The Random Walk Hypothesis and the Recent Behaviour of Equity Prices in Britain

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INTRODUCTION

Although the study of the behaviour of stock market prices has a long history, until quite recently it has been comparatively neglected as a field of academic research. Furthermore, most of the empirical work has utilized American data. There have been two main approaches in recent studies. The first has concentrated on trying to explain differences in the performance of shares by relating price to other variables such as dividends or earnings. Representative of this approach is the extensive work done by the Chicago school, and the more modest contributions made in Britain by writers such as Fisher,[1] Morgan and Taylor.[2] The second main approach has consisted of attempts to analyse the underlying statistical structure of share price movements. This paper is concerned with one of the most illustrious hypotheses examined under the second approach, namely the random walk hypothesis.[3] This hypothesis postulates that the *changes* in share prices are independent, and hence produce a random walk in price levels. Over time, however, the relative frequency of outcomes is stable. An elaboration of the hypothesis is given in the next section of this article. Recent writers have analysed not only share price movements but also other speculative price series such as those of commodity prices. Though no *universal* agreement regarding the validity of the random walk hypothesis as applied to share prices has resulted from these studies, there is a surprising dearth of instances in which it has been refuted unambiguously. Thus, as Rayner has put it, the random walk hypothesis is "in the ascendant".[4]

A danger in every empirical study is that the raw material of historical information may become an end in itself, analysis being neglected. This usually has the effect of severely limiting the implications of a study, which may spread no further than the data upon which research was

[1] G. R. Fisher, "Some Factors Influencing Share Prices", *Economic Journal*, vol. 71 (1961), pp. 121–41.

[2] E. V. Morgan and C. Taylor, "The Relationship between Size of Joint Stock Companies and the Yield of their Shares", *Economica*, vol. XXIV (1957), pp. 116–27.

[3] Many of the leading contributions have been brought together in P. H. Cootner (ed.), *The Random Character of Stock Market Prices*, Cambridge, Mass., 1964. Cootner's editorial commentaries provide a useful survey. A summary of this volume is available in I. M. D. Little and A. C. Rayner, *Higgledy Piggledy Growth Again*, Oxford, 1966, ch. 3.

[4] In Little and Rayner, *ibid.*, p. 103.

directed, and, in addition, may make the study mercilessly hard to read. In our study we have tried to avoid this pitfall by making wider methodological points which we feel are pertinent to all studies of this broad type.

On the empirical side alone, we offer what we believe to be the first study of the Random Walk Hypothesis on a random sample of British share price series. On the methodological side, we have taken pains to elaborate our viewpoints in economic terms: an aspect which has been rather neglected in the furore of statistical activity. This has led us to a detailed consideration of what constitutes a meaningful test of the hypothesis, a point which largely concerns the type of data and sample used. On the statistical side, we do not profess to have anything new to offer, though some interesting problems arose when it came to the treatment of identical adjacent observations (situations of "no change"). Finally, on the analytical side, we have again chosen to emphasize the economics of the situation, attempting to counterbalance the dependence of much of the statistical literature on physical analogies which may not be of much meaning to an economist.

The Random Walk Hypothesis

One of the basic economic models of the stock market is that which leads to the random walk hypothesis. We may distinguish two types of random walk models: the *pure* random walk model; and the *mixed* random walk model.

The pure random walk model is the earlier concept historically. It postulates that the market for a share is a perfect one in which all market operators possess full information. If follows that all investors will be receiving a profit just sufficient to keep them in the market. There is an occasional proviso in perfect competition models that information need only be complete in so far as it relates to the market. However, as so many non-economic events ranging from the weather to the war in Vietnam can affect share prices, this minor restriction is scarcely necessary in this case. We postulate, in addition, that information is received randomly by the market operators. There are several obvious mechanisms by which one could relate this random information to price changes; but one of the most concrete and convenient adopts an argument of Miller and Modigliani.[1] They consider prices to be discounted cash flows. The random acquisition of new information may affect price through several factors, such as a new evaluation of the return in a specific period, say by a change in the tax laws operative after a certain date; or through an alteration in an individual's assessment of the risks involved, and hence his subjective rate of discount; or

[1] M. H. Miller and F. Modigliani, "Dividend Policy, Growth and the Valuation of Shares", reprinted in S. H. Archer and C. A. D'Ambrosio (eds.), *The Theory of Business Finance: A Book of Readings*.

by a combination of both factors. It is through such a mechanism that the random acquisition of information could produce random price changes, this in turn producing a random walk in price levels.

The mixed random walk model is a variant suggested by Cootner.[1] He divides market operators into two classes, amateurs and experts. The amateurs are relatively poorly informed about market conditions. However, other factors such as shifts in income distribution and income levels among shareholders with different evaluations of riskiness still tend to affect their actions randomly. In the short run these amateurs will produce a random walk in prices, possibly by the mechanism suggested in the pure random walk model. The class of experts are, by contrast, far more knowledgeable. They will enter the market only when the random walk produced by the amateurs takes the price sufficiently far from what they consider to be the correct market price so as to compensate them for the opportunity cost involved in these market operations. This behaviour will produce the medium-run phenomenon of a random walk with barriers: price will tend to rebound from the upper and lower barriers in a non-random fashion as experts enter the market. Finally, in the long run, the sounder views of the experts will prevail, and produce a random walk. Though we do not necessarily support this model of Cootner, our study would probably relate to Cootner's short period in which amateurs predominate and cause a short-run random walk.

Now let us turn to the technical implications. A good clear account of the mathematics and statistical methodology is to be found in an article by Sprenkle.[2] Here we will put the matter very simply. We have as our basic data a series of observations on prices, represented by P_ts ($t=1,\ldots, 52$). Taking first differences gives the sequence P_2-P_1, $P_3-P_2,\ldots, P_{52}-P_{51}$, or more simply, ΔP_1, $P\Delta_2,\ldots \Delta P_{51}$. If the random walk hypothesis holds, the sequence of ΔP_ts ($t=1,\ldots, 51$) will be random. Statistically speaking, the best we can say is that we accept the random walk hypothesis if we cannot show this sequence to be non-random.

There has been some controversy in the literature as to what is the underlying probability distribution of price changes. The most popular current view is that it is lognormal. This view is based, first, on the observation that prices are bounded from below, but unbounded from above—a factor likely to be important in the long run. It is backed up by two further considerations. The first, suggested by Osborne,[3] is the so-called Weber–Fechner "law" of psychology, which postulates that

[1] P. H. Cootner, "Stock Prices: Random vs. Systematic Changes", *Industrial Management Review*, vol. 3 (1962), pp. 24–5, reprinted in Cootner (ed.), *op. cit.*, pp. 231 ff.

[2] C. M. Sprenkle, "Warrant Prices as Indicators of Expectations and Preferences", *Yale Economic Essays*, vol. 1 (1961), pp. 179–231, reprinted in Cootner (ed.), *op. cit.*, pp. 412 ff.

[3] M. Osborne, "Brownian Motion in the Stock Market", *Operations Research*, vol. 7 (1959), pp. 145–73, reprinted in Cootner (ed.,), *op. cit.*, pp. 100 ff.

while *levels* of sensation are not measurable, *changes* in level are measurable. Human sensibilities are capable of evaluating (and, if need be, equating) proportional changes, rather than absolute levels; and the smallest proportional change they can discriminate can be detected at all levels. In economic terms, the proposition is equivalent to the assumption that market operators are interested in proportional changes in share prices, and not simply in their absolute value. This latter justification is less metaphysical and more acceptable to an economist.

