The Value of Information in Impersonal and Personal Markets

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Preface
Mark Rubinstein

This paper was written in 1974-1975, shortly after I graduated from U.C.L.A. Even at that remove, it reflects the continuing influence of Fred Weston who provided unceasing encouragement and optimism (as well as financial assistance) about some of my earlier research which led up to this joint paper with Jeffrey Jaffe. The paper is one of the first attempts to deal with the problem of optimal trading strategies in a situation where an uninformed investor needs to trade for liquidity reasons but fears that he will be at the mercy of a better-informed investor who will take the other side of his trade.

Introduction

Until recently, theoretical research in finance has been exclusively focused on competitive securities markets in which those with whom any consumer trades are not identified to him. In this context, it has been argued, for example by Hirshleifer (1971), that information[1] about the future aggregate supply of resources in the economy is *privately* valuable to consumers since it permits informed consumers to profit at the expense of the uninformed. Moreover, if information cannot affect aggregate production decisions, then it is also *not socially* valuable since what one consumer gains another loses.[2] *Ex post*, in terms of realized consumption at the

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[1] For the purposes of this paper, it is useful to categorize three types of information:

1. *Personal information:* A consumer's knowledge of his own resources and tastes.
2. *Information about supply:* A consumer's knowledge of the aggregate supply of future consumption.
3. *Information about demand:* A consumer's knowledge of the resources and tastes of other consumers, and his knowledge of other consumers' information about supply and demand.

[2] This theme was first discussed by Samuelson (1957, p. 209) who wrote in reference to the commodities market:

> Suppose my reactions are not better than those of other speculators but rather just one second quicker…In a world of uncertainty, I note the consequences of each changing event one second faster than anyone else. I make my fortune—not once, but every day that important events happen. Would anyone be foolish enough to argue that in my absence the equilibrium pattern

same date, since the social total of consumption is fixed by assumption, the last conclusion is trivially true. However, *ex ante*, in terms of expected utility of consumption, the conclusion requires a more subtle justification.

This paper first reexamines the impersonal and competitive economy of Hirshleifer. We concur with Hirshleifer that information has private value and that the *production* of new public information does not *ex ante* have social value;[3] however, the *dissemination* of existing private information may have social value; and for the same reason the production of new private information, even if costless, can be socially harmful. In particular, if existing information is already fully reflected in security prices, then its free dissemination is *ex-ante* Pareto efficient.[4] Moreover, even if this information is not fully reflected in security prices, there always exists a way of redistributing resources concomitant with disseminating the information so that this dissemination is Pareto efficient. As a corollary, if a prior market for the sale of information can be properly organized, informed consumers will always benefit more from selling their information than from withholding it and making speculative side bets in the securities market with uninformed consumers. The dissemination (for a price) of existing private information therefore has both private and social value.

While information about supply conditions has private value and may even have social value in an impersonal and competitive pure exchange economy, this value will be substantially reduced if consumers are assumed to have direct information about demand conditions; that is, knowledge of the economic characteristics (resources, tastes and beliefs) of other consumers.[5] Such personalization is common in insurance and loan markets where the buyer has more information than the seller and the seller knows it. One may think of a continuum beginning with an impersonal market where each consumer views others as completely unidentified. Markets become more personal as each consumer learns more about those with whom he is trading. Increased personalization may occur gradually over time among a relatively small group

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> would fail to be reestablished? ... There is no necessary correspondence between the income effects realized by any person's actions and the amount of meritorious substitutions that his actions can alone bring into being.

[3] Although Hirshleifer (1971) couches his analysis of information production in an economy consisting of identical consumers, as Marshall (1974) and Ng (1975) have shown, his results are quite general carrying over to perfect and competitive economies with arbitrary heterogeneity among consumers (but with the same beliefs). In particular, Ng shows that good news tends to increase the risk-free interest rate making lenders better off and borrowers worse off.

