A Contribution to the Theory of Information Acquisition in Financial Markets

Marc-Andreas Muendler*
University of California, Berkeley

September 16, 2000

Abstract

In order to explore the incentives for information acquisition in financial markets, a model of the joint information and portfolio choice is developed. Investors are allowed to acquire a number of signals that inform about a risky asset's dividend, and "informational efficiency" is defined as a social planner's preferred signal allocation. If prices are fully revealing, a complete rational expectations equilibrium exists—contrary to a wide-held conjecture. The equilibrium entails no information acquisition and is informationally efficient. The reason is that the transmission of information through price to other investors brings market expectations closer to each investor's own expectations. This reduces the difference between expected payoffs and the asset price, a negative effect of more information. When prices are only partly informative, some investors start acquiring information as long as markets are sufficiently small so that prices reveal little information to others. However, the acquisition of more information inflicts a negative externality on uninformed investors who rationally extract information from price. Thus, markets are likely to be informationally inefficient as informed investors tend to acquire too much information.
JEL: G14, D81

*muendler@econ.berkeley.edu (URL: http://socrates.berkeley.edu/~muendler/). I have benefited from very insightful discussions with Maury Obstfeld and early explorations with Andy Rose. I am greatful to Achim Wambach, Sven Rady, and seminar participants at Ibmec Rio de Janeiro and the University of Munich for many helpful comments. Needless to say, remaining mistakes are not welcome but mine.

Grossman and Stiglitz (1976) first formulated the following paradox: If asset prices fully reveal the information of all investors, no investor wants to buy information and chooses to free-ride on other investors' information; but if no other investor buys information, at least one investor wants to acquire information. So, no rational expectations equilibrium can exist. Grossman and Stiglitz (1980) restated this paradox more formally as a conjecture, which seems to be considered true quite widely.

Is there really no equilibrium when prices are fully revealing? If it existed, a rational expectations equilibrium would involve both an equilibrium in financial markets (at Wall Street) and an equilibrium in the market for information (at the news stands). While a strain of literature has established that prices in financial markets are generically fully revealing (following Allen 1981, Jordan 1982), only few articles have investigated what that implies for investors' incentives to obtain information. The present paper sets out to embed Grossman and Stiglitz' (1980) model into a more general framework of information acquisition in financial markets, which contains their model as a special case. Grossman and Stiglitz' conjecture is proven to fail in the present framework in a particularly surprising way: No investor wants to obtain information, not even receive it for free, when prices at Wall Street are fully revealing. As a consequence, a unique equilibrium both at Wall Street and at the news stands exists in which no information is acquired. The profound reason is that information may loose the character of a good and turn into a public bad when it becomes too common. This will also be the recurring insight when prices are only partly, and not fully, informative. Information can have features of a negative externality in financial markets. Fully revealing prices are merely an extreme case.

The framework will clarify why individuals have an incentive to obtain information: More information can raise *ex ante* utility for a risk averse individual. In general, *ex ante* utility is increasing when the expected excess return of the risky asset increases. Moreover, it tends to increase when the variance of the portfolio falls. So, information is good because it sharpens our knowledge about the dividend and thus tends to reduce the variance of the portfolio. However, information also has a bad side in financial markets since it affects the excess return negatively. The excess return of an asset can be viewed as its expected dividend less the opportunity cost of acquiring it. In the notation to be adopted soon, the excess return may be defined as $\mathbb{E}[\theta] - RP$, where θ is the dividend, R the yield of a riskless bond, and P the price of the risky asset. In general, prices play a double role: They reflect the opportunity cost of an asset, and they aggregate and transmit information. This double role is precisely why more information can harm investors in financial markets. If more information gets to the market, this information is, at least partly, transmitted through prices. But then, when rational investors update their information, their expectation of the dividend gets closer to market expectations which are incorporated in the price. In other words, the excess return $\mathbb{E}[\theta] - RP$ is likely to be falling with more information! The equilibrium price P and the expected relative payoff $\mathbb{E}[\theta/R]$ get closer to each other. Under fully revealing prices, this effect is so strong that no investor wants to buy any information, and even a benevolent social planner agrees that no information is the right choice. Under partly informative prices, in which some external noise remains, this effect is still present. From the point of view of less informed investors, information behaves like a negative externality. Well informed investors feel the positive impact of a lowered variance, but still suffer from the loss in the excess return. This loss is so strong that a symmetric equilibrium, in which all investors are equally well informed, cannot exist. Not even a social planner would want everybody to become equally well informed.

1 Related Literature

The present paper uses the Lucas tree model of a financial market, reduced to two periods. It is thus essentially equivalent to the commonly used model of terminal wealth maximization. The main innovation of the paper is the addition of a second market, a market for information. Investors in the present model can choose a degree of information about a risky asset's dividend, and buy that information in a competitive market. Grossman and Stiglitz' (1980) model, in which investors could either acquire exactly one signal or no signal at all, is thus a special case of the current framework where investors are allowed to acquire $N \in \mathbb{N}_0$ signals. To make the analysis rigorous, I depart from Raiffa and Schlaifer's (1961) early contribution to decisions under uncertainty, and use what Raiffa and Schlaifer called pre-posterior analysis. Adopted to financial markets, this requires *ex ante* utility to be the criterion for information acquisition. Apart from the benefit of a coherent treatment, this will allow to apply a welfare analysis to financial markets as it has been used in many other fields of economics. "Informational efficiency" will be given a Pareto-criterion. I blend this framework of information updating with Hellwig's (1980) model of a financial market. Hellwig restricts attention to the case of one signal, too, so the present framework could be viewed as an extension in this respect. Since Hellwig's (1980) model formulation does not allow a closed-form solution, however, I make simplifying assumptions at other places to arrive at a closed-form financial market equilibrium. Still, there is no closed-form solution for the information market equilibrium.

Of course, the present paper is also related to the long literature following Grossman and Stiglitz's (1976, 1980) and Hellwig's (1980) analysis. Admati

(1991) provided an overview over this literature after a decade and a half. Recent contributions include Barlevy and Veronesi (2000), Pietra and Siconolfi (1998), Dutta and Morris (1997), or Rahi (1995), for instance. There is a similar literature on information acquisition and transmission among oligopoly firms (see Li, McKelvey, and Page 1987, Raith 1996). Investors are assumed to be price takers in the present model, which may leave an extension to imperfect competition for future investigation (compare Kyle 1989).