However, there have been some dissenters from this view, notably B. Mandelbrot. He has argued strongly for the application of Paretian distributions to economic phenomena[1]; in particular he suggests that they are more suitable for testing the random walk hypothesis. As noted below, we have avoided this controversy.

METHODOLOGY OF STUDY

Though we have been provided with a useful model, like most economic models it is lacking in empirical content. Many questions are left unanswered, such as what is the market we have discussed so glibly, or what do we mean in empirical terms by short-, medium- and long-period phenomena?

In few previous studies do we find the answers to such questions. Indeed, as Roberts has suggested,[2] it is important to know just how general we may consider the random walk hypothesis to be as an *empirical* proposition.

Concerning the type of data used, the literature offers us a rich variety: indices; averages; end-week daily closing prices; end-month daily closing prices; and many others. Although there has been a tendency for more recent writers to use time series for individual share prices,[3] the use of indices is still popular.[4] This use of indices is customarily justified by the statement that the writer is concerned more with presenting a method of analysis than with carrying out a detailed empirical investigation. However, a time for redressing the balance must come.

The use of indices is probably the most obvious weakness in many studies of the random walk hypothesis. In economic terms there seems little justification for using an index, as it is no more than a measure of general market trends; and even as such, it may be misleading. By

[1] See, for example, B. Mandelbrot, "The Variation of Certain Speculative Prices", *Journal of Business*, vol. 36 (1963), pp. 394–419, reprinted in Cootner (ed.), *op. cit.*, pp. 307 ff.

[2] H. V. Roberts, "Stock-Market 'Patterns' and Financial Analysis: Methodology Suggestions", *Journal of Finance*, vol. 14 (1959), pp. 1–10, reprinted in Cootner (ed.), *op. cit.*, pp. 7 ff.

[3] Cootner, *op. cit.*, reprinted in Cootner (ed.), *op. cit.*, pp. 231 ff.; A. B. Moore, "Some Characteristics of Changes in Common Stock Prices", abstract of doctoral dissertation, in Cootner (ed.), *ibid.*, pp. 139–61.

[4] E.g. M. Dryden, "Short-term Forecasting of Share Prices: An Information Theory Approach", *Scottish Journal of Political Economy*, Nov. 1968, pp. 227 ff. Dryden is at present engaged on more disaggregated work on British share prices.

contrast, the use of a time series of the price of a single share ensures that one is examining an understandable and clearly defined market. This procedure seems a satisfactory interpretation of the term "market" as idealized in the random walk hypothesis. There are also more technical reasons for preferring individual share price series to an index series. An index may give a completely false impression of the extent of price fluctuations in individual markets. If, in any time period, only one share in an index of 50 shows variation, the averaging will ensure that this variability (on a reduced scale) will be imputed to the whole market. This is important in a study of the random walk hypothesis, for it may conceal the frequency of "no change" observations at the level of the individual market. In our study, we included a well-known index, the *Financial Times* (*FT*) Industrial Ordinary index for the sake of comparison. This index is a geometric index of 30 leading shares in industrial markets. The base year (for which the index is 100) is 1935, and at present 400 is the "magic figure" for this index.

The use of various types of averages is less common now, but has been adopted in major studies in the past.[1] As Working has shown, the averaging of a random chain of figures will produce an auto-correlated series.[2] As a result, such series have been avoided in all subsequent tests of the random walk hypothesis.

The use of daily closing prices at intervals wider than one day is still common.[3] The economic justification for this practice is rather hard to find. Its use involves an unnecessary neglect of easily available information, and furthermore may ignore inter-observation data of fundamental importance. It seems to us that the most natural interpretation of the random walk model is to consider every single market operation.

It is perhaps worthwhile illustrating the types of reactions which daily observations can highlight. These stem from the fact that the market is not really perfect. What often happens is that a piece of information appears on day 1 and causes the price of stock x to rise. This piece of news is reported in the financial press the following day, and two results are likely. The first is that the price of the stock rises again because the information has spread to a further section of the market and new buying has taken place by those newly acquiring the information. The second possible development on day 2 is that there is profit-taking as a result of the spreading of the news of the rise in price on day 1. This often causes the price of the stock to fall below that recorded on day 1. Now these types of price movements are not likely to be reflected if weekly or monthly observations are taken. In terms of

[1] E.g. A. Cowles and H. E. Jones, "Some A Posteriori Probability Considerations of Stock Market Action", *Econometrica*, vol. 5 (1937), pp. 280–94; M. G. Kendall, "The Analysis of Economic Time Series, Part 1", *Journal of the Royal Statistical Society*, vol. 96 (1953), pp. 11–25, reprinted in Cootner (ed.), *op. cit.*, pp. 85 ff.

[2] H. Working, "Note on the Correlation of First Differences of Averages in a Random Chain", *Econometrica*, vol. 28 (1960), pp. 160–3, reprinted in Cootner (ed.), *op. cit.*, pp. 129 ff.

[3] See, e.g., Moore, *op. cit.*

Cootner's model, the views of both professionals and non-professionals are necessary before "average opinion" can be ascertained. The views of the non-professionals generally appear after those of the professionals.

Unfortunately, the ideal procedure, that of considering every transaction, cannot be performed with British data, and the closest we can get to it is to take daily closing prices. From some points of view, even this may be unsatisfactory. Alexander, for example, in his study of filters found that his initial results were affected materially by neglecting the within-day fluctuations of share prices.[1] In our own study the limitations of British data compelled us to use daily closing prices on the London Stock Exchange, though we cannot pretend that this is altogether satisfactory.

One of the consequences of the use of daily prices is that an institutional cycle might be introduced as there is no trading at week-ends. In terms of Cootner's model, one might argue that the two-day gap will mean that a good deal of relevant news will have had time to spread to amateurs and professionals alike. It might be thought that the two-day gap would give amateurs a chance to catch up with the professionals. Where amateur opinion is reinforcing professional opinion, on Monday morning the full force of their collective information will be felt at the same time. Thus Monday's dealings might be somewhat different in nature from those on other days. Inspection of the graphs of price series (both in the form of levels and changes) of the shares considered in the sample (described below) suggested that price movements on Mondays are not especially different in amplitude and direction from those on other days. In our opinion this finding is not unexpected. It is likely that professionals will always be slightly better informed than amateurs, i.e. all investors will never really be dealing on exactly the same basis. Also, it must be remembered that our data refer to *closing* prices, which will be established after the first effects of the information appearing on the Monday have been felt. And, of course, professionals as usual will be receiving such information before amateurs. Finally, on *a priori* grounds the possibility of an institutional cycle seems unlikely. If price behaviour ever became predictable in this manner, market operators would tend to level out this phenomenon in their joint efforts to profit by this regular cyclical movement.