[4] See Rubinstein (1974) on the *ex-ante* issue. Ng (1974) has extended the Pareto efficiency of free dissemination even to certain cases where existing superior information is *not* fully reflected in security prices. Starr (1972) has further shown that full dissemination of information (i.e. homogeneous beliefs) is required for *ex-post* Pareto efficiency if consumption occurs at two dates. Without homogeneous beliefs, when uncertainty is resolved at the second date for at least one state, there will be at least two consumers who will regret their consumption decisions at the first date; thus, one would realize more utility with less initial consumption (and consequently more later consumption) and the other would realize more utility with more initial consumption (and consequently less later consumption). Consequently, if only they could have forecast the future with certainty, they would have made a mutually beneficial exchange at the first date.

[5] Other recent papers modeling personal markets include those of Grossman (1977) and Rothchild and Stiglitz (1976).

of traders, as for example, among coaches and managers in the professional sports market for the allocation of athletes among competing teams.

Here we show that even under arbitrary exchange arrangements (possibly not competitive), personalization of the market limits the private value of information about supply conditions. In particular, with sufficient personalization consumers can identify and may even be able to rank Pareto-efficient allocations independent of their information about supply conditions. Consequently, if the endowed allocation is itself Pareto efficient, then information (about supply conditions) will be valueless. However, if the endowed allocation is not Pareto efficient, then information may be valuable for comparing Pareto-inefficient with Pareto-efficient allocations. But, even in this case, if exchange arrangements are competitive and "average" beliefs exist, information will be valueless. This is true even though poorly informed consumers cannot infer the beliefs of better-informed consumers from security prices.

Impersonal Markets

Consider a two-date (t = 0, 1), $E(e = 1, 2, \ldots, E)$ state, $I(i = 1, 2, \ldots, I)$ consumer perfect and competitive *pure exchange* economy, where each consumer i endowed with resources $\{\overline{W}^i_e\}$, selects state-contingent claims $\{W^i_e\}$ at present prices $\{P_e\}$ so as to maximize the expected utility $(\sum_e \pi_e U_i(W^i_e))$ of his future wealth subject to

$$\sum_e P_e W^i_e = W^i_o \equiv \sum_e P_e \overline{W}^i_e$$

with $\pi^i_e > 0, U'_i > 0$ and $U''_i < 0$. The final allocation across all consumers must also satisfy the closure conditions

$$\sum_i W^i_e = W^M_e \equiv \sum_i \overline{W}^i_e$$

for all e. Each consumer is assumed to know only his resources, tastes and beliefs, and the opportunities — prices — the market makes available to him.

Let us say, of two consumers, one has *superior information* about supply conditions if by revealing his information he can convince the other to adopt his beliefs. Consumers with superior information can benefit from it by making speculative side bets with others via the securities market or by selling it to others in a prior market for information before the securities market convenes. To see that superior information is *privately* valuable, we compare the expected utility to the same consumer of the choices he would make with and without the superior information, where *expectations are assessed with respect to beliefs which reflect the superior information*.

The simplest illustration is furnished by an economy in which all consumers have logarithmic utility functions and are identical except for their beliefs. In this instance,[6] competitive equilibrium is characterized by

$$W_e^i = \left(\frac{\pi_e^i - \pi_e}{\pi_e} \right) W_e + W_e \qquad \text{for all } i \text{ and } e$$

with $\pi_e \equiv \sum_i \pi_e^i / I$ and $W_e \equiv \sum_i W_e^i / I$. When all consumers have the same beliefs, all speculative side bets $[(\pi_e^i - \pi_e)/\pi_e] W_e = 0$ and each consumer's state-contingent future wealth is exactly the per capita amount. With differences in beliefs, consumers plan for more wealth in those states toward which they are optimistic $(\pi_e^i > \pi_e)$ and less in those states toward which they are pessimistic $(\pi_e^i < \pi_e)$.[7] Suppose that $\{\pi_e^*\}$ represents beliefs based on superior information. This information is privately valuable to consumer i if and only if