The approach in the present paper stands in a certain contrast to behaviorally inspired models on herding and informational cascades such as in Burguet and Vives (2000), Lee (1998), Banerjee (1992), or Bikhchandani, Hirshleifer, and Welch (1992). I do not want to suggest, however, that the rational expectations equilibrium in the present model describes the behavior of investors exhaustively or even correctly. I only want to draw attention to the fact that investors may not need to rely so much on observed behavior of other agents in markets where prices can contain an extremely high degree of information. An interesting question to consider would be how long informational cascades persist when individual information is at least partly transmitted through prices.

This paper proceeds in three main steps. In section 2, I consider the standard Lucas tree model of an investors' intertemporal consumption and portfolio choice and briefly investigate its implications for information acquisition in general. In section 3, I derive the closed-form solution of the market equilibrium under *fully revealing* prices. Contrary to a commonly held assertion, market prices can be fully revealing and an equilibrium exists. The properties of this equilibrium are indicative of the more general insights to follow. In section 4, I consider one possible generalization to *partly informative* prices. I derive the closed-form solution for the financial market equilibrium and analyze the characteristics of the information equilibrium, for which no closed-form exists. Section 5 concludes.

2 A Well-known Model, Extended

For simplicity, suppose that all securities loose their value after only one period. In addition, suppose that there are but two assets on Wall Street: One riskless bond and one risky stock. When Wall Street opens today at 10am, investors can choose their portfolio. Both assets will yield a payoff tomorrow once and for all. The bond is going to pay the principal plus interest $R = 1 + r$ tomorrow, whereas the stock is going to pay a risky dividend θ. Today, the bond costs exactly one dollar, while the stock will go for P dollars to be set by a Walrasian auctioneer at 10am. All investors hold prior beliefs about the distribution of the dividend θ. Newsstands open at

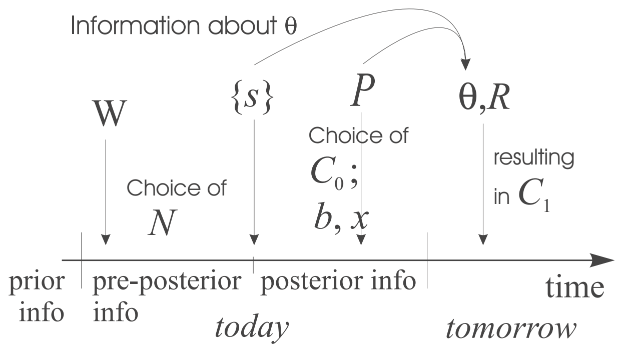

Figure 1: **Timing of Information Revelation and Decisions**

9am today.

Investors can, of course, choose to completely ignore newspapers and only observe security prices at Wall Street. Let's call these guys "price watchers." After all, price watchers know that the price P will convey market information about tomorrow's dividend since P is a function of all other investors' asset demands which in turn reflect their information. If at least some of the other investors have purchased newspapers, the pure price watcher can free-ride on their information by merely looking at the price. In the most extreme case, the stock price at 10am will *fully reveal* all investors' information (not θ, of course, but the content of all newspapers that others have read). This is one possibility, to be revisited in section 3. Alternatively, the price may contain noise. Then it only *partly informs* about other investors' knowledge—a more realistic possibility analyzed at large in section 4. A pure price watcher combines his prior knowledge about θ with the information that he can extract from the price, and then makes his portfolio choice.

However, investors may also choose to buy newspapers at 9am. Let's call investors who do so "news watchers." Besides reading newspapers, news watchers still use the price P to extract additional information (unless they consider it redundant to the information in the newspapers). To become a news watcher requires a fixed but not sunk cost F for wasting time with the sales person at the news stand, for reading the newspaper, and for taking time to interpret the information.[1] Each newspaper goes at a unit cost c. How many different newspapers should a news watcher buy? When standing in front of the news stand at 8.55am, each news watcher knows that she will base her portfolio decision, to be taken at 10am today, on the information

[1] The fact that F is fixed and not sunk allows each news watcher to become a price watcher by buying $N^i = 0$ newspapers. I will also consider the case of $F = 0$.

that she is about to get out of the newspapers at 9am. She also knows the statistical distribution of the information in the newspaper, which is more informative than her own prior beliefs. Taking all this into account, she rationally evaluates what a newspaper is worth to her and makes her best choice. Formally, a news watcher maximizes her (pre-posterior expected) indirect utility, or *ex ante* utility, with respect to the number of newspapers—given her information at 8.55am, her anticipation of how signals are likely to affect her beliefs in five minutes, and her expected portfolio choice to be taken at 10am. This timing of decisions is illustrated in figure 1. Given her wealth W_0^i and her prior information, a news watcher first chooses the number of newspapers to buy, or, in the language of a statistician, the number of signals N^i. A news watcher then receives the realizations $\{s_1^i, ..., s_{N^i}^i\}$ of these N^i signals $\{S_1^i, ..., S_{N^i}^i\}$ (she gets to know the newspaper content). Given this information, she finally chooses consumption today, C_0^i, and decides how many bonds b^i and how many risky x^i assets to hold for consumption tomorrow. This is the model in a nutshell.

What is the right timing of information acquisition? Or, why the time difference between 8.55am when investors have to choose the number of newspapers and 9am when news stands open? Note that an investor cannot know what is written in the newspaper when she takes her decision on information acquisition. Otherwise she would not need to acquire the information anymore. It would rather be part of her prior beliefs already. In other words, there must always be a logical second between the acquisition of a newspaper and the revelation of its content, on which the portfolio decision will be based. I extended the logical second to five minutes in the above description (or to an hour and five minutes if you so want). Formally, this logical second makes all the difference between *pre-posterior* beliefs and *posterior* beliefs. In her pre-posterior beliefs a news watcher rationally anticipates how the N^i signal realizations to arrive will most likely affect her portfolio choice. Her posterior beliefs, however, incorporate the signal realizations themselves. Pre-posterior beliefs, in turn, are different from mere prior beliefs exactly because of the anticipation of more precise future information. In more economic language, the logical second takes us from pre-posterior or *ex ante* expected indirect utility, not knowing the newspaper content, to posterior expected indirect utility, knowing the newspaper content.[2]

To make things concrete and formal, let investor i maximize additively separable utility under a discount factor $\delta < 1$ and an instantaneous utility

[2]To all my knowledge, Raiffa and Schlaifer (1961) first proposed the notion of *pre-posterior* analysis. When talking about utility, I will generally keep the more common economic term of *ex ante* utility, but use the adjective pre-posterior for the according distributions at this stage of beliefs.

