Bachelier's Predecessors*

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Abstract

Recent papers by Jean-Michel Cortault et al. (2000) and Murad S. Taqqu (2001) have cast new light on Bachelier's work and his times (see [1], [S3], [S21]). In both articles Bernard Bru's research seems to be the most influential. In the case of Louis Bachelier and his area of activity the dominant French point of view is the most natural thing in the world and every body is convinced by the results. The aim of the present note is to add a few tesseras also from other countries to the picture which is known about the birth of mathematical finance and its probabilistic environment.

1. C.F. Gauss (1777 - 1855)

Under Bachelier's predecessors who have prepared tools and first steps to stochastic finance Carl Friedrich Gauss should be mentioned first. Bachelier cited "la loi de Gauss" which was introduced as an error distribution by Gauss in 1809 (see [1], [19]). We enlighten a fewer known side of the "mathematicorum princeps", his work as a successful investor. Since the 1830s, in defiance (or because) of his modest salary, Gauss invested his savings in bank stocks and obligations and leaved an estate of more than 150000 thalers (see [20], [21]). His great experience with financial operations on a bourse took effect also in public affairs. He reorganized the fund for professor's widows at the university of Göttingen during 1845 - 1851. For this purpose he calculated special life-insurance tables (see [22]).

2. G.T. Fechner (1801 - 1887)

Gustav Theodor Fechner learned as a Physicist from Jean B. Biot and Georg Simon Ohm mathematical modelling. His own work in psychology and statistics included the foundation of psychophysics and the theory on the measurement of collectives. The Weber-Fechner law based on the assumption that the sensation as a function of a stimulus is Lipschitz continuous (Bachelier and Einstein worked later with smooth functions and the Taylor formula). It found applications also in finance by M.F.M. Osborne (see [15] - [17], [31] - [34], [S5], [S17], [S18]).

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3. H.D. Macleod, Ch. Castelli, F.Y. Edgeworth

In the 1860s Henry Dunning Macleod showed in his main work the "principles of Currency and Banking, their progressive development in practice, and the laws at present affecting them" (see [28]). He gave juridical definition of the terms used in monetary science.
Bachelier refers in his Thesis to specialized works on options. Probably his translator A. J. Boness added in [S4] the title of a book written by the stock and share broker Charles Castelli in 1877. In this little book Castelli explains very clearly the modes practised in the Stock Exchange in relation to the "call" and "put" operations for brokers and clients. A French translation was published in 1882 (see [9], [10]).

Francis Ysidro Edgeworth found in the 1880s a connection between finance and probability. He investigated the problem of bankruptcy and postulated the law of error as the foundation of banking. Edgeworth tested his hypothesis with real data, the amount of returns of Bank of England notes in the hand of the public in the period of 1833 to 1844. Upon this foundation he computed the probability that the demand will not exceed any proposed limit. Furthermore, he presented a complex model of banking as a new game of chance formulated as a certain portfolio problem with three possibilities to handle a disposable fund (see [12], [13]).

4. J. Bresson, J. Regnault, H. Lefévre

One of the first important books about operations on the Stock Exchanges in Paris is written by Jacques Bresson. It followed later the handbook by Ambroise Buchère.
Probably the most important predecessors of Bachelier with respect to finance were Henri Lefévre (private secretary to Baron James de Rothschild) and Jules Regnault. Franck Jovanovic and Philippe Le Gall investigated their work in a series of papers [S11 - S15].

We refer especially to Lefévre's booklet comparable with Castelli's (see [27], [9]), which is only the third part (Livre III) of the book [26]. A graphical depiction of a long position in a European call option long before Bachelier should be emphasized (see [27], [S11], [S13], [S20]).
Regnault's book [38] contain an abundance of ideas for modelling the price behavior of a financial market. He empirically tested the law : "the deviation of the prices increases with the square root of time" evaluating the mean value of the French 3% bond. This mean-value approach analyzing time series was extended significantly by Thorwald Nicolai Thiele (see [40], [S16], [S8], [S9]). The use of the Gaussian normal law in Regnault's research based on a visual approach (see [S14]).

5. L. Bachelier's Thesis

There were a few dissertations about the markets of the exchange, but all presented to faculties of law (see [23]). The first thesis on speculation from a stochastic point of view was written by Louis Bachelier. It has been seen and allowed for publication in January 1900 by Jean Darboux the dean of the Paris Faculty of Science and was published inmediately in February (see [S4], [1]).
It is interesting how Bachelier derived the law of probability of relative prices in comparison with Rayleigh's approach for vibrations and Einstein's model of Brownian motion (see [1], [39], [14]).

6. The time after

Henry Poincaré suggested in his report on Bachelier's thesis to study further into the details of Fourier's analysis the relationship of stochastic processes with partial differential equations. Bachelier realized this hint.

In the year 1906 three attemps were published to handle dependent random phenomena. They were given in the case of discrete time by Andrei Markov in St. Petersburg and following - G.T. Fechner's ideas - by Heinrich Bruns in Leipzig, but in continuous time by Louis Bachelier in Paris (see [29], [7], [2]). Bachelier presented a framework which covered not only the Wiener process as in [1] but also the Ornstein-Uhlenbeck process. Furthermore, he disclosed a connection with the work of Pierre Simon Laplace (see [2], p. 275, [25] II 14, §17). Unfortunately, Bachelier's approach failed the necessary rigour for a general acceptance. In the special case of the Ornstein-Uhlenbeck process Markov's method of moments was successful yet. The general classical case in Poincaré's sense has been solved by Andrei Kolmogorov (see [30], [24]).

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