Equilibrium, Price Formation, and the Value of Private Information

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An economy is analyzed in which agents first choose whether to acquire costly information about the return to a risky asset, and then choose demand functions that determine the allocation of assets. It is a well-known paradox that if agents are price-takers and prices are fully revealing, then an equilibrium with costly information acquisition does not exist. It is shown that if the price formation process is modeled explicitly and agents are not price-takers, then it is possible to have an equilibrium with fully revealing prices and costly information acquisition.

Rational expectations models have been used to analyze the allocation of goods in economies in which agents initially possess private and diverse information. [For surveys see Admati (1989), Allen (1985), Grossman (1981), and Jordan and Radner (1982).] In the rational expectations paradigm, traders understand that prices convey information. They condition their expectations upon prices and, thus, the information revealed by prices is used in the formation of demand functions. At the same time, however, agents

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are assumed to be price-takers. This latter assumption is generally justified as representing competitive behavior. The combination of these assumptions leads in some instances to discontinuities and nonexistence of equilibria.

The focus of this article is the nonexistence result identified by Grossman (1976) [see also Grossman and Stiglitz (1980)]. Grossman considers a situation in which agents are price-takers and prices fully reveal the information possessed by the different agents in the economy. In such a scenario, no agent will wish to purchase information since all relevant information is revealed by prices. However, if no one purchases information, then prices reveal nothing and an agent will wish to purchase information (for some range of cost). Therefore, there cannot exist an equilibrium that has fully revealing prices and costly information acquisition.[1]

The most common method of circumventing this difficulty has been to add noise to the system so that prices are not fully revealing. This has been done in various ways: by adding noise traders, by assuming that the aggregate endowment is imperfectly observed, or by considering uncertainty, which has dimension greater than that of price. In these cases private information is not redundant with partially revealing prices. Equilibria then exist in which agents pay to acquire information and the Grossman (1976) result is avoided.

This article takes another approach. Fully revealing prices are maintained, but the price-taking assumption is dropped. It is shown that the costly acquisition of information can arise in equilibria with fully revealing prices. This illustrates that the Grossman result depends critically on agents' price-taking behavior. Thus an important distinction is made. Informationally efficient markets are not in general inconsistent with information acquisition. It is only markets in which all agents are price-takers and prices are fully revealing that make costly information acquisition impossible.

The reason the price-taking assumption is critical to the Grossman paradox is that it permits a definition of equilibrium without an explicit price formation process. A closer look at the Grossman (1976) model clarifies how the absence of price formation leads to his result, and why it is essential to model price formation in order to obtain costly information acquisition with fully revealing prices. Grossman first describes a rational expectations equilibrium given that agents have observed signals, and then discusses the existence of an equilibrium when the acquisition of information is endogenized. In the equilibrium (given that signals have been observed) each agent demands a risk tolerance weighted share of the aggregate endowment, indepen-

[1] Since agents are atomless, this result is not altered by the consideration of mixed strategies.

dent of the price or the signal observed.[2] The strange characteristic of this equilibrium is that each agent is submitting a constant demand function, and yet somehow their signals become incorporated into the price. It is unclear how the price is formed.[3] The price is not measurable with respect to the agents' demand functions, as the price incorporates more information than is present in the demands of the agents. This presents a difficulty for an agent deciding whether to acquire information, since the agent must consider how information acquisition will impact the price. This requires that the process through which agents' demands determine prices be known.

Price formation can be modeled simply by requiring that the equilibrium price (as a function of information) be measurable with respect to agents' demands. This would be consistent with a Walrasian auctioneer story, and would maintain the price-taking assumption. As Beja (1977) points out, however, fully revealing prices are not possible under this measurable rational expectations definition as agents are price-takers.[4] Therefore, it is necessary to drop the price-taking assumption in order to model price formation explicitly and to obtain fully revealing prices.