function $u(\cdot)$ that is increasing and concave. That is, let her maximize

$$U^i = u(C_0^i) + \delta \mathbb{E}\left[u(C_1^i)\,\middle|\mathcal{F}^i\right] \tag{1}$$

with respect to consumption today, C_0^i, and tomorrow, C_1^i, and a portfolio choice. $\mathcal{F}^i$ denotes the information set available to investor i at the time of her portfolio choice. Besides the price P, it contains the realizations $\{s_1^i, ..., s_{N^i}^i\}$ of the N^i signals $\{S_1^i, ..., S_{N^i}^i\}$ that she has acquired. For ease of notation, I will usually abbreviate investor i's conditional expectations $\mathbb{E}\left[\cdot\,|\mathcal{F}^i\right]$ with $\mathbb{E}^i\left[\cdot\right] \equiv \mathbb{E}\left[\cdot\,|\mathcal{F}^i\right]$. These expectations are different for each investor in general unless *all* information is publicly available and commonly known.

The intertemporal budget constraint of investor i is

$$b^i + Px^i = W_0^i - C_0^i - F^i - cN^i \tag{2-a}$$

so that

$$C_1^i = Rb^i + \theta x^i \tag{2-b}$$

will be available for consumption tomorrow. The investor is endowed with initial wealth W_0^i, and decides about her consumption C_0^i and C_1^i in each period, her holdings of the riskless bond b^i, her holdings of the risky stock x^i, and how much information N^i she wants to buy. If an investor acquires at least one newspaper, she has to incur the fixed cost F. To indicate this, we can use the shorthand $F^i \equiv \mathbf{1}(N^i \geq 1) \cdot F$. While assets are assumed to be perfectly divisible, signals have to be acquired in discrete amounts.[3]

On the second stage, after having received the realizations of her N^i signals $\{s_j^i\}_{j=1}^{N^i}$, each investor decides on asset holdings and consumption given the asset price P. A price watcher receives no signals and simply relies on the price. In any case, at the second stage every investor has updated his or her beliefs about the dividend's distribution to a posterior distribution, given the signal realizations. The Euler conditions for the problem at this stage are therefore

$$\frac{1}{\delta} = R\,\mathbb{E}^i\left[\frac{u'(C_1^{i,*})}{u'(C_0^{i,*})}\right] \tag{3-a}$$

and

$$\frac{P}{\delta} = \mathbb{E}^i\left[\theta\,\frac{u'(C_1^{i,*})}{u'(C_0^{i,*})}\right], \tag{3-b}$$

[3]For a signal to contain information, its distribution has to depend on θ. So, a continuum of signals (or an infinite number of them), will a.s. reveal the exact realization of θ to news watchers. For markets to clear, P must equal θ/R in this case, otherwise news watchers want to reshuffle their portfolio. But then the price fully reveals θ itself and removes all uncertainty—an unrealistic case of little interest.

where expectations $\mathbb{E}^i[\cdot]$ are conditional on the realizations of the signals and the asset price. The optimal choices $C_1^{i,*}$, $b^{i,*}$ and $x^{i,*}$ are decision rules depending on the price P, on the chosen number of signals N^i (which has been decided earlier), and on the information transmitted through the signal realizations and the price P. The choices of $C_1^{i,*}$, $b^{i,*}$ and $x^{i,*}$ imply a level of posterior indirect utility, which we can denote by $U^{i,*} = u(C_0^{i,*}) + \delta\mathbb{E}^i\left[u(C_1^{i,*})\right]$.

On the first stage, the investor chooses the number of signals she wants to receive. She does this by maximizing *ex ante* utility given her information before the realizations of the signals arrive. At this time she cannot know more than the prior parameters of the respective distributions, but she builds her new pre-posterior beliefs by taking into account how signals will most likely change her beliefs in the next stage. *Ex ante* utility is $\mathbb{E}^i_{pre}\left[U^{i,*}\right] = \mathbb{E}^i_{pre}\left[u(C_0^{i,*})\right] + \delta\mathbb{E}^i_{pre}\left[u(C_1^{i,*})\right]$ by the law of iterated expectations. The optimal number of signals $N^{i,*} \in \mathbb{N}_0$ maximizes *ex ante* utility $\mathbb{E}^i_{pre}\left[U^{i,*}\right]$.

These observations immediately imply

Lemma 1 *Suppose signals are costly. Then an investor acquires no information*

- *if she is risk neutral, or*
- *if the prior distribution is insensitive to changes in the number of signals. That is, if the pre-posterior distribution of the fundamental and the prior distribution coincide.*

Proof. Suppose the investor is risk neutral. Then there is no benefit from a signal. *Ex ante* utility degenerates to $C_0^{i,*} + \delta\mathbb{E}^i_{pre}\left[C_1^{i,*}\right]$. For a risk neutral investor to neither demand a positively nor a negatively infinite number of assets, $\mathbb{E}^i\left[\theta\right] = RP$ and $R = 1/\delta$ in a financial market with no arbitrage (or in equilibrium). Thus, *ex ante* utility becomes $C_0^{i,*} + \delta\mathbb{E}^i_{pre}\left[C_1^{i,*}\right] = W_0^i - F^i - cN^i$ by (2-a) and (2-b). *Ex ante* utility of a risk neutral investor is independent of the portfolio composition. As a result, signals only cause costs, but do not have a benefit, which proves the first statement. To prove the second statement, suppose the prior distribution of the fundamental is insensitive to the number of signals. Then an additional signal weakly reduces both $u(C_0^{i,*})$ and $u(C_1^{i,*})$, and strictly reduces at least one of the two, for *any* future realization of the dividend. Since the prior distribution of the fundamental is supposed not to change, a signal cannot have a benefit in this case either. ∎

A risk neutral investor is indifferent whether she holds a risky stock or a riskless bond in her portfolio. Hence, she would never act upon information, which makes information useless to her. As immediate as lemma 1 may seem, it has important consequences. It clarifies that the incentive to purchase costly information is closely linked to risk aversion and higher-order moments of the risky asset's distribution. Risk averse investors do care about their portfolio composition, whereas their risk neutral colleagues don't. The fact that information has no value for risk neutral investors also highlights that information is not a good or bad in its own right. It has only value if it affects our decisions.

For the remainder of this paper, I will make the following assumptions.