It is worth emphasizing that the random walk hypothesis does not tell us that one method is better than the other. It is for the individual investigator to put the hypothesis into what he considers to be a significant empirical context. An important consequence of using both individual share price series *and* daily prices is that we minimize the risk of discovering a greater apparent variability in prices than actually

[1] S. Alexander, "Price Movements in Speculative Markets: Trends or Random Walks", *Industrial Management Review*, vol. 2 (1961), pp. 7–26, reprinted in Cootner (ed.), *op. cit.*, pp. 199 ff; and "Price Movements in Speculative Markets Trends or Random Walks, No. 2", in Cootner (ed.), *op. cit.*, pp. 338 ff.

exists. In the case of economic time series in particular it seems likely that the greater the interval between observations, the greater the probability (barring seasonal and "institutional" cycles) of successive observations being different. Wide observation intervals may completely conceal the phenomenon of "no change".

An important factor which might affect price variability and which is relevant in this context is the price "weight" of a share. It is known that dealers in stocks and shares prefer to deal in easy fractions of a shilling, probably the most popular being one-eighth (i.e. 1½d.). The high frequency of dealings in such fractions tends to produce the "lattice-effect" noted by Osborne.[1] Accordingly, it is much easier to record very small percentage changes in shares priced 80/- or so than in those priced 1/- or so. There would seem to be a stronger possibility of an "inertia" effect with shares priced in the latter range.

The time series chosen has often been of considerable length, though the early study of Cowles and Jones[2] used some rather short series, as has the recent study of Niederhoffer and Osborne.[3] Provided the small-sample properties of a statistical test have been worked out, there is nothing invalid in applying it to short series. There may, however, be a loss of power. In our study we used 52 observations for each share considered, and also for the *FT* index. The period covered is from October 28, 1968 to January 10, 1969: i.e. exactly five "accounts", including the "long" account over Christmas. A time period of this length is, in the first place, interesting from an economic point of view. It may be thought by some to be representative of Cootner's short-period. But probably it is more interesting as the type of time horizon on which a short-term speculator might operate. As is well known, the institutional arrangements of the Stock Exchange may promote the activities of bulls and bears within the Account. Normally, settlement day is the Tuesday in the second week after the end of a particular account. In addition, the contango arrangement makes it possible for payment to be transferred until the following settlement day—at a price. Thus the time span considered is significant from an economic point of view. In the second place, we simply come up against a resource limitation. We used a basic sample of 50 shares, one further share, and the *FT* index; and this presents a data-processing problem of some magnitude both in terms of man hours and computer time. We felt that from the point of view of our study it would be more useful to take a wide spread of shares rather than fewer shares with longer series. The use of *longer* series does not in any case answer the question of how share prices move over a *shorter* period.

[1] M. Osborne, "Periodic Structure in the Brownian Motion of Stock Prices", *Operations Research*, vol. 10 (1962), pp. 345–79, reprinted in Cootner (ed.), *op. cit.*, pp. 262 ff.

[2] Cowles and Jones, *ibid.*

[3] Niederhoffer and Osborne, "Market Making and Reversal on the Stock Exchange", *Journal of the American Statistical Association*, vol. 61 (1966), pp. 879–916.

The use of relatively short periods does leave one open to the valid criticism that one may, by chance, have selected an atypical period in Stock Exchange history. Obviously we do not feel this to be so in our case; but the criticism must be met. If any criterion guided us at all in our selection of time span, it was that we felt that general market activity was neither particularly bullish nor particularly bearish over the chosen period. Whether or not this is so, should have no effect on the testing of the random walk hypothesis, since random walks with upward or downward drift are both consistent with the hypothesis.[1] However, our guiding criterion did ensure that we did not select an unusual period in which the whole market was stampeding madly in one direction or the other, leaving us with very little to talk about. Even if it were true that we are examining very unusual segments of some time series, the mere fact that such segments are so rare makes it unlikely that many of them are contained in our total sample. Given time and resources, the ideal procedure probably would be to select a random sample of such time periods as ours from the past twenty years or so, thus strengthening the generality of any findings about these time periods.

The data themselves may not be in a suitable form for testing the random walk hypothesis if they are confounded by effects such as ex-dividend price falls, capitalization issues and the like. We adjusted for all such effects. This was done on the economic grounds that all effects which could not be attributed to the actions of market operators, instigated by the random acquisition of information, should be ironed out. As yet there is not much economic theory available to help one make decisions of this kind. Miller and Modigliani, working with a formal model, have come to the conclusion that dividend policy does not affect share prices.[2] Their model works on a rarified set of assumptions, including one to the effect that the future dividend policy of a firm is known. From our point of view this is not very helpful, as it assumes away our problem. A strong opponent of this abstract view is Gordon, who claims to have both theoretical and empirical grounds for refuting the Miller–Modigliani hypothesis.[3] On a more empirical level Campbell and Beranek found that on the New York Stock Exchange the "stock exchange practice of marking down open bids and stop orders to sell by the full amount of the dividend tends to cause the ex-dividend drop-off to be larger than it otherwise would be and definitely larger than it logically should be".[4] Though many writers have just passed over

[1] Osborne, *op. cit.*, pp. 100–28, points out that the assumption of a lognormal distribution for price changes implies a long-term upward drift in price levels. This consideration should not be relevant to the length of time series we are considering.

[2] Miller and Modigliani, *op. cit.*

[3] M. J. Gordon, "Optimal Investment and Financing Policy", reprinted in *The Theory of Business Finance*, *op. cit.*, pp. 366 ff.

[4] J. A. Campbell and W. Beranek, "Stock Price Behavior on Ex-Dividend Dates", *Journal of Finance*, vol. 10 (1955), pp. 425–9.

such problems, or given them perfunctory attention,[1] we felt it was necessary to adjust our data to be correct methodologically; although frankly, it seems unlikely that our results would have been radically affected had we not taken such pains. The chief data irregularity was the occurrence of ex-dividend days at some point or other for about half our sample. The general effect is that closing price is reduced by the amount of the net dividend (after tax) when a share goes ex-dividend. We merely reversed this process where appropriate, and added on the net dividend throughout the rest of the observations. A less common occurrence was a capitalization issue. This required the adjustment of all prices before the free issue to make them compatible with post-issue prices. For a one-for-three issue, for example, previous prices have to be corrected by three-fourths.

Sampling Method

The Share Information Service list of closing prices provided in the *Financial Times* provided the basic population.[2] Several major sections of the list were excluded: British Funds, International Bank, Corporation Loans, Commonwealth and African Loans, Foreign Bonds and Rails, Americans, Canadians, South Africans, Trusts, Finance and Land, and Utilities. The general principle was, therefore, to exclude investment trusts, government and other fixed interest securities, and to concentrate on individual shares. Many companies have more than one class of share quoted on the London Stock Exchange. In such cases, only the ordinary share quotation was included in the sample.

We considered three sampling methods as being feasible: judgment sampling; simple random sampling; and proportional stratified random sampling. None of these techniques is complicated. We were not so much concerned with making a strict statistical inference from the sample, as with getting a sample which was in some sense "representative" of our population.

With this in mind, the first method, judgment sampling, commended itself to us. This procedure was used by Cootner, for example, who seems to us to have selected his shares on a very casual basis.[3] Fisher, studying a rather different aspect of share price behaviour, considered only shares which were "active".[4] It is true that some shares, generally of very large firms, are traded more frequently than those of small firms. But the distinction between active and non-active shares on the London

[1] Cootner, in Cootner (ed.), *op. cit.*, pp. 231–52, does outline his adjustment procedure, but gives no justification for it.