To do this, a game is examined in which a finite number of agents submit demand functions and the price is determined to clear markets. Agents recognize that their actions influence the price and the resulting allocations. An agent compares the expected utility of the allocation with an additional signal to the expected utility of the allocation without an additional signal, in deciding whether to acquire a signal. Under specific parametric assumptions, equilibria are identified that have fully revealing prices and costly information acquisition. This shows that agents are willing to pay to acquire information that they know will be completely revealed to other agents through the price.

The equilibria provide an interesting notion of the value of information. Since the price is fully revealing, acquiring information does not give an agent an advantage in forming expectations. In fact, agents who acquire information are worse off than those who do not. How-

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[2] This may be verified by substituting Grossman's price equation (14) into his expectations equation (26), and that in turn into his demand function (13) [Grossman (1976)].

[3] Grossman (1976) recognizes this fact in his note 1. He explains that the demands described are only equilibrium demands. The process by which agents come to these demands remains unmodeled. It may be, however, that information is valuable to agents during this process.

[4] The Beja intuition is as follows. A price-taking agent's private information is redundant given a fully revealing price and, therefore, is not used in the formation of demand. However, in that case, the price cannot incorporate any private information. For examples of measurable rational expectations equilibria and further discussion of the existence of fully revealing prices, see Diamond and Verrecchia (1981), Anderson and Sonnenschein (1982), Verrecchia (1982), and McAllister (1989).

ever, they would be even worse off if they did not acquire information, given that other agents expect them to. The value of information to an agent is a function of the beliefs of the other agents concerning the acquisition of information, since these beliefs and the actual distribution of information help to determine equilibrium allocations. In equilibrium, the advantage of acquiring more (or less) information than other agents anticipate is just balanced against the cost of information.

The competitive case is analyzed by letting the number of agents in the economy become arbitrarily large. In specific cases, the results are noncompetitive: a small number of agents acquire information and submit demands different from those of other agents, regardless of the size of the economy.

There are several limitations of the analysis. First, only an example is offered: agents are risk-neutral and the return to the risky asset is exponentially distributed. Second, information comes only in the form of independent and identically distributed signals. This does not permit agents to fine-tune the precision of their information, or to obtain the same information as other agents. [For models in which agents may observe the same information, see Grossman and Stiglitz (1980), Allen (1983), and Caballé (1988).] Third, there is only one risky asset in the model, so agents cannot infer information from things they know about correlated assets [as in Admati (1985) or Caballé and Krishnan (1989)]. Finally, there exist a multiplicity of equilibria for demand submission games. The analysis only provides a "possibility" result. It shows that the Grossman "paradox" depends critically on the price-taking assumption: it is possible to have fully revealing prices and valuable information. The analysis does not indicate whether there always exist equilibria with fully revealing prices or whether these equilibria should be singled out.

The model is related to several previous papers. The case of fully revealing prices is examined as a limit of noisy rational expectations economies (as the noise disappears) by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), among others. In each case, information is valuable when there exists some noise in the economy. It is also true, however, that if the amount of noise in the economy becomes too small, then there no longer exists an equilibrium. Thus, without sufficient noise, the Grossman paradox is not avoided.

An alternative "noisy" approach is taken by Kyle (1989) [see also Kyle (1985a)]. In his model, equilibria are well-defined for arbitrarily small (but nonzero) amounts of noise. Kyle drops the price-taking assumption and looks at an imperfectly competitive market with noise-traders. Informed agents trade increasingly small amounts as the noise vanishes, so the prices do not become fully revealing and the Grossman paradox is avoided. This is in contrast to our work in which prices are fully revealing.[5]

Another approach that overcomes the Grossman paradox is to consider fully revealing prices, but not to permit agents to condition their demands on these prices. In such cases, private information is valuable in forming demand, since the information in the price cannot be exploited. Models of this type include Hellwig (1982), Kyle (1985b), Dubey, Geanakoplos, and Shubik (1987), and Milgrom (1981). In the models of Hellwig and Kyle, trade is repeated, but it is assumed that agents can only condition on past prices; thus, current private information is valuable for choosing demand. The same intuition holds in the work of Dubey et al. and Milgrom. In those models, trade is modeled as games in which agents must choose actions that do not condition on the price.[6] Our contribution is to show that the Grossman result can be overcome even when the price is fully revealing and agents can condition their actions upon the price.