Assumption 1 (Risk) *The prior variance of the risky asset return is strictly positive and finite.*

Assumption 2 (Common risk aversion) *Investors are risk averse and share a common and certain degree of risk aversion, all else equal.*

Assumption 3 (Common priors) *Investors hold the same prior beliefs about the distributions of the risky asset return, the signals, and the supply of the risky asset.*

Assumption 4 (Conditional independence) *Signals are conditionally independent given the dividend's realization. Formally,* $S_j^i|\theta \overset{i.i.d.}{\sim} f(s_j^i\,|\theta)$.

Assumption 5 (Equal precision) *All signals have a constant common precision* $1/\sigma_S^2$.

Assumption 6 (No borrowing constraint) *Investors can carry out unlimited short sales.*

Assumption 7 (Price taking) *Investors are price takers in all markets.*

Assumption 8 (Rationality) *Investors are fully aware of the correlation between signals and the asset price in equilibrium.*

Assumptions 2 through 6 are made for convenience. They also happen to be necessary conditions for prices to become fully revealing. However, an uncertain asset supply will ultimately prevent prices from being fully revealing (section 4). As Hellwig (1980) observed, assumptions 7 and 8 stand in a certain conflict. Investors are assumed not to take into account how their asset demand affects price. Yet, they are assumed to perceive how the equilibrium price correlates with their own information through their demand. Hellwig called investors of this kind "schizophrenic." Kyle (1989) has provided a way out by allowing that investors behave like "monopsonists" when demanding assets. However, to enhance tractability of the model, I retain assumption 7 throughout this paper. Finally, to obtain closed-form solutions, let's suppose the following.

Assumption 9 (CARA) *Investors have CARA utility with* $u(C) = -e^{-\gamma C}$.

Assumption 10 (Normality) *Random variables are Gaussian.*

This last assumption implies that the dividend realization can be negative or positive. Consequently, it entails the more profound assertion that investors are prevented from rejecting a negative payoff through a well-working legal system.

3 Fully Revealing Prices

In this section, I reconsider the benchmark case of fully revealing prices both to revisit Grossman and Stiglitz's (1976, 1980) famous paradox and "no equilibrium conjecture," and to clarify the guts of the model. For this, I make the assumption that supply of the risky asset is certain and known to all investors. It takes the value $\bar{x}$. I will first establish the financial market equilibrium at Wall Street. Second, I will prove the existence of a unique rational expectations equilibrium both at Wall Street and at the news stands under fully revealing prices. I finally discuss its efficiency properties.

3.1 The financial market equilibrium

Suppose that, at 10am, all investors have possibly different information about the two parameters μ^i and τ^i of the risky asset's distribution. Dividends are normally distributed. So, $\theta \sim \mathcal{N}(\mu^i, (\tau^i)^2)$. As for 8.55am, however, all investors have been assumed to share the same priors about the distribution of θ (assumption 3). So, $\mu^i_{prior} = \bar{\mu}_\theta$ and $\tau^i_{prior} = \bar{\tau}_\theta$. We would like the distribution of the signals to be such that both the prior and the posterior distribution of θ are normal. In fact, assuming a normal distribution of the signals (assumption 10) along with conditional independence of the signals (assumption 4) has already done the job. Concretely, let each signal be independently normally distributed conditional on θ with $S^i_j|\theta \sim \mathcal{N}(\theta, \sigma^2_S)$. So, every signal has exactly the same precision $1/\sigma^2_S$ (assumption 5). Then we obtain

Fact 1 *Suppose that the prior distribution of* θ *is a normal distribution with given mean* $\bar{\mu}_\theta$ *and variance* $\bar{\tau}^2_\theta$. *Suppose also that the signals* $S^i_1, ..., S^i_{N^i}$ *are independently drawn from a normal distribution with unknown mean* θ *and conditional variance* σ^2_S . *Then the posterior distribution of* θ, *given the realizations* $s^i_1, ..., s^i_{N^i}$ *of the signals, is a normal distribution with a mean-variance ratio*

$$\frac{\mu^i}{(\tau^i)^2} = \frac{\bar{\mu}_\theta}{\bar{\tau}^2_\theta} + \frac{1}{\sigma^2_S}\sum_{j=1}^{N^i} s^i_j$$

and variance

$$(\tau^i)^2 = \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \frac{1}{\sigma_S^2} N^i}.$$

Proof. Apply fact 2 in appendix A (p. 41) to conditionally independent signals. ∎

The mean-variance ratio $\mu^i/(\tau^i)^2$ will play an important role for investors' decisions. The posterior mean μ^i can be inferred by multiplying the mean-variance ratio with $(\tau^i)^2$. Since a sum of normal variables is normally distributed, fact 1 implies that the pre-posterior expectation of the posterior mean is $\mathbb{E}^i_{pre}[\mu^i] = \bar{\mu}_\theta$. It is independent of the number of signals as it has to be in general by the law of iterated expectations. While the posterior mean is a random variable, the normal-normal pair of distributions has the rare property that the posterior variance $(\tau^i)^2$ is certain given the chosen number of signals. In light of lemma 1, it is important that the pre-posterior variance is changing in the number of signals N^i. Indeed, fact 1 is good news for risk averse individuals: News watchers can lower the pre-posterior variance of the risky asset $\mathbb{E}^i_{pre}[(\tau^i)^2] = (\tau^i)^2 = 1/\left(\frac{1}{\bar{\tau}_\theta^2} + \frac{1}{\sigma_S^2} N^i\right)$ by purchasing more information.

For now, let's only focus on the financial market equilibrium, that is let's restrict attention to the equilibrium at Wall Street and disregard the market for newspapers for a moment. So, we have taken a time jump to 10am. For CARA utility, the marginal utility ratios in first order conditions (3-a) and (3-b) become $u'(C_1^i)/u'(C_0^i) = e^{-\gamma(C_1^i - C_0^i)}$. Since

$$C_1^i - C_0^i = (1+R)b^i + (\theta + P)x^i - W_0^i + F^i + cN^i$$

by (2-a) and (2-b), the first order conditions (3-a) and (3-b) simplify to

$$\frac{1}{\delta} = R\,\mathbb{E}^i\left[e^{-\gamma(C_1^i - C_0^i)}\right] = R \cdot H^i\,\mathbb{E}^i\left[e^{-\gamma x^i \cdot \theta}\right] \quad \text{(4-a)}$$

$$\frac{P}{\delta} = \mathbb{E}^i\left[\theta\, e^{-\gamma(C_1^i - C_0^i)}\right] = H^i\,\mathbb{E}^i\left[\theta \cdot e^{-\gamma x^i \cdot \theta}\right] \quad \text{(4-b)}$$