[2] As this population is so easily available, we have not included it here. The figures are available from the authors on application.

[3] Cootner, *ibid.*, pp. 231–52.

[4] Fisher, *op. cit.* It is important to note that Fisher was concerned with ensuring the homogeneity of his sample as he was working with cross-section data. We are not so constrained in our study, and merely develop the notion of "activity" as one of the possible criteria to use in selecting a judgment sample.

Stock Exchange must be arbitrary to some extent. The fact that a share has a quotation in the *FT* Share Information Service is in itself some guarantee of marketability, as persistently low-activity shares are liable to have their quotation omitted. It could be argued that the fact that some shares are traded much more than others is not particularly important from the point of view of testing the random walk hypothesis, provided there is *an opportunity* available to deal in all quoted shares. It is not immediately clear what effect this phenomenon of differential activity has on the movement of share prices. It seems likely that a larger volume of buying or selling will be required with a very active share, as compared with an inactive share, to produce the same proportional effect on price. Although on first sight "activity" may seem a plausible criterion for making a judgment sample, the preceding reasoning does not suggest that a sample heterogeneous with respect to the attribute of activity is unsuitable for testing the random walk hypothesis. This is perhaps just as well, for the data available on volume are weak; and without such data any definition of activity must be highly subjective. We have only aggregate monthly turnover figures and "marks" to help us. Only the latter could be useful in this instance; but as brokers are not obliged to mark business done, the markings in the Official List may give a distorted picture of the true volume of business. In brief, lacking suitable criteria, we found the task of selecting a judgment sample altogether too demanding.

The most obvious alternative was to pick a random sample. As the sample (50) was very small in proportion to the population (2,000+), we feared that simple random sampling would carry the risk of obtaining an unrepresentative sample. We therefore selected a proportional stratified random sample.[1] We divided the population into groups which we felt represented particular *economic* activities, rather than, say, according to company size. The result (continuing to use the headings employed by the *FT*) was as in Table 1.

Each stratum was allocated a quota of the total sample in proportion to its stratum size. Some rounding was involved, and the final quota of 50 was made up of 11, 6, 7, 7, 5 and 14 for the six strata indicated in Table 1. The requisite quota was selected randomly for each stratum. The final sample is shown in Appendix 1. It was noted that one member of the sample (Clarke Chapman) had ceased to be quoted some time after the end of the period of observation. This means that the population would have been slightly different for a longer period covering the day when the quotation of Clarke Chapman was discontinued. The tables of random numbers dictate that in this case Harland and Wolff would have been used instead. Although Clarke Chapman is the correct company for inclusion in the present study, we also computed all tests

[1] Statistical analysis shows that typically the variance of stratified random sampling is less than or equal to the variance of simple random sampling. See Cochran, *Sampling Techniques*, 2nd ed., theorem 5.8, p. 98, and also pp. 99–100. It may be thought at first sight that the attribute "random" or "not random" does not have a variance; but, see again, Cochran, *ibid.*, pp. 49 ff.

for Harland and Wolff as well, in case our study should be extended at a later date.

TABLE 1

Strata	Number of Share quotations	
1. Engineering and Metal	246	
Electrical and Radio	90	
Motor, Aircraft	88	
	424	424
2. Banks and Hire Purchase	64	
Insurance	42	
Property	117	
	223	223
3. Drapery and Stores	115	
Cinemas, Theatres and TV	70	
Food, Groceries	18	
Hotels, Caterers	23	
Beers, Wine, Spirits	31	
Shoes, Leather	26	
Newspapers, Publishers	15	
	298	298
4. Primary Commodities	281	281
5. Building Industry, Timber and Roads	220	220
6. Industrials (Miscellaneous)	403	
Textiles—General	52	
Textiles—Wool	33	
Paper, Printing and Advertising	43	
Steels	5	
Shipping	20	
	556	556
Total		2,002

STATISTICAL METHOD

In our investigation of the random walk hypothesis we were primarily concerned with adopting procedures which were methodologically sound. Thus we preferred to avoid the debate as to whether the distribution of price changes is normal, lognormal or otherwise. Since the publication of an important paper by Osborne,[1] academic opinion has favoured the hypothesis of lognormality, as outlined above. If one were to accept this view, then it would be necessary to study the logarithm of price changes, $\Delta \log P$, if parametric tests are used, because parametric tests generally assume that the population is normally distributed (e.g. Anderson's test for auto-correlation).[2] Several recent writers, have used a logarithmic transformation.[3] However, we have chosen to stand aside from the controversy, and have used non-parametric tests.

As is well known, non-parametric tests tend to be less efficient than parametric tests. However, a preliminary examination of the data suggested that non-parametric statistics would be adequate. On the principle of Occam's Razor there seemed little point in assuming a particular type of distribution if our general conclusions would hold without such an assumption.

Our approach was to submit the data to successively more specific tests. Because most non-parametric tests will detect only a particular form of non-randomness, such as the presence of too few or too many runs, we subjected our data to a series of different tests.

Our first test was based simply on the number of runs.[4] This test is ideally suited to detecting non-randomness in a series of two kinds of elements which constitute a natural dichotomy (such as man/woman, or day/night). It was necessary to introduce this dichotomy artificially by considering runs above and below the median. Although this is one of the least powerful distribution-free tests, it was sufficient to detect six cases of significant ($\alpha = 0{\cdot}05$) non-randomness in our sample.

The second test used was the Wallis–Moore test for cycles.[5] This test is based on the number of turning-points in the series; or, equivalently, on the number of runs of like signs (+ or −) in the first differences of the series. We worked with the first differences of the price-change series, which is the same as working with the second differences, $\Delta^2 P$, of the basic series on price levels. By comparing the actual number of runs of a specific length with the number of that length expected from a random series, a statistic similar to χ^2 may be computed. This statistic, christened $\chi_p{}^2$ by Wallis and Moore, is calculated from the formula

$$\chi_p{}^2 = (u_1 - U_1)^2/U_1 + (u_2 - U_2)^2/U_2 + (u_3 - U_3)^2/U_3,$$

where the u ($i = 1, 3$) are actual number of runs of length one, two, three or more; and the U_i ($i = 1, 3$) are the expected number of runs of length

[1] Osborne, "Brownian Motion . . .", *op. cit.*, pp. 145–73, reprinted in Cootner (ed.), *op. cit.*, pp. 100–27.

[2] R. L. Anderson, "Distribution of the Serial Correlation Coefficient", *Annals of Mathematical Statistics*, vol. 13 (1942), pp. 1 ff.

[3] E.g. Moore, in Cootner (ed.), *op. cit.*, pp. 139–61.

[4] See J. V. Bradley, *Distribution-Free Statistical Tests*, pp. 260 ff.

[5] W. A. Wallis and G. H. Moore, "A Significance Test for Time Series Analysis", *Journal of the American Statistical Association*, vol. 30 (1941), pp. 401 ff.

1, 2 and 3 or more. For example, from the first differences in the series of Lombard Banking we obtained the results:

	Actual no. of runs	Expected no. of runs
Length 1	5	19·583
Length 2	11	8·433
Length 3 or more	8	2·983
	24	30·999

This gives $\chi_p^2=20{\cdot}077$ for the series of length $N=51$. This value is greater than the critical value of 6·898 given in the Wallis–Moore tables[1] for $\alpha=0{\cdot}05$ and $N>12$. Hence this series is significantly non-random. It is clear that in this case there are too few runs of length 1, and too many of length 3 or more.