1. A Simple Framework

The framework is similar to that of Grossman (1976), except that agents are risk-neutral and the return to the risky asset is exponentially distributed. We make this change because it provides equilibria with perfectly revealing prices and linear demand functions (which do not exist under the negative exponential–normal formulation).

The economy consists of N traders. There are two traded assets: a risk-free asset with known gross return $R > 0$, and a risky asset with unknown return P, which is exponentially distributed with mean $\bar{P} > 0$. Information concerning the return to the risky asset comes in the form of a signal. A signal is the value of a random variable, y_k, which is jointly distributed with P. The values of y_k and y_j, $k \neq j$, are independently and identically distributed with an exponential distribution having mean P^{-1}. With this information structure, the degree to which an agent is informed depends only on the number of signals observed. Agent i chooses a number of signals, n_i, to observe prior to trading, and incurs a cost $C_i(n_i)$, which is assumed to be nonnegative. The sum of the signals observed by agent i is denoted $\bar{y}_i$. Define n to be the total number of signals observed across all agents, and let y be the vector of all signals.

[5] Gale and Hellwig (1987) look at a variation of the Kyle (1989) model and solve for a fully revealing equilibrium, but do not examine the value of information.

[6] In Milgrom (1981), agents are bidding in a Vickrey auction. Agents condition on whether their bid is at least as high as the kth highest bid of the other agents. In the case of a tie (the important case for determining an optimal bid), this fully reveals the information of the other agents. An agent's own information is then valuable in forming expectations.

Agents know only their own n_i, prior to trade.[7] At time $t = 1$, trade occurs and then, at $t = 2$, the return P is revealed and final wealth is realized. Agent i's wealth available for trade at $t = 1$ is the market value of w_i and e_i, the endowment of the risk-free and risky assets, respectively. The trading prices of the risk-free and risky assets are 1 and p, respectively. After trading is completed, agent i's risk-free and risky asset holdings are denoted by z_i and x_i, respectively. Without loss of generality, the budget constraint holds with equality, and so the demand for the riskless asset z_i is implicitly determined by the demand for the risky asset x_i:

$$z_i(x_i) = w_i + pe_i - px_i - C_i(n_i).$$

After P is revealed, agent i's wealth is $Rz_i(x_i) + Px_i$.

Since agents are risk-neutral, the initial endowments are Pareto efficient and there are no gains from trade. To generate trade, we add agents who obtain utility from consumption of the risk-free asset at $t = 1$. These agents are called "sellers," since they desire to trade their endowment of one unit of the risky asset for units of the risk-free asset. The first N_B agents are referred to as "buyers" ($i \leq N_B$), and the remaining N_S agents are the sellers ($i > N_B$), where $N_B + N_S = N$. It is assumed that $N \geq 3$ and, to keep the actions of the sellers as simple as possible, that $N_S = N_B - 1$.[8] Sellers do not care about the return to the risky asset and thus do not acquire information. They are modeled to complete the equilibrium and permit a welfare analysis.

The demand submission game. The set of possible trading actions for each agent is the set of all (demand) functions mapping the price of the risky asset into units of the risky asset. Agents simultaneously submit trading functions, denoted $D_i \in D$, $i \in \{1, \dots, N\}$. The allocation received by agent i, (x_i, z_i), is determined according to

(i) $(x_i, z_i) = (e_i, w_i - C_i(n_i))$,

[7] Alternatively, we could examine situations in which information acquisition is a public event. That is, where $(n_1, \dots, n_N)$ is common knowledge prior to trade, but agents only observe the values of their own signals. Under such an assumption there is an equilibrium with no information acquisition. [See Jackson (1988). No information acquisition is the unique subgame perfect equilibrium, while there are additional Nash equilibria. With private information, the Nash and subgame perfect equilibria coincide.] This follows since if an agent were to deviate and acquire information, then the other agents could react and take advantage of the fully revealing price. In contrast, if n_i is known only to i, then other agents cannot react to such a deviation and so $n_i = 0$, for each i is no longer an equilibrium (for some cost structures).