$$\sum_e \pi_e^* \log W_e^* - \sum_e \pi_e^* \log W_e^i > 0$$

with $W_e^* = (\pi_e^* / \pi_e) W_e$, the choices based on superior information, and $W_e^i = (\pi_e^i / \pi_e) W_e$, the choices based on inferior information $\{\pi_e^i \neq \pi_e^*\}$.[8] Consequently, superior information is privately valuable if and only if

$$\sum_e \pi_e^* \log \pi_e^* - \sum_e \pi_e^* \log \pi_e^i > 0$$

To see that this must be positive, solve the following programming problem by choosing $\{\pi_e'\}$ such that

$$\max_{\{\pi_e'\}} \sum_e \pi_e^* \log \pi_e' - \lambda(\sum_e \pi_e' - 1)$$

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[6] To derive this sharing rule

$$\max_{\{W_e^i\}} \sum_e \pi_e \log W_e^i - \lambda_i (\sum_e P_e W_e^i - W_0)$$

which has first order conditions $W_e^i = (\pi_e^i / P_e) W_0$ for all e and i. Summing this over all i and dividing by I, $W_e = (\pi_e / P_e) W_0$.

[7] Note that

$$\sum_i \left(\frac{\pi_e^i - \pi_e}{\pi_e} \right) W_e = \sum_e P_e \left(\frac{\pi_e^i - \pi_e}{\pi_e} \right) W_e = 0$$

for all *i* and *e*.

[8] This analysis is somewhat simplified since it ignores the likely second-order effect of the conversion of the beliefs of a single consumer on $\{\pi_e\}$, the "average" beliefs. Were this second-order effect considered, our conclusion concerning the private value of information would be unaffected.

Observe that at the maximum $\pi'_e = \pi^*_e$ for all *e*.

To see that the dissemination of information may also be *socially* valuable, suppose that, before dissemination, security prices fully reflect the superior information. In that case, revealing the information leaves prices, and hence "consensus beliefs."[9] unchanged. In the logarithmic utility illustration, consensus beliefs $\pi_e \equiv \sum_i \pi^i_e / I$ for all *e* and the difference in expected utility with and without the dissemination of the information is again $\sum_e \pi_e \log \pi_e - \sum_e \pi_e \log \pi^i_e$ for consumer *i*. Clearly, this is positive for all uninformed consumers $\{\pi^i_e \neq \pi_e\}$ and zero for all informed consumers $\{\pi^i_e = \pi_e\}$. Note that Pareto efficiency has been assessed with respect to beliefs that reflect the superior information. This type of Pareto efficiency will be termed "full information efficiency"; it defines a situation that would be Pareto efficient after all private information has been released. Hereafter, the term "Pareto efficient" will refer unambiguously to this full information definition of the concept.

Observe that if the informed consumers were to sell their superior information at a positive (but not too high) price, then even informed (as well as uninformed) consumers would be better off as a result of the dissemination of the information. To derive the maximum price[10] an uninformed consumer would pay for the superior information (assuming it were disseminated to all consumers), find that proportion γ_i of initial wealth for which

$$\sum_e \pi_e \log[(1-\gamma_i)\, W_e] - \sum_e \pi_e \log\left(\frac{\pi^i_e}{\pi_e} W_e\right) = 0$$

It follows that

$$\gamma_i = I - \prod_e \left(\frac{\pi^i_e}{\pi_e}\right)^{\pi_e}$$

a fraction between zero and one.

This analysis suggests that even if superior information is not fully reflected in security prices, there always exists a way of redistributing resources concomitant with disseminating the information so that this dissemination would be socially valuable. In particular, suppose a prior market for information, if it were utilized, did not use up aggregate resources. That is, for each state *e*, W^M_e would remain the same before and after the sale of private information. Moreover, suppose the sale of private information were to bring about agreement in beliefs. Without a prior market for information, uninformed consumers would make disadvantageous side bets. With a prior market for information, while no side bets would later be taken in the securities market, poorly informed consumers would instead deplete their wealth by purchasing superior information. They would then enter the securities market poorer but wiser.