An auxiliary test is also available, based on the total number of runs in the first differences. For large samples,[2] this is normally distributed with mean $(2N-7)/3$ and variance $(16N-29)/90$. Wallis and Moore suggest[3] that N should be no less than ten. One would expect intuitively that this test is less sensitive than the χ_p^2 test, because it uses less information. But very often we found it to be more powerful, that is, more capable of rejecting the null hypothesis of randomness. This seems to be a result of two factors.

(1) The first factor is that the Wallis–Moore test does not seem to be very sensitive to relatively minor divergences between expected and actual run frequencies in each of the three classes. However, the cumulative result of such divergences may make the sum of runs significantly different from that expected with a random series. The number of runs-up-and-down test can be easily fooled, however, where the total number of runs is close to the expected, but where the pattern of lengths of runs is significantly different from that expected with a random series. A good example of such a situation was found with the full Blaw Knox series. The configuration of runs was as follows:

	Actual no. of runs	Expected no. of runs
Length 1	26	19·583
Length 3	2	8·433
Length 3 or more	3	2·983
	31	30·999

This gives $\chi_p^2=7{\cdot}010$, which is greater than the critical value of 6·898

[1] Wallis and Moore, *op. cit.*, p. 404.
[2] *Ibid.*, p. 405.
[3] *Ibid.*, p. 405.

for $\alpha=0{\cdot}05$ and $N>12$. We reject the null hypothesis of randomness apparently because there are too many runs of length 1, and too few of length 2. By contrast, for the runs-up-and-down test we get a value for the normal deviate of 0·000. We cannot reject the null hypothesis of randomness, the reason being that the total number of runs is equal to the expected, namely 31. However, this case was not typical. A more common case was that the Wallis–Moore test could not reject the null hypothesis, whereas the runs-up-and-down test could. Representative of this situation is the full series of G. H. Downing:

	Actual no. of runs	Expected no. of runs
Length 1	16	19·583
Length 2	4	8·433
Length 3 or more	2	2·983
	22	30·999

This gives $\chi_p^2=3{\cdot}310$, and for $\alpha=0{\cdot}05$ and $N>12$ we cannot reject the null hypothesis of randomness. However, for the runs-up-and-down test, the normal deviate is $-3{\cdot}075$ and, for $\alpha=0{\cdot}05$, we reject the null hypothesis. When working with the full series, the typical case seemed to be that we had too few runs. This leads us to what looks like the more important causal factor in producing this peculiar reversal in the expected power of the two tests: the length of the series.

(2) A conspicuous feature of many of the time series was the frequent occurrence of "no change" between successive observations. This presents difficulties in the application of many non-parametric tests. A typical assumption is that the data to which the test statistic is applied contain no difference scores of zero. This is a consequence of a very common assumption about the population from which the observations are drawn, namely, that the probability of adjacent observations being identical is infinitesimally small. This is an assumption of continuity, and would be quite acceptable for many economic variables. A well-known assumption of classical supply and demand analysis, for example, is that price and quantity sold per unit time are continuously variable. In practice, of course, difference scores of zero are quite common. This may be the result of either of two situations: (a) The true situation is that the population is continuous, but measurement is necessarily made in terms of discrete approximations; or (b) the true situation is that the population is discrete. The former case would be appropriate to time, and the latter, say, to motor car production levels.

To break down this impasse of zero difference scores, one may do either of two things: ignore zeros; or find some procedure for assigning a plus or minus to each zero. The first method may be valid if there are only a very few zeros in a very long series, or if the zeros are "non-critical" from the point of view of the test. As an example of the latter case, suppose a test based on the number of runs in the first differences of a series produced the following results:

$$+++--++--0++-+++.$$

The zero is non-critical in this case, as the total number of runs will still be seven, whether we replace the zero by a plus or a minus. The second method, that of imputing a plus or minus to a zero, is one that we usually have to adopt. There are several available procedures.[1] The method adopted here has been to eliminate the zeros from the data and to base the test on the shortened series. If the population is continuous, this is equivalent to the assumption that the ratio of actually positive to actually negative difference scores in the set of observations is the same as the ratio of measurably positive to negative. If this assumption is met, the test is exact and valid on the sample of reduced size. If the population is not continuous, then this procedure is the only meaningful one available, and avoids imputing a spurious sign to a zero which we know to be the result of differencing two identical, discrete, successive observations.

In the Wallis–Moore test, some account has been taken of the possibility of ties[2] in calculating the distribution of χ_p^2; but in the runs-up-and-down test, no such considerations have been made, and the presence of ties definitely violates the assumptions of the test.[3] To cover all possible interpretations, both tests were re-computed for reduced sample sizes.

But again we found that the null hypothesis was rejected more often with the runs-up-and-down test, though, as one would expect, there appeared to be an all-round loss of power. It seems plausible, therefore, that the types of non-random features in most of our series were more easily detected by a runs-up-and-down test. This is merely a particular feature of our data, and cannot be used to draw wider conclusions about the relative effectiveness of the two tests.

The fourth test applied was the Wald–Wolfowitz non-parametric test for auto-correlation. This test takes into account all the information available. It is based on the test function:

$$R=\sum_{t=1}^{N-1}\Delta P_t\Delta P_{t+1}+\Delta P_N\Delta P_1.$$

The mean and variance of R depend on complex algebraic expressions.[4] The auto-correlation coefficient has a circular definition. For this reason one must be cautious in applying it to economic time series, particularly if there is a strong trend. For if there were, the $\Delta P_n\Delta P_1$ term may tend to swamp the value of R. However, for our data such a

[1] For a full discussion of these procedures, see Bradley, *op. cit.*, ch. 3.
[2] See Wallis and Moore, *op. cit.*, pp. 402–3.
[3] See Bradley, *op. cit.*, p. 278.
[4] See A. Wald and J. Wolfowitz, "An Exact Test for Randomness in the Non-Parametric Case based on Serial Correlation", *Annals of Mathematical Statistics*, vol. 14 (1943), pp. 378 ff.

possibility is highly unlikely. Visual inspection of the graphs of all price change series suggested that all series were stationary. It may be noted that cases of significant negative auto-correlation were discovered as well as the more commonly expected positive auto-correlation. This feature is discussed in more detail below. The main purpose of computing the statistic was to find a more detailed measure of the extent of dependence between observations than the previous statistics could provide. It has the advantage of attaching a sign and magnitude to the degree of dependence and of making full use of the data. In the final analysis, however, it did not add a great deal to our knowledge; and in only one case, that of Brooke Bond Liebig, did it reveal a case of non-randomness which none of the previous tests had revealed.

Basic Results

The results of the statistical tests are summarized in Table 2, and are set out in more detail in Appendix 2. Our results strongly support the point of view that the random walk hypothesis has been over-generalized. Considering the summary column 7(a) of Table 2, about 80 per cent. of the sample was found to be significantly non-random. This column summarizes the series of tests, excluding those in which "no changes" were removed from the data. Column 7(b), as explained in the footnote to the table, takes cognizance of tests in which "no changes" were removed from the data. The initial conclusion is weakened slightly, and about 50 per cent. of the sample was found to be significantly non-random. However, whichever set of assumptions one operates on, the results are an obvious caution to those who would hold up the random walk hypothesis as a relatively universal empirical generalization.