[8] This can be generalized to situations in which $N_S \geq N_B$ (even a continuum of atomless sellers), by assuming that short sales are not possible. This complicates the analysis without changing the results.

$$\text{if}\left\{p \,\middle|\, \sum_i D_i(p) = \sum_i e_i,\ p \geq 0\right\} = \emptyset$$

and

(ii) $(x_i, z_i) = (D_i(p^*), w_i - p^*[D_i(p^*) - e_i] - C_i(n_i))$, otherwise.

Here p^* is chosen from $\{p|\Sigma_i D_i(p) = \Sigma_i e_i,\ p \geq 0\}$ according to any deterministic rule. Thus, if agents submit demands for which there is no market-clearing price, then (i) applies and there is no trade. If agents submit demands for which there is at least one market-clearing price, then (ii) applies, a market-clearing price is chosen, and allocations are determined according to the demands submitted. The payoff to each agent is the utility obtained from the resulting allocation.

A (pure) strategy for agent i is a choice of n_i and a map from the signals observed by i to a trading action. An *equilibrium* is defined to be a (Bayesian) Nash equilibrium of the game. In other words, an equilibrium is a set of strategies such that, for each i: (a) agent i's choice of a trading action maximizes i's expected utility, given the signals observed by i and the strategies of the other agents;[9] and (b) n_i maximizes i's ex ante expected utility, given the strategies of the other agents and i's map from signals to a trading action.

2. Equilibria with Fully Revealing Prices and Valuable Information

We first look at the choice of trading actions, given that signal acquisition has already taken place. Once we have examined this part of the game, we can step back and analyze the decision concerning how much information to acquire prior to trade. The following proposition is partially derived from the analysis of a share auction in Wilson (1979).

Proposition 1. *If n signals have been acquired by buyers (and none by sellers), then the trading actions*

$$D_i(p) = e_i + 1 - 2Rp\left(\frac{\bar{y}_i\bar{P} + N_B^{-1}}{\bar{P}(n + 1)}\right),$$

[9] Information can be inferred from the market-clearing restriction implicit in the structure of the game. For example, consider an economy with two agents each having one signal. Suppose $D_2(y_2) = y_2 - p$. Then agent 1 can deduce that $y_2 = e_1 + e_2 - x_1 + p^*$.

for buyers, and $D_i(p) = 0$, for sellers, form an equilibrium of the trading game. The fully revealing market-clearing price is $p^* = E[P \mid y]/2R$.

The proof of Proposition 1 is presented in the Appendix. The method of proof is straightforward. Taking the actions of the other agents as given, an agent can infer the information of the other agents from the market-clearing price and his or her demand. This information is used in calculating expected utility. It is then verified that the above actions maximize expected utility, taking into account the agent's influence on the price.

Since agents are not price-takers, they understand that it is the *equilibrium* price that is fully revealing. In forming expectations, given the strategies of the other agents, agents view the price as a function of their own demand, and hence understand that they must make use of their own signals as well as the price. If an agent does not use the value of his or her signals in forming a demand, then the price will only reveal the value of the signals of the other agents.

Having dropped the price-taking assumption, the nonexistence result of Beja (1977) no longer holds. The equilibrium price is fully revealing, and yet the price is explicitly determined by the actions of the agents. The value of information when prices are fully revealing can now be analyzed.

The following proposition shows that there exist equilibria with costly signal acquisition and fully revealing prices.

Proposition 2. *Consider the game in which signal acquisition is private. Suppose that C_i is increasing and convex[10] for each buyer. Signal acquisition choices, which form an equilibrium together with the trading actions described in Proposition 1, are characterized by $n_i = 0$, for each seller, and, for each buyer,*

$$\frac{\bar{P}}{2}\left(\frac{n - n_i + 2(N_B - 1)/N_B}{(n+1)(n+2)(n+3)}\right) \leq R[C_i(n_i + 1) - C_i(n_i)],$$

and, when $n_i \geq 1$,

$$\frac{\bar{P}}{2}\left(\frac{n - n_i + 2(N_B - 1)/N_B}{(n+2)(n+1)^2}\right) \geq R[C_i(n_i) - C_i(n_i - 1)].$$

[10] The assumption of convex costs says that the marginal cost of information is nondecreasing. This is stronger than the condition we need. It may be that costs are initially concave: for example, there may be a fixed cost to learning. The proposition can be extended to cover such situations, in which costs are only convex after some point.