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[9] Consensus beliefs are those beliefs that, if held by all consumers in an otherwise similar economy, would generate the same equilibrium prices as in the actual economy (see Rubinstein, 1975).

[10] For an extended analysis, considering choice of information structures, see Morris (1974).

Is it possible to organize a prior market for information so that its use is preferred by all consumers in the economy to the alternative of making side bets in an impersonal competitive market?

Theorem 1. Impersonal Markets. In an impersonal competitive market, given at least some disagreement, there always exists a way of redistributing resources through a prior market for information such that all consumers will be better off.

Proof. In an impersonal competitive market, each consumer i

$$\max_{\{W_e^i\}} \sum_e \pi_e^i U_i(W_e^i) - \lambda_i(\sum_e P_e W_e^i - W_0^i)$$

Since U_i is concave, $U_i'(W_e^i)/U_i'(W_s) = P_e \pi_s^i/(P_s \pi_e^i)$, for any two states e and s and all i, are the necessary and sufficient conditions for an equilibrium. Consequently, consumers have the same beliefs if and only if for any two states e and s, $U_i'(W_e^i)/U_i'(W_s^i)$ is the same for all i. Therefore, the equilibrium allocation under heterogeneous beliefs cannot be the same as the equilibrium allocation under homogeneous beliefs. Only the equilibrium under homogeneous beliefs can be Pareto efficient (with respect to the beliefs reflecting the superior information). If consumers were to trade without convening a prior market for information, they would not have the same beliefs and thus would not reach a Pareto-efficient allocation. Therefore, all consumers can be made better off by holding a prior market for information, and then all entering the securities market with the same beliefs.

In short, the benefits from buying and selling information in a properly organized prior market for all information exceed the benefits from speculative side bets for all consumers in the economy.[11] This may *partially* explain the empirical evidence supporting the speed with which the securities market digests new information, since disclosure for a price is more profitable than taking a speculative position and waiting for nature to reveal the true state. Taking a speculative position and then disclosing the information for a price will also *not* be preferred to pure disclosure for a price, since the prior speculative position, even by its slight effect on prices in a large market, will diminish the benefits to poorly informed individuals from later disclosure, causing them to pay less for the information.

The theorem only asserts that it is *possible* to redistribute resources via a prior market for information so that all consumers are better off. However, due to the special characteristics of information as a "commodity," such a prior market will be difficult to design. For an informed

[11] Hirshleifer (1971) has argued that given the opportunity to purchase information in a prior market, consumers would actually pay *not* to have information released and therefore such a market would be inactive. Each consumer would view the release of information new to him as creating a fair gamble and risk averters are willing to pay to avoid fair gambles. However, the prior market for information envisioned here contains a special feature that mitigates this behavior. Concomitant with the release of information is a *redistribution of resources*. The theorem implies it is possible, by redistributing resources, to guarantee that all consumers will be better off. As a result, all consumers want the information to be released. Either by itself, releasing the information or redistributing resources, would not be Pareto efficient.

consumer to benefit more from selling information than from speculating, its value (the benefits its disclosure confers upon others) must be sufficiently appropriable. However, because of the Pareto inefficiency of heterogeneous beliefs, this value need not be *completely* appropriable to its owner.

Since heterogeneous beliefs are full information inefficient and the production of private information tends to increase the heterogeneity of beliefs, then the production of private information, even if costless, is socially harmful. The society therefore has an incentive to implement full information efficiency by establishing regulatory bodies to prevent the production of private information. For example, insider trading laws, which strive to control profits earned by corporate officials on their information, may reduce the amount of private information produced. Similarly, the public dissemination of private information helps to create homogeneous beliefs and therefore full information efficiency. Hence authorities may create disclosure laws, even if the dissemination of information is costly. The key to full information efficiency is the homogeneity of beliefs, not the amount of information. It is important to stress that these public policy implications have not been justified in this paper by recourse to equity arguments. Rather, they have been justified by considerations of Pareto efficiency alone.

