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EFFICIENT CAPITAL MARKETS: COMMENT

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STUDENTS OF FINANCE are indebted to Eugene F. Fama for his summary [1] of the literature on efficient capital markets. However, Fama's discussion of the theory of efficient capital markets, as distinct from his able review of tests of the theory, contains several important passages that are, at best, very misleading. In view of the prominence Fama's article has achieved, it is of some importance that these errors be corrected.

In his Section II.A (p. 384), Fama's concern is to provide a formal representation of the intuitive notion that prices will "fully reflect" available information in an efficient capital market. He correctly points out that such a representation is necessary if testable implications of the theory are to be derived. Most studies, Fama notes, are "based only on the assumption that the conditions of market equilibrium can (somehow) be stated in terms of expected returns" ([1], p. 384). Such "expected return theories" are held to be representable in the form

$$E(\tilde{p}_{j,t+1}|\phi_t)=[1+E(\tilde{r}_{j,t+1}|\phi_t)]p_{jt}, \tag{1}$$

where E is the expectations operator, tildes indicate random variables, ϕ_t is the information set at time t, and p_j and r_j are the price and rate of return on the jth security. Fama states that equivalent representations of expected return theories can be made in terms of "fair game" variables. Thus, define $x_{j,t+1}$ by

$$x_{j,t+1}=p_{j,t+1}-E(\tilde{p}_{j,t+1}|\phi_t), \tag{2}$$

and $z_{j,t+1}$ by

$$z_{j,t+1}=r_{j,t+1}-E(\tilde{r}_{j,t+1}|\phi_t). \tag{3}$$

Then

$$E(\tilde{x}_{j,t+1}|\phi_t)=E(\tilde{z}_{j,t+1}|\phi_t)=0. \tag{4}$$

Or, more generally, let $V_{t+1}(\phi_t)$ be the excess market value (i.e., the difference between actual market value and the conditionally expected market value) of any collection of the securities generated by any trading system based on ϕ_t. Then Fama asserts that

$$E(\tilde{V}_{t+1}|\phi_t)=0 \tag{5}$$

can be derived as a "testable implication" of the efficient markets model.

The difficulty with these definitions is that they are true as tautologies; any stochastic processes $\{r_{jt}\}$, $\{p_{jt}\}$, $\{x_{jt}\}$ and $\{z_{jt}\}$ related by (2), (3), and the rate of

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return definition

$$r_{j,t+1} = \frac{p_{j,t+1} - p_{jt}}{p_{jt}}$$

will obey (1) and (4). It follows that these equations cannot properly characterize an efficient capital market, however defined, since they are as true of the most naive Dow theory model as of a pure random walk. Because the equations imply no restrictions on the data, they cannot possibly generate testable implications, contrary to Fama's clear implication.

The correct definition, that which underlies the tests Fama reports, is that prices conform to

$$E(\tilde{p}_{j,t+1} | \phi_t) = p_{jt}(1 + g_j(t)) \qquad \text{for all } \phi_t \tag{6}$$

or, for rates of return,

$$E(\tilde{r}_{j,t+1} | \phi_t) = g_j(t) \qquad \text{for all } \phi_t. \tag{7}$$

Equations (6) and (7) say that all the information needed to predict the (conditional) expected value of $p_{j,t+1}$ is reflected in p_{jt}. If $\{p_{jt}\}$ is stationary, then $\partial g / \partial t \equiv 0$, but nothing in the intuitive notion of an efficient capital market requires that this be the case. For example, if investors are risk-neutral[1] and a safe asset generates a rate of return r_t^* which can vary over time, the expected price of the risky asset will be given by

$$E(\tilde{p}_{j,t+1} | \phi_t) = p_{jt}(1 + r_{t+1}^*) \qquad \text{for all } \phi_t$$

and its expected rate of return by

$$E(\tilde{r}_{j,t+1} | \phi_t) = r_{t+1}^*.$$

Fama's difficulty with the theoretical implementation of the notion of an efficient capital market becomes compounded in his discussion of the problem of nonstationarity. He writes (p. 392):

But the "fair game" model does not necessarily imply that the serial covariances of *one-period returns* are zero. In the weak form tests of this model the "fair game" variable is

$$z_{jt} = r_{jt} - E(\tilde{r}_{jt} | r_{j,t-1}, r_{j,t-2}, \ldots) \tag{8}$$

... and (8) does not imply that $E(\tilde{r}_{j,t+1} | r_{j,t}) = E(\tilde{r}_{j,t+1})$: In the "fair game" efficient markets model, the deviation of the return for $t+1$ from its conditional expectation is a "fair game" variable, but the conditional expectation itself can depend on the return observed for t.

In the random walk literature, this problem is not recognized, since it is assumed that the expected return (and indeed the entire distribution of returns) is stationary through time.

[1] The martingale restriction can be derived from the general equilibrium theory of asset choice only under risk-neutrality. For this derivation, see Samuelson [4], [5]; for a demonstration that the martingale property does not carry over under risk-aversion, see LeRoy [3]. Definitions and discussions of martingales and fair game variables are found in Feller [2], esp. p. 210.

This statement is literally correct, since as already noted Fama's definition of the efficient markets model implies nothing at all about $\{r_t\}$. Further, if only (1) is assumed (that is, if nothing is assumed) Fama's statement that rates of return may be correlated is correct even if rates of return are stationary, so the correlatedness or uncorrelatedness of $\{r_t\}$ has nothing to do with its stationarity or nonstationarity. But the martingale model (7) does imply that $E(\tilde{r}_{j,t+1}|r_{jt}, r_{j,t-1}, \ldots) = E(\tilde{r}_{j,t+1})$, and therefore population rates of return are in fact uncorrelated whether or not they are stationary.

Nonstationarity poses a problem, not because population rates of return are correlated (under the martingale, they are not), but because the usual statistic for correlation assumes a constant mean. Therefore if no correction for nonstationarity is made, a test for uncorrelatedness based on the sample correlation coefficient will be biased toward rejection. However, as Fama notes, even with this bias most tests for uncorrelatedness accept that hypothesis at the usual significance levels.

The problems noted in this comment are not minor, particularly for the student who seeks an understanding of the theory underlying the empirical studies of capital market efficiency. However, corrections are easily made, and subject to these corrections Fama's summary article is a valuable addition to the literature.

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