where $H^i \equiv \exp\left(-\gamma\left[(1+R)b^i + Px^i - W_0^i + F^i + cN^i\right]\right)$ is certain. The expected values in (4-a) and (4-b) have simple closed-form solutions for a normally distributed dividend. They are reported as facts 3 and 4 in appendix A (p. 41). Applying these facts to (4-a) and (4-b), and dividing one by the other, yields demand for the risky asset

$$x^{i,*} = \frac{1}{\gamma}\,\frac{\mathbb{E}^i[\theta] - RP}{(\tau^i)^2} \quad (5)$$

with $\mathbb{E}^i[\theta] = \mu^i$. As is well known, demand for the risky asset is independent of wealth for CARA utility. Throughout this paper, the term $\mathbb{E}^i[\theta - RP]$ in (5) will be key. It denotes investor i's expected excess return of the risky asset over the opportunity cost of one unit of the risky asset. A news watcher will go short in the risky asset whenever $\mathbb{E}^i[\theta] = \mu^i < RP$, that is whenever her posterior expectation of the dividend falls short of opportunity costs RP, and go long otherwise.

In equilibrium, asset supply equals asset demand, that is $\sum_{i=1}^{I} x^{i,*} = \bar{x}$, where $\bar{x}$ has been assumed to be certain for this section. Thus, the equilibrium price P of the risky asset is implicitly given by

$$RP = \frac{1}{\frac{1}{I}\sum_{i=1}^{I}\frac{1}{(\tau^i)^2}}\left[\left(\frac{1}{I}\sum_{i=1}^{I}\frac{\mu^i}{(\tau^i)^2}\right) - \frac{\gamma\bar{x}}{I}\right]. \tag{6}$$

This relationship sheds light on the double role of prices in financial markets. On the one side, RP is the opportunity cost of one unit of the risky asset, indicating its scarcity or value to investors. On the other side, prices aggregate all investors' information. Neglecting $\bar{x}$, RP can also be viewed as "market expectations" of the risky asset return (where market expectations are the average expected dividend, weighted by subjective variances). Looking back at (5), we could also have stated *cum grano salis* that a news watcher will go short in the risky asset whenever her posterior expectation of the dividend falls short of market expectations RP, and go long otherwise. This already hints at the fact to be established later that investors will reduce their asset demand in situations in which their own information is very similar to the market information.

Given optimal asset demand $x^{i,*}$, *posterior* indirect utility of investor i can be shown to equal

$$U^{i,*} = -\tfrac{1+R}{R}(\delta R)^{\frac{1}{1+R}}\ e^{-\gamma\frac{R}{1+R}(W_0^i - F^i - cN^i)}\ \mathbb{E}^i\left[e^{-\gamma x^{i,*}(\theta - RP)}\right]^{\frac{1}{1+R}} \tag{7}$$

for CARA utility, irrespective of the distribution of the risky asset return (see appendix B, p. 43). Only investors who buy a positive amount of signals have to pay the fixed cost F. To express this, I have used the short hand $F^i \equiv \mathbf{1}(N \geq 1)\cdot F$ in (7) again. For a normal distribution of the dividend, the last factor in (7) becomes

$$\mathbb{E}^i\left[\exp\left\{-\gamma x^{i,*}(\theta - RP)\right\}\right] = \exp\left\{-\frac{1}{2}\left(\frac{\mu^i - RP}{\tau^i}\right)^2\right\}$$

by fact 3 and asset demand (5). Note that RP is certain from a posterior point of view. Using this in (7), posterior indirect utility for a normally distributed dividend can be written

$$U^{i,*} = -k^i \cdot \exp\left\{\gamma \frac{R}{1+R}(F^i + cN^i)\right\} \cdot \exp\left\{-\frac{1}{2}\frac{(\tau^i)^2}{1+R}\left(\frac{\mu^i - RP}{(\tau^i)^2}\right)^2\right\} \quad (8)$$

for $k^i \equiv \frac{1+R}{R}(\delta R)^{\frac{1}{1+R}} \exp\left\{-\gamma \frac{R}{1+R} W_0^i\right\} > 0$. The last factor in (8) contains the posterior variance $(\tau^i)^2$ twice, while it in fact cancels. Writing it this way will simplify later calculations.

This posterior indirect utility finally hints at one of the most important insights to be established soon: investors may suffer a utility loss when their own information gets very similar to the market information. So, information obtained by news watchers and transmitted to price watchers through prices may in fact have features of a negative, and not a positive externality for those price watchers.

Investors $i = 1, ..., I$ choose their portfolios, b^i and x^i, given their respective information sets $\mathcal{F}^i = \{RP; s_1^i, ..., s_{N^i}^i\}$ if they are news watchers and $\mathcal{F}^i = \{RP\}$ if they are price watchers. If prices are fully revealing, however, then the average of all signal realization received by any investor will become known to everyone through the price. So, if prices are fully revealing, all information sets become equivalent and $\mathcal{F}^i = \mathcal{F}$.[4]

Then fact 1 implies that the equilibrium price P of the risky asset is implicitly given by

$$RP = \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \frac{1}{\sigma_S^2}\frac{1}{I}\sum_{i=1}^{I} N^i}\left(\frac{\bar{\mu}_\theta}{\bar{\tau}_\theta^2} + \frac{1}{\sigma_S^2}\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{N^i} s_j^i - \frac{\gamma \bar{x}}{I}\right), \quad (9)$$

where RP is the opportunity cost of the risky asset. In equilibrium, every investor chooses an optimal number of signals, given the information choice of all other investors. So, $\sum_{k \neq i} N^k$ is known to every investor i. Since everything else in RP but the sum of signal realizations is also known to every investor, the sufficient statistic $\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{N^i} s_j^i$ becomes fully revealed to everyone through price. The following lemma explicitly restates necessary conditions for this to occur.

Lemma 2 *The equilibrium price of the risky asset P fully reveals all market participants' information $\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{N^i} s_j^i$ only if*

- *assumptions 1 through 5 are satisfied,*

[4]Formally, all investors' information sets become $\mathcal{F}^i = \mathcal{F} = \{\frac{1}{I}\sum_{i=1}^{I}\sum_{j=1}^{N^i} s_j^i\}$. A complete derivation of this result can be found in appendix I (p. 57). There the fully revealing equilibrium is treated as a special case of the most general model.

