason for studying the filter rule arises from the fear that the dependence in price changes is of such a complicated form that standard statistical tools, such as serial correlations, may provide misleading measures of the degree of dependence in the data. We shall now see, however, that for our samples this does not seem to be the case; the rather strong correspondence between the filter results and serial correlation tests indicates that, if indeed the serial correlations fail to uncover some of the dependence in the changes, this same dependence has also remained hidden from the scrutiny of the filter tests.

In another study [4, pp. 72, 73] one of the authors has computed serial correlation coefficients for the data used in this report. The first-order coefficients for the daily price changes of the individual securities are positive in twenty-two out of thirty cases, and the average value of the coefficients is .026. Such results are entirely consistent with the small degree of persistence on a very short-term basis

[10] From the results in Table 3 it would seem that using short signals to initiate long positions would be even more profitable for the very largest filters than for the 5 per cent filter, especially since transactions costs will be very low for these filters. This cost savings, however, is probably more than counterbalanced by the fact that the proportion of time when funds are idle will be greater for the larger filters. That transactions are infrequent for the largest filters is a very mixed blessing, since it means that funds received when positions are closed may stand idle for long periods.

that was uncovered by the filter tests. Similarly, for four- and nine-day price changes the first-order serial correlation coefficients are negative in twenty-one and twenty-four out of thirty cases. Again, however, the coefficients are extremely close to zero; for the four- and nine-day changes the average values are −.038 and −.053, respectively. These results are entirely consistent with the small degree of negative dependence in intermediate size price movements that was uncovered by the filter results.[11]

Even though standard statistical tools such as serial correlations cannot provide exact estimates of the expected profits from mechanical trading rules such as the filter technique, the discussion above suggests that for measuring the direction and degree of dependence in price changes, the standard tools are probably as powerful as the Alexandrian filter rules.

[11] In fact, there are indications that the relationships between the filter results and the serial correlations are even more formal than is implied by the discussion in the text. The rank correlation between the first-order daily serial correlations for the different securities and the returns before commissions from the 0.5 per cent filter is .76. Thus the small degree of persistence in very small price movements affects the serial correlations in the same direction as the filter results. For the different securities, the rank correlation between the first-order serial correlations for four-day price changes and the returns before commissions on the 5 per cent filter is .45. Although the formal relationship between the filter results and the serial correlations is not as strong for the intermediate size price movements, there is still a definite correspondence between the results provided by the two measures.

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