Personal Markets

Assume the economy is described as before except that the securities market is completely personal with arbitrary exchange arrangements. The securities market is said to be *completely personal* if, in addition to his own resources, tastes, and beliefs, each consumer knows (1) the resources of all other consumers, (2) the tastes of all other consumers, (3) the "type" of information he and all others have. By knowledge of type of information, we mean all consumers agree on the ranking of informativeness of all consumers in the economy, although they do not know the content of this information. Although the exchange arrangements are arbitrary,[12] in a completely personal market it seems reasonable to impose the restriction that the final allocation be Pareto efficient (with respect to the beliefs reflecting superior information $\{\pi_e^*\}$). As we have seen in the preceding section, any Pareto-inefficient allocation could not be stable since an informed consumer can always sell his superior information at a positive price. More precisely, we require that the final allocation $\{W_e^i\}$ be feasible so that $\sum_i W_e^i = \sum_i \overline{W}_e^i$ for all *e* and there exists no other feasible allocation $\{\hat{W}_e^i\}$ for which

$$\sum_e \pi_e^* U_i(\hat{W}_e^i) - \sum_e \pi_e^* U_i(W_e^i)$$

be greater or equal to zero for all consumers *i* and greater than zero for some consumers *i*. In addition to Pareto efficiency, we require that each consumer perceive his final allocation as superior to his endowed allocation. That is, by whatever beliefs $\{\pi_e^i\}$ he chooses to use

[12] In particular, we no longer require that there exist prices $\{P_e\}$ such that $\sum_e P_e W_e^i = \sum_e P_e \overline{W}_e^i$ for all i.

$$\sum_e \pi_e^i U_i(W_e^i) - \sum_e \pi_e^i U_i(\overline{W}_e^i)$$

be greater than or equal to zero.

With this preamble, we are prepared for the following theorem.

Theorem 2. Personal Markets. In a completely personal market,[13] consumers can identify Pareto-efficient allocations independent of their beliefs.

Proof. It is well known that for concave utility, an allocation is Pareto efficient if and only if it maximizes a positively weighted sum of consumer utilities subject to closure conditions:

$$\max_{\{W_e^i\}} \sum_i k_i \sum_e \pi_e^i U_i(W_e^i) - \sum_e \lambda_e (\sum_i W_e^i - \sum_i \overline{W}_e^i)$$

where $k_i > 0$ for all i and $\{\lambda_e\}$ are Lagrangian multipliers. The necessary and sufficient conditions for efficiency are then $k_i \pi_e^i U_i'(W_e^i) = k_1 \pi_e^1 U_1'(W_e^1)$ for all i and e and the closure conditions. Assessing Pareto efficiency with respect to beliefs which reflect superior information amounts to setting $\pi_e^i = \pi_e^1 = \pi_e^*$ for all e so that $k_i U_i'(W_e^i) = k_1 U_1'(W_e^1)$ for all i and e. These equations and the closure conditions describe the exhaustive set of Pareto-efficient allocations (by varying $\{k_i\}$) and are independent of $\{\pi_e^*\}$. Moreover, since the market is completely personal, all consumers can calculate this set.

Information, in a completely personal market, therefore, does not acquire private value because it assists in identifying ending portfolio positions; irrespective of the beliefs an uninformed consumer attributes to an informed consumer, the set of full information efficient allocations is the same. The following corollary emphasizes this result.

Corollary. In a completely personal market, if the endowed allocation is Pareto efficient, then no consumer will trade and the private value of information is zero.