- *supply of the risky asset is certain,*
- *the number of investors is certain,*
- *the total number of all other investors' signals* $\sum_{j\neq i}^{I} N^i$ *is known to each investor* i *at the time of the portfolio choice, and*
- *assumption 6 is satisfied.*

Proof. The first four conditions follow by inspection of the general solution for the market price

$$RP = \frac{1}{\frac{1}{I}\sum_{i=1}^{I}\frac{1}{\gamma^i}\left(\frac{1}{\bar{\tau}_\theta^2}+\frac{1}{\sigma_S^2}N^i\right)}\left(\frac{1}{I}\sum_{i=1}^{I}\frac{1}{\gamma^i}\left(\frac{\bar{\mu}_\theta}{\bar{\tau}_\theta^2}+\frac{1}{\sigma_S^2}\sum_{j=1}^{N^i}s_j^i\right)-\frac{\bar{x}}{I}\right).$$

For the last necessary condition, consider the case in which some investors cannot go short in the risky asset due to the borrowing constraint. Then another investor will not know whether the equilibrium price is low because many relatively poor investors received bad signals and hit their borrowing constraint or whether only a few relatively wealthy investors received extremely bad signals. As a consequence, some uncertainty remains and the price cannot be fully revealing. ∎

In the light of lemma 2, fully revealing asset prices seem unlikely to occur in practice. They are still an important theoretical benchmark case. Yet, ever since Grossman and Stiglitz's (1976, 1980) seminal articles, fully revealing market prices have been dismissed with the theoretical argument that no equilibrium existed. In this and related arguments, some important features of informational externalities appear to have been overlooked as results in the following subsection may clarify.

3.2 The information market equilibrium

A complete market equilibrium both at Wall Street and the news stands can be defined as follows.

Definition 1 (Rational Expectations Equilibrium) *A rational expectations equilibrium is an allocation of* $x^{i,*}$ *risky assets,* $b^{i,*}$ *riskless bonds, and* $N^{i,*}$ *signals to investors* $i = 1, ..., I$*. It involves an asset price* P*, a signal price* c *and a fixed cost of news watching* F *along with a set of beliefs such that*

1. *asset demands* $x^{i,*}$ *and* $b^{i,*}$ *are optimal given opportunity cost* RP *and the respective information sets* $\mathcal{F}^i$ *for investors* $i = 1, ..., I$

2. *the choice of signals $N^{i,*}$ is optimal for investors $i = 1, ..., I$ given the sum of all other investors' signal choices $\sum_{j \neq i} N^{j,*}$, and given the costs c and F*

3. *the market for the risky asset clears, $\sum_{i=1}^{I} x^{i,*} = \bar{x}$, and*

4. *investors' beliefs are consistent with the equilibrium outcome.*

This could be called a 'hybrid' equilibrium. On the first stage of the game, investors choose the number of signals given the choice of all other investors and a Bayesian Nash equilibrium results. The equilibrium can be considered Bayesian since investors anticipate their Bayesian updating of beliefs when signal realizations arrive. On the second stage, investors do not perceive their impact on price by assumption 7 and a Walrasian competitive equilibrium results, given the Bayesian Nash equilibrium on the first stage of the game. The mixture of elements of a Bayesian Nash equilibrium with those of a Walrasian equilibrium makes the definition 'hybrid.'

The market for the riskless bond clears by the assumption of a perfectly elastic world supply given the world interest factor $R = 1+r$. The market for information clears under the implicit assumption that there is an infinitely elastic supply of information. That is, any number of signals can be produced at unit cost c. Given their anticipation of a financial market equilibrium as outlined in the previous subsection, investors choose their level of information on the first stage. Since the cost of becoming a news watcher is fixed, but not sunk, a choice of $N^i = 0$ signals means that an investor decides to become a price watcher.

To make her choice of information, each investor maximizes pre-posterior indirect or *ex ante* utility. If prices are fully revealing, the posterior parameters are the same for all investors, that is $\mu^i = \mu$ and $\tau^i = \tau$ for all investors i. Therefore, $RP = \mu - \frac{\gamma\bar{x}}{I}\tau^2$ for fully revealing prices by (6) and posterior utility is certain. Hence, *ex ante* utility $\mathbb{E}^i_{pre}[U^{i,*}]$ simply equals the certain posterior indirect utility

$$\mathbb{E}^i_{pre}\left[U^{i,*}\right] = -k^i \exp\left\{\gamma\frac{R}{1+R}(F^i + cN^i)\right\} \cdot \exp\left\{-\frac{1}{2}\frac{\tau^2}{1+R}\left(\frac{\gamma\bar{x}}{I}\right)^2\right\} \quad (10)$$

for

$$\tau^2 = \frac{1}{\frac{1}{\bar{\tau}^2_\theta} + \frac{1}{\sigma^2_S}\frac{1}{I}\sum_{k=1}^{I} N^k}.$$

Even though investors may only choose a discrete number of signals, it does no harm in the present context if we differentiate *ex ante* utility (10) with respect to N^i. Taking the derivative and multiplying it by the positive factor $-(1+R)/\mathbb{E}^i_{pre}[U^{i,*}]$ yields

$$-\frac{1+R}{\mathbb{E}^i_{pre}[U^{i,*}]}\frac{\partial \mathbb{E}^i_{pre}[U^{i,*}]}{\partial N^i} = -\gamma R\, c - \frac{1}{2}\left(\frac{\gamma \bar{x}}{I}\right)^2 \frac{\tau^4}{I\sigma_S^2} < 0. \tag{11}$$

So, no matter whether investors add a discrete or real number of signals, each additional signal lowers their *ex ante* utility! Therefore, the unique rational expectations equilibrium involves zero information. No investor wants to acquire any signal even if nobody else acquires a signal. Moreover, even if newspapers were free of charge ($c = F = 0$), investors would refuse to open them and throw them away unread.

Theorem 3 *Suppose that the asset price is fully revealing. Then there is a unique rational expectations equilibrium. No investor acquires information in this equilibrium even if signals are for free.*

Proof. By inspection of (10). ■

Since investors in Grossman and Stiglitz' (1980) model only have a choice between one signal or no signal, their model is a special case of the present framework which allows for any finite number of signals to be acquired ($N^i \in \mathbb{N}_0$). Grossman and Stiglitz (1980, *Conjecture 6*) wrote: "In the limit, when there is no noise, prices convey all information, and there is no incentive to purchase information. Hence, the only possible equilibrium is one with no information. But if everyone is uninformed, it clearly pays some individual to become informed. Thus, there does not exist a competitive equilibrium." This and similar conjectures can be found in the literature ever since. Recent examples include Romer (1993) and Barlevy and Veronesi (2000). The latter authors remark: "Finally, as Grossman and Stiglitz point out, we need to prevent prices from being fully revealing; otherwise an equilibrium will fail to exist." This "no equilibrium conjecture" is proven to be wrong in the present more general framework. The reason is that, even if everyone is uninformed, it does not pay any individual to become informed. Under fully revealing prices, any signal reduces the expected excess return of the risky asset $\mathbb{E}^i[\theta - RP] = \frac{\gamma\bar{x}}{I}\tau^2$. Consequently, information is not a public good, but a public bad under fully revealing prices.