The expressions on the left-hand side in Proposition 2 represent the change in expected utility for an agent (not considering acquisition costs) due to acquiring one more or fewer signals, respectively. The expressions on the right-hand side indicate the corresponding changes in the information acquisition cost. The proof of Proposition 2 appears in the Appendix. The proof involves making an expected utility calculation as a function of the number of signals an agent acquires, holding constant the number of signals the other agents anticipate that the agent will acquire. Given other agents' anticipations, the agent expects to receive a larger allocation if he or she acquires more signals. This benefit is weighed against the cost of acquiring signals. The equilibrium conditions identify signal acquisition choices, such that given the anticipations of other agents, each agent would not gain from acquiring any more (or fewer) signals.

It is interesting to note that agents would, in fact, prefer not to acquire signals, *if* they could demonstrate to other agents that they had not acquired any signals. However, since signal purchase is private, an agent cannot always convince the other agents that he or she will not acquire information. If other agents were to believe that an agent was not going to acquire information, then the agent might benefit from acquiring it.

For a given economy, there may exist multiple signal acquisition choices that satisfy the conditions in Proposition 2. With additional assumptions on cost curves, more can be said about the distribution of information, as in the following three remarks.

Examining the change in expected utility from acquiring an additional signal, not considering cost, it is clear that the value of a signal decreases rapidly in n (at a rate proportional to n^{-2}). This is because the value of a signal is related to the marginal reduction in uncertainty that it provides to the economy. As the total amount of information acquired in the economy goes up, acquiring information that other agents do not anticipate provides less of an advantage. Furthermore, the relative gain from acquiring unanticipated signals is larger for less-informed agents (smaller n_i). This will tend to even out the number of signals acquired by different agents.

Remark 1. *If the buyers in Proposition 2 have identical*[11] *cost curves, then in an equilibrium of the type described in Proposition 2,* $|n_i - n_j| \leq 1$*, for any buyers i and j.*

Remark 1 results from the facts that agents are initially identical and that the gain from acquiring an unanticipated signal is propor-

[11] The remark is true for sufficiently similar cost functions. In other words, there exists ϵ such that if $\sup_{i,j,n}|C_i(n) - C_j(n)| \leq \epsilon$, then the remark holds. The same is true of Remarks 2 and 3.

tional to how informed an agent is anticipated to be. Remark 1 can be verified by checking that the conditions in Proposition 2 cannot be simultaneously satisfied by n_i and n_j, such that $n_i \geq n_j + 2$, when costs are identical and convex.

Remark 2. *If buyers have identical linear signal costs, then all equilibria of the type described in Proposition 2 have the same total number of signals.*

Remark 2 shows that, with identical linear cost structures, the equilibria of the type in Proposition 2 are unique, up to a permutation of the buyers. For instance, consider an equilibrium satisfying the inequalities of Proposition 2, where $n = 3$ and $N_B = 4$. Remark 2 states that all equilibria of the type in Proposition 2 must have $n = 3$. There are four such equilibria: one with each of the buyers not acquiring a signal. Remark 2 is verified by noting that if the inequalities in Proposition 2 are satisfied for n and n_i, then they cannot be satisfied by $\bar{n} > n$ and $\bar{n}^i > n^i$.[12]

Remark 3. *Consider a sequence of economies for which buyers face identical convex information cost structures. There exists a number $\bar{N}$ of agents, such that if there is an equilibrium of the type described in Proposition 2, with a total number of signals n (for the economy $\bar{N}$), then n is an equilibrium number of signals for all economies with $N > \bar{N}$.*

In equilibria of the type described in Remark 3, $n \leq \bar{N}_B$, and as the economy becomes larger, the same fixed number n of buyers acquires one signal each, while other agents do not acquire signals. For sufficiently large economies, the influence on the equilibrium offered by a signal on the margin is related to the marginal reduction in uncertainty it offers, not to the size of the economy. To verify the remark, notice that for large N_B the dependence of the inequalities in Proposition 2 on N_B becomes negligible [i.e., $(N_B - 1)/N_B \to 1$].