The summary statistics tend to conceal a conspicuous feature of the data, namely, their diversity of form. On plotting all the time series of ΔP_ts graphically, we were impressed with the obvious individuality of each share. This is a feature which is completely concealed by using an aggregate index. The graph of daily changes for the *FT* index is shown in Figure 1. This series was not found to be non-random under any of the tests—a result typical of previous tests of the random walk hypothesis. Contrast this with the series for Hanipha Ceylon in Figure 2, which shows a marked lack of variation. Though this is an extreme example, several other shares were very static over this period, notably Henriques, Taylor Pallister, Odex Racasan, Lotus and Lanka. The results on the auto-correlation statistic (column 6, Table 2) were interesting, and are summarized in Table 3. The cases of significant negative auto-correlation are particularly interesting in view of the controversy surrounding the interpretation of this phenomenon. Osborne, for example, has suggested that this may be attributed to the fact that prices tend to stick at certain most-favoured levels.[1] A more

[1] Osborne, *op. cit.*, in Cootner (ed.), *op. cit.*, p. 286.

TABLE 2
SUMMARY OF RESULTS

CODE \ TEST	(1)	(2)	(3)	(4)	(5)	(6)	(7) (a)	(7) (b)
01	R	NR	NR	NR	NR	R	NR	NR
02	R	R	NR	R	NR	NR	NR	NR
03	R	R	NR	R	R	R	NR	R
04	NR	R	NR	R	R	NR	NR	NR
05	R	NR	NR	R	R	R	NR	R
06	R	R	NR	R	NR	R	NR	NR
07	R	R	NR	R	R	R	NR	R
08	R	NR	NR	R	R	R	NR	R
09	R	NR	NR	R	NR	R	NR	NR
10	R	NR	NR	R	R	R	NR	R
11	R	R	NR	R	R	R	NR	R
12	NR	R	NR	R	R	NR	NR	R
13	R	NR	R	NR	NR	NR	NR	NR
14	R	R	R	R	R	R	R	R
15	R	R	NR	R	NR	R	NR	NR
16	R	NR	NR	NR	NR	NR	NR	NR
17	R	NR	NR	R	NR	R	NR	NR
18	R	NR	NR	NR	NR	NR	NR	NR
19	R	R	R	R	R	NR	NR	R
20	NT	NR	NR	NR	NR	NR	NR	NR
21	R	R	R	R	NR	R	R	NR
22	R	NR	NR	R	NR	R	NR	NR
23	R	R	R	R	R	R	R	R
24	R	R	NR	NR	NR	R	NR	NR
25	R	R	NR	R	R	R	NR	R
26	R	NR	NR	R	R	R	NR	R
27	NR	R	R	R	NR	R	NR	NR
28	NT	NR	NR	NR	NR	R	NR	NR
29	R	R	R	R	NR	R	R	NR
30	R	R	R	R	R	R	R	R
31	R	R	R	R	R	R	R	R
32	R	NR	NR	R	R	R	NR	R
33	R	R	R	R	R	R	R	R
34	R	NR	R	NR	NR	R	NR	NR
35	R	R	NR	R	R	NR	NR	R
36	NR	R	NR	R	R	R	NR	R
37	R	NR	NR	R	R	R	NR	R
38	R	NR	NR	NR	NR	NR	NR	NR
39	R	R	NR	R	R	R	NR	R
40	NR	NR	NR	R	NR	NR	NR	NR
41	R	NR	NR	R	R	R	NR	R
42	NT	NR	NR	R	NR	R	NR	NR
43	R	NR	NR	R	R	R	NR	R
44	R	NR	NR	R	NR	R	NR	NR
45	NT	NR	NR	NT	NT	NT	NR	NR
46	R	NR	NR	R	NR	R	NR	NR
47	R	R	R	R	NR	R	R	NR
48	R	R	R	R	R	R	R	R
49	R	R	R	R	R	R	R	R
50	R	R	R	R	R	R	R	R
51	NR	R	NR	R	R	NR	NR	NR
52	R	R	R	R	R	R	R	R

Key at top of page 45

Key to Column Headings
Code See Table 4, in Appendix 2.
Column (1). Runs above and below median, performed on ΔP_t series in original form.
Column (2). Wallis–Moore test, performed on ΔP_t series in original form.
Column (3). Runs-up-and-down test, performed on ΔP_t series in original form.
Column (4). Wallis–Moore test, performed on ΔP_t series with zero difference scores removed.
Column (5). Runs-up-and-down test, performed on ΔP_t series, with zero difference scores removed.
Column (6). Wald–Wolfowitz test for auto-correlation performed on ΔP_t series in original form.
Column (7). Column (a) summarizes the results for each row, ignoring the entries in columns (4) and (5). Column (b) summarizes the results for each row, ignoring the entries in columns (2) and (3). The appearance of at least one NR in any row is sufficient to warrant an entry of NR in column (a) or (b) of Column (7). For example, for row 21 (Charrington, Gardner & Locket), the following entries (1)—R, (2)—R, (3)—R, (6)—R, call for an entry of R in (7(a)); and the entries (1)—R, (4)—R, (5)—NR, (6)—R, call for an entry of NR in (7(b)).

Key to Column Entries.
R =the test did not reject the null hypothesis of randomness.
NR=the test rejected the null hypothesis of randomness.
NT =no test performed. This arises because of a peculiarity of the data, typically an extreme lack of variation.

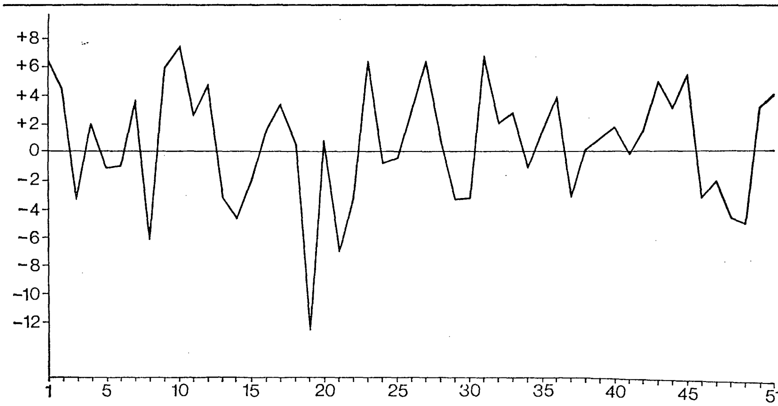

FIGURE 1 *FT* Industrial Ordinary Index

economic explanation could run in terms of the diffusion of information through the market. It is well known that a piece of information takes some time to penetrate to all potential buyers and sellers, the professionals acquiring the news first and the amateurs later.[1] Positive auto-correlation may be produced in the following manner. When a piece of information induces, say, a price rise in share x on day 1, the subsequent spread of the information on day 2 to a further group of potential buyers/sellers causes the price of the stock to rise further on that day. When there is negative auto-correlation, the sequence will be as follows. A piece of information appearing on day 1 causes the price of share y to (say) rise. On day 2, when the information has spread

[1] Moore, *op. cit.*, reprinted in Cootner (ed.), *op. cit.*, pp. 152–3, uses the term "inept traders" in his discussion of the auto-correlation question.