Somewhat separately, a number of articles in mathematical economics has investigated fully revealing equilibria, too (see e.g. Allen 1981, Rahi 1995, Pietra and Siconolfi 1998). These articles establish that, generically, a fully revealing rational expectations equilibrium exists at Wall Street. Beyond those articles, the present framework explicitly analyzes the incentives for information acquisition and incorporates a market for information at the news stands. Without claiming generality, the present framework presents an example in which a fully revealing equilibrium exists but prices will contain no information.

3.3 Informational efficiency

So, a complete financial and information market equilibrium exists under fully revealing prices. However, investors choose to acquire no information at all so that nothing can get revealed in fact. Is this informationally efficient?

The present consumption maximization framework allows for a classical welfare analysis, applied to information. Think of a benevolent social planner who can dictate every investor i to buy exactly $N^{i,**}$ signals. This social planner maximizes $\sum_{i=1}^{I} \mathbb{E}^i_{pre}\left[U^{i,**}\right]$ with respect to $\{N^1, ..., N^I\}$, where $U^{i,**}$ denotes posterior indirect utility after the social planner has implemented an allocation of newspapers to investors.[5] Thus, similar to Samuelson's (1954) seminal condition for public good provision, a benevolent social planner's does not consider condition (11) for signal acquisition but rather

$$-\frac{1+R}{\mathbb{E}^i_{pre}\left[U^{i,**}\right]}\frac{\partial \sum_{k=1}^{I} \mathbb{E}^{k,**}_{pre}\left[U^k\right]}{\partial N^i} = -\gamma R c - \frac{1}{2}\left(\frac{\gamma \bar{x}}{I}\right)^2 \frac{\tau^4}{I\sigma_S^2}\left(1+\sum_{k\neq i}^{I}\frac{\mathbb{E}^k_{pre}\left[U^{k,**}\right]}{\mathbb{E}^i_{pre}\left[U^{i,**}\right]}\right) \tag{12}$$

for $k = 1, ..., I$ (pretending again that signals are divisible for simplicity's sake). Thus, the second term in every investor's condition is scaled up by a factor of $1 + (1/\mathbb{E}^k_{pre}\left[U^{k,*}\right]) \cdot \sum_{i\neq k}^{I} \mathbb{E}^i_{pre}\left[U^{i,*}\right] > 1$ for every single investor i. Since information is not beneficial but undesirable for each and every individual investor under fully revealing prices, the benevolent social planner emphatically agrees with the private market solution: No news watchers under fully revealing prices, please. If one price watcher bought a signal and became a news watcher, he would not only reduce his own excess return $\mathbb{E}^i\left[\theta - RP\right]$, but that of any other investor, too. The unique rational expectations equilibrium is therefore informationally efficient.

Theorem 4 *Suppose that the asset price is fully revealing. Then the unique rational expectations equilibrium, in which no investor acquires information, is informationally efficient.*

Proof. By inspection of the sum of (10), or (12). ■

One might conjecture that the result would turn out to be less extreme under dividend distributions that are not symmetric. The same result, however, can be shown to hold for the gamma-Poisson pair of distributions, too.[6]

[5] Passing by, I have confined the social planner to leaving every investor with his or her beliefs. The social planner cannot transfer knowledge between investors.

[6] For a derivation, see the author's web page http://socrates.berkeley.edu/ muendler/.

Theorem 4 also sheds some new light on Grossman and Stiglitz' (1980) more general assertion that financial markets are unlikely to be informationally efficient in general. Information need not have the character of a public good in all circumstances. It may actually be a public bad, and no information acquisition can be socially desirable! Under fully revealing prices, information is a perfect strategic substitute. No matter which price watcher dares to buy a signal, he harms himself and everybody else in the market.

4 Partly Informative Prices

The previous section has shown that investors rationally choose not to become news watchers when prices are fully revealing. To arrive at a more realistic information market equilibrium, suppose that the asset price is only partly informative about tomorrow's dividend. For this, only one of the necessary conditions in lemma 2 needs to fail. To make things concrete and to keep the tradition of the previous literature, suppose that supply of the risky asset is uncertain. To keep things simple, assume that investors cannot buy information about asset supply. In particular, let the asset supply be normally distributed with $X \sim \mathcal{N}(\bar{x}, \omega_x^2)$ and independent of any other random variable in the model.

The derivation of the financial market and information market equilibrium is based on an extension of Hellwig (1980). Whereas Hellwig's general model does not have a closed-form solution, I aim at obtaining a closed-form solution for my extension and make two additional assumptions. First, each investor has to choose the membership in either of two groups. She can either become a news watcher and do what the group representative mandates, or become a price watcher. This will affect the equilibrium definition. Second, I make an additional assumption.

Assumption 11 (Perfect copies) *All signals are sold in perfect copies.*

This is not so unrealistic considering that the large majority of investors obtains information from publicly accessible media in practice. Assumption 11 rules out, however, that an investor may talk in private to the CEO of the stock-issuing firm.

I proceed in similar steps as in the previous section. First, I derive the complete financial market equilibrium in closed-form and discuss, second, its immediate implications for information acquisition. Third, I analyze the information equilibrium which does not have a closed-form solution. Finally, I discuss its welfare properties.

4.1 The financial market equilibrium

Investors can update their information both through newspapers and through observation of the asset price. Then investors act upon these signal realizations, and signal realizations make their way into asset price. Therefore, one signal, the asset price, is no longer conditionally independent of the other signals. As a consequence, to derive a complete financial and information market equilibrium under partly informative prices, we need a generalization of fact 1 to the case of correlated signals. The according property is reported as fact 2 in appendix A (p. 41). Rational investors, who know the correlation in equilibrium, update their beliefs accordingly. They infer a conditional distribution of θ—given the signal realizations that they received, given the equilibrium price that they observed, and given their respective correlation as it occurs in equilibrium.