Note that Remark 3 depends on the fact that agents can acquire a number of "unit" signals. If, instead, agents could acquire a single signal with a variable degree of precision, then as the economy became large, each agent would buy a signal of decreasing accuracy (provided there is no fixed cost). This would provide a symmetric equilibrium. However, if there is any fixed cost to acquiring a signal, then a small number of agents would purchase signals, as in Remark 3.

[12] If $\bar{n} > n$, then it follows that at least one agent i is gathering more signals in the equilibrium leading to $\bar{n}$ than in the equilibrium leading to n.

Information acquisition and welfare issues. The notion of ex ante Pareto efficiency is used as the measure of welfare in order to take into account the signal acquisition decisions. Equilibria in which information is acquired at a positive cost are ex ante Pareto inefficient, since resources are lost in the acquisition of information, while there is no change in the aggregate supply of the risky asset. In this simple model, information is only important in determining the distribution of wealth among identical risk-neutral agents; therefore, it is not surprising that costly information acquisition is inefficient. We should be careful to point out that this inefficiency does not imply that the fully revealing prices are inferior to partially revealing prices. In our setting, since prices reveal information, only a few agents end up acquiring information. With partially revealing prices, it might be the case that more agents spend resources acquiring information, and that in total more resources are expended. This would agree with the intuition derived from Verrecchia (1982).

Notice that there are other settings in which costly signal acquisition might be efficient. For instance, information could influence the production process, either directly or through investment. In that case, greater information acquisition might lead to an increase in $\bar{P}$. Efficiency would depend upon the extent to which n affects $\bar{P}$ and on the cost structure. Alternatively, agents might have utility that depends on each good consumed (rather than on wealth). Given a difference in tastes, information could improve the allocation of the goods between agents. Finally, some agents may be "endowed" with information that they could sell to less-informed agents. [For discussions along these lines see Admati and Pfleiderer (1986) and Shubik (1976).] In such a scenario, the efficiency issues are no longer so clear.

3. Concluding Remarks

A simple example has been analyzed that demonstrates that if price formation is explicit and agents influence prices, then it is possible to have fully revealing prices and a value to information in a static setting. This isolates a value of information that differs from the value of information that arises when prices are not fully revealing. The value of information to an agent derives from the perceptions of other agents concerning the amount of information that the agent has acquired (rather than an advantage it provides the agent in forming expectations). Equilibrium conditions balance the cost of signals against the advantage of acquiring more (or fewer) signals than other agents anticipate.

The "competitive" market is analyzed without making an artificial (price-taking) assumption, simply by letting the number of agents become arbitrarily large. This results in an interesting outcome when agents have sufficiently similar increasing and convex costs to acquiring information: A fixed number of agents acquire information as the economy grows, while the remaining agents "free-ride" on this information.

There are many issues associated with the approach taken here that were not addressed. First, this analysis does not suggest that equilibria with fully revealing prices exist generally. It is easy to show the existence of a nonrevealing equilibrium: let each agent demand their endowment for all prices. The existence of such a trivial equilibrium makes the question of the existence of a nontrivial equilibrium all the more difficult, since fixed-point theory can no longer be directly applied. Second, the multiplicity of equilibria for demand submission games was not discussed.[13] Insomuch as our objective was to show that it is possible to have a value to information with fully revealing prices, the approach taken here is useful. The multiplicity of equilibria for the demand submission game can, more generally, be a problem, since the behavior associated with the different equilibria can vary dramatically. This appears to be due (at least in part) to the extreme size of the action space in the demand submission game. It is not necessarily the strategic approach that is to blame, but rather the description of the game. A demand submission game was analyzed because it is similar to the rational expectations paradigm, thus allowing a direct comparison to the Grossman paradox. For other applications, it would be appropriate to use a model that pays closer attention to the institutional detail of exchange.