Proof. From the theorem, since each consumer knows the resources and tastes of all consumers, even though he does not know the beliefs of the best-informed consumer, he can calculate the set of Pareto-efficient allocations. Therefore, each consumer knows the endowed allocation is Pareto efficient. Since (1) no well-informed consumer will offer to trade with a poorly-informed consumer unless the well-informed consumer will benefit *ex ante*, (2) the poorly-informed

[13] For this theorem to hold, we can weaken the information requirements of a completely personal market and require in place of (1) that each consumer knows only the aggregate resources $\{W_e^M\}$ available in each state and not their distribution among consumers. In addition, there are some special cases for which these information requirements can be further weakened. For example, with no aggregate uncertainty (i.e. W_e^M is the same for all e), in place of (2) consumers only need know that all other consumers are risk averse. In this case, irrespective of tastes, all Pareto-efficient allocations are characterized for each consumer i by W_e^i the same for all e.

consumer must lose *ex-ante* in this exchange (since the endowed allocation is already Pareto efficient), and (3) the poorly-informed consumer knows this and that he is poorly informed (since the market is personal), he will refuse to trade. Superior information about supply conditions, i.e. the probability distribution of $\{W_e^M\}$, clearly has no value in a personal market if the endowed allocation is Pareto efficient. Even if it is publicized, there will be no trade.

Therefore, the private value (if it has any) of information in a personal market must arise from the necessity of *ranking* Pareto-efficient allocations or *comparing* Pareto-efficient with Pareto-inefficient allocations. As long as the endowed allocation is not itself Pareto efficient, in the process of trading consumers will move from a Pareto-inefficient to a Pareto-efficient allocation. In this process, consumers must choose among alternative Pareto-efficient allocations. However, in general a poorly-informed consumer does not know which direction of movement along the contract curve will be to his advantage and information will therefore have private value. None the less, there are special situations where Pareto-efficient allocations can be ranked, as well as identified, independent of beliefs.

Theorem 3. Personal Markets. In a completely personal market, consumers can rank Pareto-efficient allocations independent of their beliefs if either

1. there are only two consumers in the market, or

2. all consumers have H.A.R.A. (linear risk tolerance) utility functions with the same cautiousness.[14]

Proof. Consider first the case of two consumers. From the observation in the previous proof, Pareto efficiency with respect to homogeneous beliefs, requires that

$$\frac{U_1'(W_e^1)}{U_2'(W_e^2)} = \frac{U_1'(W_s^1)}{U_2'(W_e^2)} = \dots \qquad \text{for all states } e \text{ and } s$$

Consider any other Pareto-efficient allocation; it must likewise satisfy

$$\frac{U_1'(\hat{W}_e^1)}{U_2'(\hat{W}_e^2)} = \frac{U_1'(\hat{W}_s^1)}{U_2'(\hat{W}_s^2)} = \dots \qquad \text{for all states } e \text{ and } s$$

[14] The H.A.R.A. class of utility functions is described by the solution to the differential equation $-U_i'(W_e^i)/U_i''(W_e^i) = A_i + B_i W_e^i$ where A_i and B_i are constants. Requiring identical cautiousness implies $B_i = B$ (independent of *i*) for all consumers. The solution includes most popular utility functions including quadratic, exponential, logarithmic, and power utility. These utility functions possess a surprising constellation of properties some of which are developed in Rubinstein (1974, 1981). In particular, Brennan and Kraus (1975) have recently shown that in a perfect and competitive financial market with homogeneous beliefs, condition (2) is necessary and sufficient for all consumers to have parallel linear Engle curves.

Together with $W_e^1 + W_e^2 = \hat{W}_e^1 + \hat{W}_e^2 \equiv W_e^M$ for all *e*, these conditions are necessary and sufficient for Pareto efficiency. Therefore, if $\hat{W}_e^1 > W_e^1$, then since U_i is concave $U_1'(\hat{W}_e^1) < U_1'(W_e^1)$; also by closure, $\hat{W}_e^2 < W_e^2$ so that similarly $U_2'(\hat{W}_e^2) > U_2'(W_e^2)$. In short, if $\hat{W}_e^1 > W_e^1$, then

$$\frac{U_1'(\hat{W}_e^1)}{U_2'(\hat{W}_e^2)} < \frac{U_1'(W_e^1)}{U_2'(W_e^2)}$$