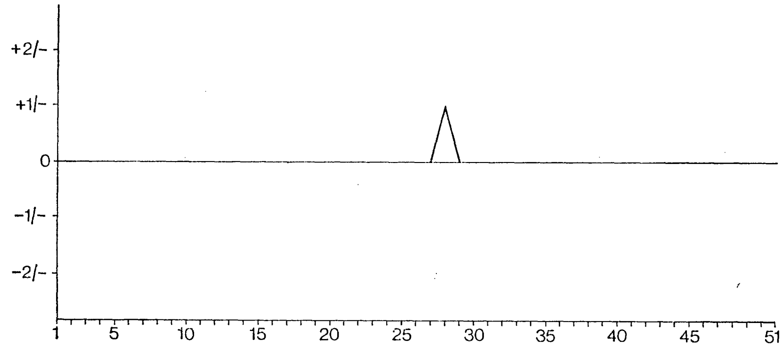

FIGURE 2 Hanipha Ceylon.

further, there is a reaction to this higher price of sales (profit-taking, normally) by the amateurs, causing price to fall below that achieved on day 1, though probably not below that ruling on day 0. The amateurs are taking a different view from the professionals as to what constitutes a "reasonable price". The graph of the ΔP_ts for Blaw Knox was one of the most unusual of the sample, and is shown in Figure 3. Particularly noteworthy are the marked saw-tooth character of the series, and the unusually wide fluctuation around period 30. Concerning the latter, the extreme magnitude of the reversal may have swamped the statistic to some extent.[1]

Because non-parametric tests detect very specific types of non-randomness, it is very hard to generalize over-all and to say, "Non-randomness was by and large of type x". Probably the auto-correlation statistic would have come closest to providing an appropriate generalization; but unfortunately the test is not very powerful and detected only twelve cases of non-randomness. However, Table 3 is at least

TABLE 3
POSITIVE AND NEGATIVE AUTO-CORRELATION

Positive	Negative
Wagon Finance	Blaw Knox
Downing (G. H.)	Greening (N.)
Harland & Wolff	Taylor Pallister
Brooke Bond Liebig 'B'	McKay Securities
British Syphon	
Mills and Allen	
Ropner Holdings	
Malayan Tin Dredging	

[1] These are the results stated in their simplest form. In a follow-up study which the writers are at present completing, a simple economic model is developed, and wider conclusions based on these results are derived.

suggestive of the general pattern of non-randomness exhibited, namely some sort of positive serial dependence, which of course is the overwhelmingly prevalent type in economic time series.

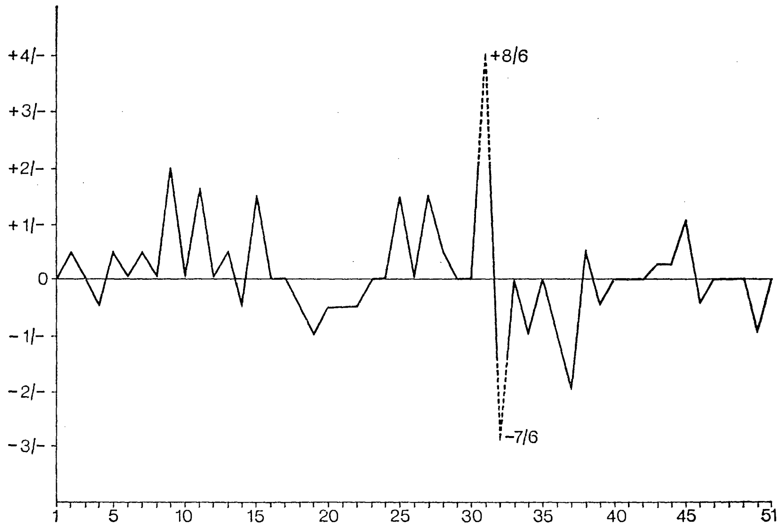

FIGURE 3 Blaw Knox

SUMMARY

In this paper our approach has been largely empirical. We have developed a critique of previous tests of the random walk hypothesis and suggested what we consider to be the most significant way to carry out such tests.

We worked only with daily prices. We also worked with relatively short series, having a particular interest in the investors' behaviour over short time horizons; but at the same time, we used a relatively large sample of shares to avoid the danger of observing isolated atypical behaviour. In our statistical analysis we used non-parametric tests to avoid unnecessary assumptions about the underlying distribution of price changes.

Our conclusion is that share price movements were conspicuously non-random over the period considered. This result, we feel, should be a caution to those who have been startled by the apparent finding of randomness in share price movements in their studies. Their findings are often as much a product of the method of analysis as they are an intrinsic feature of the data.

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TABLE 4

Code No.	Company Name	Unadjusted Data: Standard Normal Runs Test (1)	Unadjusted Data: χ_p^2 for $N=51$ (2)	Unadjusted Data: Standard Normal Runs up and down (3)	Adjusted Data: χ_p^2 for variable N (4)	Adjusted Data: Standard Normal Runs up and down (5)	Test Function (R) (6)	Standard Normal for R (7)	Mean (8)	Standard Deviation (9)
01	Lombard Banking	−0·12	20·077	−2·392	20·456 (48)	−1·978	774·000	1·893	0·15	7·97
02	Wagon Finance	−1·10	5·241	−4·100	2·630 (25)	2·298	117·000	2·299	0·35	2·80
03	Guinness	0·63	5·372	−3·075	4·405 (34)	0·697	127·125	0·820	0·81	4·97
04	Downing (G.H.)	−3·47	3·310	−3·075	0·963 (36)	0·135	1494·000	3·123	1·41	8·43
05	Hills F.	−1·44	7·818	−4·783	3·958 (24)	1·678	27·000	0·595	−0·12	3·14
06	Howarth	−1·45	5·454	−3·758	4·677 (26)	2·411	−49·500	−1·238	0·00	2·32
07	Mucklow	0·57	5·324	−4·100	1·492 (29)	0·910	85·500	0·570	1·06	3·23
08	Symes A.E.	1·55	12·467	−6·492	2·774 (19)	0·953	−18·000	−1·222	−0·09	1·41
09	Henriques	0·35	16·387	−7·517	6·979 (11)	3·310	−0·563	−0·120	0·12	0·85
10	Edinburgh Industrial Holdings	0·31	9·071	−5·467	3·021 (21)	1·805	−0·563	0·005	0·15	1·30
11	Clarke Chapman	−1·22	6·523	−3·417	4·817 (41)	−1·515	−1350·000	−1·357	−3·29	13·74
12	Harland & Wolff	−2·48	2·160	−2·733	0·158 (37)	0·267	942·750	5·099	0·03	5·35
13	Blaw Knox	−1·32	7·010	0·000	11·357 (39)	2·852	−9243·000	−3·578	1·29	20·82
14	Carrier Engineering	0·17	1·571	−1·025	2·414 (39)	1·685	−215·250	−0·729	0·82	6·49
15	Edibrac	1·10	3·777	−2·733	4·429 (30)	2·382	−51·000	−1·393	−0·17	2·22
16	Greening (N)	−0·44	7·900	−3·075	16·472 (24)	4·196	−105·688	−2·247	0·50	3·15
17	Johnson Con.	−1·44	14·569	−7·175	3·548 (13)	2·600	6·750	1·299	−0·06	0·94
18	Taylor Pallister	−1·41	16·106	−7·175	15·000 (11)	3·912	−27·000	−2·456	−0·35	1·52
19	Brooke Bond Liebig 'B'	−0·67	1·571	−1·025	1·906 (41)	1·137	157·500	2·173	0·18	3·36
20	British Syphon	No test	15·179	−6·833	19·300 (11)	4·695	288·000	3·017	1·29	3·27
21	Charrington, Gardner & Locket	−1·18	3·752	−1·708	6·497 (33)	2·690	2·250	0·176	0·18	1·87
22	Cuthbert (R.G.)	−0·93	9·420	−5·467	5·206 (19)	2·670	0·000	0·152	0·00	0·79