Appendix

The density of a signal y_k conditional on P and the unconditional density of P are

$$f(y_k|P) = Pe^{-Py_k}I_{[0,\infty)}(y_k) \quad \text{and} \quad g(P) = \bar{P}^{-1}e^{-P\bar{P}^{-1}}\, I_{[0,\infty)}(P), \tag{A1}$$

respectively, where I is an indicator function and $\bar{P} > 0$. An application of Bayes's rule and a bit of calculation show that the expectation of P conditional on y is

$$E[P|y] = \frac{(n+1)\bar{P}}{\Sigma_{k=1}^{n} y_k\bar{P} + 1}. \tag{A2}$$

[13] This issue (as well as the existence issue) may be easier to address for equilibrium concepts other than Nash equilibrium. It is, of course, necessary to make sure that the solution is reasonable in this setting.

Proof of Proposition 1

Proposition 1 is the special case of the statement below in which $k = 0$. This more general statement is useful in the proof of Proposition 2.

Proposition 1*. *Suppose that there exists an agent i and integer $n \geq 0$, such that the strategies of agents $j \neq i$ have trading actions described by*

$$D_j(p) = e_j + 1 - 2Rp\left(\frac{\bar{y}_j\bar{P} + N_B^{-1}}{\bar{P}(n+1)}\right), \tag{A3}$$

for some integer $n \geq 0$ if j is a buyer and $D_j(p) = 0$ if j is a seller. If overall strategies involve a total of $n + k$ signals being acquired by buyers, then the trading action

$$D_i(p) = e_i + \left(\frac{n+k+1}{n+1}\right)\left[1 - 2Rp\left(\frac{\bar{y}_i\bar{P} + N_B^{-1}}{\bar{P}(n+k+1)}\right)\right] \tag{A4}$$

is a best response for buyer i, while $D_i(p) = 0$ is a best response for a seller i. The corresponding market-clearing price fully reveals the information of the buyers and is $p^* = E[P|y_B]/2R$, where y_B is the vector of signals observed by buyers.

Proof. The expression for p^* follows directly from market-clearing.

Consider the situation in which i is a buyer. It is shown that if agents $j \neq i$ submit demands according to Proposition 1*, then the response (A4) maximizes agent i's expected utility, conditional upon all of the signals observed by buyers. This implies that expected utility conditional upon the agent's signals, the price, market-clearing, and the other agents' demands (as specified in Proposition 1*) is also maximized.

Under the market-clearing restriction, the expected utility (conditional on y_B) of agent i given demand x_i is

$$E[P|y_B]x_i + Rw_i - Rp(x_i)[x_i - e_i], \tag{A5}$$

where $p(x_i)$ is the market-clearing price as it depends on the choice of x_i, given the strategies in (A3). The market-clearing condition is $\Sigma e_i = x_i + \Sigma_{j \neq i} x_j$. Substituting for x_j, $j \neq i$, we solve for the market-clearing price as a function of x_i:

$$p(x_i) = \frac{(x_i - e_i)}{2R}\left(\frac{\bar{y}_{B-i}\bar{P} + (N_B - 1)/N_B}{\bar{P}(n+1)}\right)^{-1}, \tag{A6}$$

where $\bar{y}_{B-i}$ is the sum of the signals of the buyers other than i. The necessary first-order conditions for maximizing (A5) are

$$E[P|y_B] - Rp(x_i) - Rp'(x_i)[x_i - e_i] = 0. \tag{A7}$$

Substituting from (A2) and (A6), (A7) is rewritten as

$$x_i = e_i + \frac{n + k + 1}{n + 1}\left(\frac{\bar{y}_{B-i}\bar{P} + (N_B - 1)/N_B}{\bar{y}_i\bar{P} + 1}\right).$$

This is exactly the allocation that results given the demands in Proposition 1*.