For any other state *s*, this implies

$$\frac{U_1'(\hat{W}_s^1)}{U_2'(\hat{W}_s^2)} < \frac{U_1'(W_s^1)}{U_2'(W_s^2)}$$

Repeating the same reasoning backwards, this in turn implies $\hat{W}_s^1 > W_s^1$ and $\hat{W}_s^2 < W_s^2$. In brief, any two Pareto-efficient allocations must either be characterized by $\hat{W}_e^1 > W_e^1$ and $\hat{W}_e^2 < W_e^2$ for all states *e* or by $\hat{W}_e^1 < W_e^1$ and $\hat{W}_e^2 > W_e^2$ for all states *e*. Consequently, the nonsatiation property $(U_i' > 0)$ of consumer tastes allows consumers to rank Pareto-efficient allocations independent of their beliefs.[15]

Consider now the case of H.A.R.A. utility functions. As developed in Rubinstein (1974), a necessary and sufficient condition for a Pareto-efficient allocation is

$$W_e^i = \frac{A_M \ W_0^i - A_i W_0^M}{A_M\phi + BW_0^M} + \frac{A_i\phi + BW_0^i}{A_M\phi + BW_0^M} W_e^M \quad \text{for all } e \text{ and } i$$

with $\phi \equiv \sum_e P_e$, $W_0^M \equiv \sum_e P_e W_e^M$, and $A_M \equiv \sum_i A_i$. Although this condition was derived in the context of a competitive market (as in the preceding section), it applies even in the absence of a competitive market. This follows since all Pareto-efficient allocations can be spanned by an appropriate redistribution of resources (i.e. redistribution of $\{W_0^i\}$). Moreover, from his resource distribution irrelevancy theorem, Rubinstein (1974) proved that the price system is independent of the distribution of resources; therefore, ϕ and W_0^M remain fixed through any redistribution of resources. This motivates rewriting the condition for Pareto efficiency as

$$W_e^i = \alpha_e A_i + \beta_e W_0^i \qquad \text{for all } e \text{ and } i$$

with

$$\alpha_e \equiv \frac{\phi W_e^M - W_0^M}{A_M\phi + BW_0^M}$$

[15] We are indebted to David Ng for teaching us that this proof does not generalize to economies with more than two consumers.

and

$$\beta_e \equiv \frac{A_M + BW_e^M}{A_M \phi + BW_0^M}$$

As the set of Pareto-efficient allocations is spanned by redistributing resources, α_e, A_i and β_e remain fixed. Since, as is easy to show, $\beta_e > 0$ for all e, for any given consumer i, Pareto-efficient allocations must either be characterized by $\hat{W}_e^i > W_e^i$ or $\hat{W}_e^i < W_e^i$ for all e. Again, the nonsatiation property $(U_1' > 0)$ of consumer tastes allows consumers to rank Pareto-efficient allocations independent of their beliefs.

However, even if information is not needed to identify and rank Pareto-efficient allocations, it may still be useful in comparing Pareto-efficient with Pareto-inefficient allocations. In the process of moving from a Pareto-inefficient to a Pareto-efficient allocation, each consumer will insist that he be made at least as well off as his endowed allocation. That is, by whatever beliefs $\{\pi_e^i\}$ he chooses to use, he will require that

$$\sum_e \pi_e^i U_i(W_e^i) - \sum_e \pi_e^i U_i(\overline{W}_e^i)$$

be greater than or equal to zero. Unlike the ranking of Pareto-efficient allocations, information will generally affect the perceived sign of this expression. The consumer with information $\{\pi_e^*\}$ superior to all others in the economy will be in the enviable position of "knowing" under what allocations its sign is positive. An uninformed consumer, even though he knows he is uninformed, cannot in general evaluate the sign of this expression independent of the beliefs he attributes to informed consumers.