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
23	Glaxo Group	−0·08	5·638	1·367	6·294 (48)	1·861	−2883·375	−1·074	2·83	19·77
24	Hensher 'A'	−0·99	4·398	−2·392	7·000 (30)	2·829	−6·750	−0·618	−0·09	1·17
25	Hestair	0·05	6·783	−3·758	5·415 (30)	1·042	99·000	1·601	0·12	3·14
26	Inter City 'A'	0·14	12·596	−6·492	3·722 (18)	1·375	0·750	0·185	−0·11	0·85
27	Nairn Williamson	−2·57	2·664	−1·708	3·832 (35)	2·058	54·000	0·662	0·24	5·57
28	Odex Racasan	No test	20·838	−8·541	11·933 (7)	3·818	0·000	−0·309	0·91	5·07
29	Photo Me International	1·20	3·236	−1·367	5·622 (35)	2·470	−87·750	−0·795	0·53	3·97
30	British Leyland	1·28	4·780	1·025	5·028 (49)	1·266	−0·188	−0·500	0·93	3·17
31	Seddon Diesel	1·01	2·755	−1·025	3·197 (42)	0·873	31·500	0·242	0·72	2·73
32	Brown Brothers	0·09	8·946	−5·125	4·287 (24)	1·175	81·188	0·588	0·14	5·14
33	Daily Mail and General Trust	−0·55	1·374	−0·683	1·917 (42)	1·247	171·000	0·338	1·25	10·73
34	Smith (W. H.) 'A'	0·24	7·416	−0·342	14·515 (35)	3·705	626·000	1·629	2·49	5·51
35	Mill & Allen	−1·45	3·093	−3·075	1·474 (31)	1·610	180·000	2·145	0·59	3·58
36	Hallmark Securities	−3·28	2·457	−2·733	0·614 (37)	0·267	33·813	0·344	0·70	2·99
37	Land & House	−0·09	8·143	−5·125	3·193 (22)	1·935	114·750	1·029	0·90	3·84
38	McKay Securities	0·98	13·297	−6·492	9·945 (14)	3·397	−11·250	−2·137	0·21	0·94
39	Oddeninos	−1·25	4·247	−3·758	0·870 (30)	1·042	180·750	1·763	0·48	4·00
40	Ropner Holdings	−3·45	7·818	−4·783	5·199 (22)	2·463	69·000	2·274	−0·47	2·04
41	Lennards Deferred 'A'	0·49	17·344	−7·858	2·470 (12)	1·734	−4·500	−0·823	0·06	0·89
42	Lotus	No test	23·817	−9·225	12·200 (5)	3·985	0·000	−0·204	0·09	0·46
43	Jute Industries Ordinary	0·91	8·414	−5·125	2·377 (27)	0·158	20·250	1·144	0·20	1·64
44	Uttley	−1·41	17·621	−7·858	6·042 (10)	3·039	2·250	0·660	−0·06	1·54
45	Hanipha Ceylon	No test	27·204	−9·908	No test (3)	No test	0·000	No test	0·24	1·66
46	Lanka	−0·44	17·313	−7·858	2·917 (11)	2·347	0·000	0·031	0·18	1·38
47	Buffels	−0·98	3·901	−1·916	6·497 (33)	2·690	6696·000	1·741	4·72	23·67
48	Rand Selection Corporation	0·27	0·354	−0·225	0·419 (48)	0·465	4492·000	0·230	8·75	31·82
49	Nchanga	0·37	1·739	−1·708	1·838 (37)	1·466	99·000	0·990	−0·12	4·16
50	Metals Exploration	−1·78	5·509	−1·025	5·562 (48)	−0·582	−4878·000	−0·675	3·29	3·22
51	Malayan Tin Dredging	−2·30	6·710	−4·783	1·083 (26)	0·964	292·500	2·779	0·10	4·03
52	'F.T.' Industrial Ordinary Index	0·01	0·909	0·342	0·860 (51)	0·113	125·650	1·027	0·64	4·14

APPENDIX 1

The sample was as follows:

Stratum Number	Companies	
(1)	Clarke Chapman	Taylor Pallister
	Blaw Knox	Edinburgh Industrial Holdings
	Carrier Engineering	British Leyland
	Edibrac	Seddon Diesel
	Greening (N)	Brown Brothers
	Johnson Con.	
(2)	Lombard Banking	Land and House
	Wagon Finance	McKay Securities
	Hallmark Securities	Oddeninos
(3)	Henriques	Lotus
	Brooke Bond Liebig 'B'	Daily Mail and General Trust
	Guinness	Smith (W. H.) 'A'
	Lennards Deferred 'A'	
(4)	Hanipha Ceylon	Nchanga
	Lanka	Metals Exploration
	Buffels	Malayan Tin Dredging
	Rand Selection Corporation	
(5)	Downing (G. H.)	Mucklow
	Hills (F.)	Symes A. E.
	Howarth	
(6)	British Syphon	Nairn Williamson
	Charrington, Gardner and Locket	Odex Racasan
	Cuthbert (R. G.)	Photo Me International
	Glaxo Group	Jute Industries Ordinary
	Hensher 'A'	Uttley (Wm.)
	Hestair	Mills and Allen
	Inter City 'A'	Ropner Holdings

In addition, all statistics were calculated for Harland and Wolff, and for the *Financial Times* Industrial Ordinary Index, as explained in the section "Sampling Method".

APPENDIX 2

Table 4 gives more detailed statistical results than Table 1 of the main text. Some comment on the more detailed set of results is in order.

Column (1) gives the normal deviate calculated by using a normal approximation for the distribution of the number of runs.[1] However, it was frequently found that the Swed–Eisenhart tables[2] had to be used because of the low number of runs and few observations above and below the median. Thus the figures in column (1) are included largely as a gage of good faith.

In column (4) [χ_p^2 for variable N], the figure in brackets after the χ_p^2

[1] Bradley, *op. cit.*, p. 262, gives the mean and variance for the asymptotic normal distribution of the total number of runs.

[2] These are available in Bradley, *ibid.*, p. 362.

value refers to the length of the series of ΔP_ts after removal of "no changes". Column (5) likewise applies to the same set of data, this time showing the results for the runs-up-and-down test. It should be noted that in three cases (Odex Racasan, Lotus, Hanipha Ceylon) the series with "no changes" removed were so short as to make the computed test statistics of very little meaning.

Column (7) gives the calculated approximate standard normal deviate for R. The actual value of the test function R, though not necessarily very informative in itself, is shown in column (6), again largely as a gage of good faith.