To verify that the first-order conditions are indeed sufficient, it is shown that the second derivative of expected utility is less than zero. Since $p''(x_i) = 0$, the second derivative of (A5) is simply $-2Rp'(x_i)$. Since $\Sigma_{k\in S_i} y_k \geq 0$, it follows from (A6) that this second derivative is negative.

Next, it is shown that $D_i(p) = 0$ maximizes a seller's expected utility, conditional upon y. This implies that expected utility, conditional upon the agent's signals, the price, market-clearing, and the other agents' demands (as specified in Proposition 1*), is also maximized.

The seller's expected utility (conditional upon y) is

$$w_i + p(x_i)[x_i - e_i] - C_i(n_i), \tag{A8}$$

where $p(x_i)$ is the market-clearing price. Market-clearing implies that

$$0 = x_i - 1 - (N_S - 1) + N_B - 2Rp\left(\frac{\bar{y}_B\bar{P} + 1}{\bar{P}(n + 1)}\right),$$

where $\bar{y}_B$ is the sum of all buyers' signals. The price is solved as a function of agent i's demand:

$$p(x_i) = (x_i + 1)\left[2R\left(\frac{\bar{y}_B\bar{P} + 1}{\bar{P}(n + 1)}\right)\right]^{-1}. \tag{A9}$$

The first-order necessary conditions for maximizing (A8) are $-p(x_i) - p'(x_i)[x_i - e_i] = 0$, or, substituting from (A9),

$$(x_i + 1 + x_i - e_i)\left[2R\left(\frac{\bar{y}_B\bar{P} + 1}{\bar{P}(n + 1)}\right)\right]^{-1} = 0.$$

Since $e_i = 1$, these are satisfied by $x_i = 0$. The second derivative, $-2p'(x_i) - p''(x_i)[x_i - e_i]$, is always negative, since $p''(x_i) = 0$ and $p'(x_i) > 0$. The first-order necessary conditions are, therefore, also sufficient. ■

Proof of Proposition 2

Consider the situation in which agents $j \neq i$ anticipate that agent i will acquire n_i signals and that a total of n signals are to be acquired.

Let agent i deviate by acquiring $n_i + k$ signals (where it is possible that $k < 0$), so that a total of $n + k$ signals are acquired. Define $V_i(n_i, n, k)$ to be the ex ante expected utility of agent i in this situation, given that the agents use trading strategies described by Proposition 1* (where i's strategy is a best response).

For buyers this expected utility is as follows. Let $x_i^* = D_i(p^*)$, where D_i is described by (A4):

$$V_i(n_i, n, k) = E_0[Px^* - Rp^*(x_i^* - e_i)] + Rw_i - RC_i(n_i + k),$$

where E_0 indicates expectation before the signals are observed. Substitution from Proposition 1* and calculation of the expectation shows that

$$V_i(n_i, n, k) = \left(\frac{n + k + 1}{n + k + 2}\right)\frac{\bar{P}}{2}\left[\frac{n - n_i + 2(N_B - 1)/N_B}{n + 1}\right] + \bar{P}e_i + Rw_i - RC_i(n_i + k). \quad \text{(A10)}$$

For sellers it is easily verified that

$$V_i(n_i, n, k) = w_i + \frac{\bar{P}}{2} - C_i(n_i + k). \quad \text{(A11)}$$

To prove Proposition 2, the choice of n_i is examined, given that the trading actions are described by Proposition 1*. Consider a deviation by agent i of k signals from the situation in which n signals were to be acquired in total and i was to acquire n_i signals. An equilibrium will be a situation in which there are no profitable deviations. Equilibria are characterized by

$$V_i(n_i, n, 0) \geq V_i(n_i, n, k) \qquad \text{for all } k \geq -n_i, \quad \text{(A12)}$$

for each i. It follows directly from (A11) that for sellers this requires that $n_i = 0$. For the buyers, since C_i is convex and nonnegative, (A12) is equivalent to $V_i(n_i, n, 0) \geq V_i(n_i, n, k)$, for $k = 1$ and $k = -1$ (when $n_i \geq 1$). Direct calculation of these expressions, using (A10), establishes Proposition 2. ∎

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