While information would appear to derive positive private value from its assistance in comparing Pareto-inefficient with Pareto-efficient allocations, the absolute size of its private value will depend on the bargaining arrangements among consumers (for example, see Harsanyi and Selton, 1972). Except for the special competitive case, we have little to add. However, we speculate if uninformed consumers tend to overestimate the private value of their endowed resources $(\sum_e \pi_e^i U_i(\overline{W}_e^i) > \sum_e \pi_e^* U_i(\overline{W}_e^i))$, informed consumers may find it in their interest to disclose their superior information to quash the unduly tough bargaining stance of uninformed consumers. Similarly, if $\sum_e \pi_e^i U_i(\overline{W}_e^i) < \sum_e \pi_e^* U_i(\overline{W}_e^i)$, then informed consumers should tend to keep their superior information to themselves. Unfortunately, the consequent ability of an uninformed consumer to infer the undue optimism or pessimism of his own beliefs from the decision of informed consumers to reveal or not reveal their information complicates this speculation.

Although private information appears generally to have some value in a personal market with arbitrary exchange arrangements, this value can fall to zero in a completely personal market with competitive exchange opportunities, even though the endowed allocation is not Pareto efficient. The simplest illustration is furnished by an economy in which all consumers have logarithmic utility functions but are otherwise different. In this instance, competitive equilibrium is characterized by

$$W_e^i = \underbrace{\left(\frac{\pi_e^i - \pi_e}{\pi_e}\right)\left(\frac{W_0^i - W_0}{W_0}\right)W_e}_{\textbf{(WB)}} + \underbrace{\left(\frac{\pi_e^i - \pi_e}{\pi_e}\right)W_e}_{\textbf{(B)}} + \underbrace{\left(\frac{W_0^i - W_0}{W_0}\right)W_e}_{\textbf{(W)}} + W_e$$

with

$$\pi_e \equiv \sum_i \left(\frac{W_0^i}{W_0}\right)\pi_e^i$$

$$W_0 \equiv \frac{\sum_i W_0^i}{I}$$

and

$$W_e \equiv \frac{\sum_i W_e^i}{I}$$

When all consumers have the same wealth and beliefs, then each consumer's state-contingent future wealth is exactly the per capita amount (W_e). With differences in initial wealth, rich consumers $(W_0^i > W_0)$ plan for more future wealth under all states (term **W**); with differences in beliefs, optimistic consumers $(\pi_e^i > \pi_e)$ plan for more wealth in those states (term **B**). The additional joint demand created by both differences in wealth and beliefs is captured by the remaining term (**WB**). Clearly, informed consumers will be most anxious to take on side bets (terms **B** and **WB**) with uninformed consumers. However, the uninformed consumers have an easy way of preventing such unfavorable trades: they can act as if they have the same beliefs as the informed consumers without knowing what these beliefs are. This follows since in a personal competitive market they can calculate the above sharing rule and make their choices satisfy

$$W_e^i = \left(\frac{W_0^i - W_0}{W_0}\right)W_e + W_e$$

what they would have been had all consumers had the same information.[16]

Kihlstrom and Mirman (1975) have derived a similar result for personal competitive markets. They show that private superior information is valueless when a one-to-one correspondence exists between security prices and the beliefs based on superior information. Superior private information then leaks out through the price system. The above example suggests that in complete markets, a necessary and sufficient condition for this leakage is the existence of consensus beliefs $\{\pi_e\}$. As noted elsewhere (Rubinstein 1975), their existence is a necessary condition for security prices to fully reflect all available information. That is, security market

[16] Observe also that even though only a few consumers are sophisticated, their information is fully reflected in security prices.

"efficiency" demands that there exist some set of beliefs that, if commonly held by all consumers, is capable of explaining actual security prices.

Perhaps unfortunately, consensus beliefs do not generally exist. The only examples of which the authors are aware are:

1. all consumers have logarithmic utility, or
2. all consumers have exponential utility, or
3. all consumers are identical except for their beliefs.

This last case we owe to David Ng. It, of course, is already covered by the corollary to Theorem 2.

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