After correctly inferring the correlation between their signals and opportunity cost RP in equilibrium, rational investors base their portfolio choice on this knowledge (assumption 8). Thus, a rational expectations equilibrium as in definition 1 is a fixed point that results in no excess asset demands and consistent beliefs (see Hellwig 1980 for a general argument). Due to the mutual dependence of asset demands on equilibrium beliefs and beliefs on equilibrium asset demands, a complete financial market and information market equilibrium can be complicated to characterize, and often has no closed-form solution.[7]

To obtain a closed-form solution for the financial market equilibrium in this section, let's consider a subclass of equilibria (for the general equilibrium see appendix I, p. 57). As before, there are two groups of investors: Price watchers and news watchers. Now, however, let's require that news watchers be a homogeneous group. They must not independently decide on different amounts of information. Instead, they must jointly pick a number of identical newspapers, acquire them and read them or not. Or, in more intuitive words, a "news watcher representative" enters an agreement with all news stands at 8.55am to offer exactly N different newspapers and to sell one copy of each to every news watcher at 9am. If the group representative determines that a strictly positive number of newspapers be purchased, all news watchers agree to go to a news stand at 9am, to pay the fixed cost F, and to buy N newspapers at a cost of c each. If the group representative happens to mandate that no newspaper be purchased, news watchers jointly become

[7]For the derivation of the equilibrium under fully revealing prices in the previous section, we were able to take a shortcut at this point. In particular, we could use the fact that the information sets of all investors had to coincide under fully revealing prices. As a result, we never needed to explicitly consider the correlation of signals and RP (but investors implicitly evaluated this correlation correctly). A detailed derivation can be found in appendix I (p. 57).

price watchers and do not pay the fixed cost F. Among the I investors, a share $\lambda \equiv I^{NW}/I$ chooses to be news watching in equilibrium.

An according equilibrium definition is

Definition 2 (Two-Group Rational Expectations Equilibrium) *A two-group rational expectations equilibrium is an allocation of $x^{i,*}$ risky assets and $b^{i,*}$ riskless bonds to investors $i = 1, ..., I$, a share λ of news watchers, and an allocation of N signals to each news watcher. It involves an asset price P, a signal price c and a fixed cost of news watching F along with a set of beliefs such that*

1. *asset demands $x^{i,*}$ and $b^{i,*}$ are optimal for all investors $i = 1, ..., I$ given opportunity cost RP and their respective information sets $\mathcal{F}^{NW}$ for a news watcher and $\mathcal{F}^{PW}$ for a price watcher*
2. *(a) the choice of N signals is optimal for every news watcher given that there are λI news watchers, and given the costs c and F*

   (b) receiving no signal is optimal for every price watcher given that there are λI news watchers receiving N signals,
3. *the market for the risky asset clears, $\sum_{i=1}^{I} x^{i,*} = \bar{x}$, and*
4. *investors' beliefs are consistent with the equilibrium outcome.*

Condition 2 is the main requirement in this definition. First of all, in equilibrium a news watcher must not want to object to the group representative about the choice of N. Or, in other words, she must want to read the N newspapers that she is required to buy and not want any further newspaper. Similarly, a price watcher must not have an incentive to switch group. If $N^* = 0$ or $\lambda^* = 0$ or both, everybody is a price watcher in equilibrium.

In the previous section on fully revealing prices we have seen that the equilibrium asset price (9) is a linear function of the signals $\sum_i \sum_j s_j^i$ and the certain asset supply $\bar{x}$. In the current framework, the supply X of the risky asset is uncertain and all news watchers buy copies of the same N newspapers (assumption 11). Yet, suppose that there is a unique financial market equilibrium under partly informative prices, in which the price will satisfy a very similar linear structure. Suppose,

$$RP = \pi_0 + \pi_S \sum_{j=1}^{N} S_j - \pi_X X \tag{13}$$

for three coefficients π_0, π_S, π_X to be determined. That this guess is right will be confirmed soon.

To make his portfolio choice at 10am, each price watcher takes into account how θ and RP are jointly distributed from a posterior perspective. At this time, he extracts all possible information from his observation of RP and infers the most likely realization of the dividend θ applying fact 2 (appendix A, p. 41). To update his beliefs to posterior beliefs, a price watchers departs from his pre-posterior knowledge. At 9am he knows that there are λI news watchers and that they read N newspapers. So, from a price watcher's perspective, the joint pre-posterior normal distribution of θ and RP has a vector of means $\bar{\mu}^{PW} = (\bar{\mu}_\theta; \pi_0 + \pi_S N \bar{\mu}_\theta - \pi_X \bar{x})^T$ and a variance-covariance matrix

$$\bar{\mathbf{\Sigma}}^{PW} = \begin{pmatrix} \bar{\tau}_\theta^2 & \pi_S N \bar{\tau}_\theta^2 \\ \pi_S N \bar{\tau}_\theta^2 & \pi_S^2 N \left(N \bar{\tau}_\theta^2 + \sigma_S^2\right) + \pi_X^2 \omega_X^2 \end{pmatrix}.$$

Recall that signals are conditionally normally distributed $S_j|\theta \sim \mathcal{N}(\theta, \sigma_S^2)$ so that $\mathbb{V}(S_j) = \mathbb{V}_\theta\left(\mathbb{E}\left[S_j \,|\theta\right]\right) + \mathbb{E}_\theta\left[\mathbb{V}\left(S_j \,|\theta\right)\right] = \bar{\tau}_\theta^2 + \sigma_S^2$.

When Wall Street opens, the price watcher observes RP, updates his pre-posterior to posterior beliefs applying fact 2, and arrives at the updated expected value of the dividend

$$\mathbb{E}\left[\theta \,|RP; \lambda, N\right] = \mu^{PW} = m_0^{PW} + m_{RP}^{PW} RP \tag{14}$$

and the updated variance of the dividend $\mathbb{V}\left(\theta \mid RP; \lambda, N\right) = (\tau^{PW})^2$, where

$$m_0^{PW} = \frac{(\pi_S^2 N \sigma_S^2 + \pi_X^2 \omega_X^2)\bar{\mu}_\theta - \pi_S N(\pi_0 - \pi_X \bar{x})\bar{\tau}_\theta^2}{\pi_S^2 N(N\bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \omega_X^2}, \tag{15-a}$$

$$m_{RP}^{PW} = \frac{\pi_S N \bar{\tau}_\theta^2}{\pi_S^2 N(N\bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \omega_X^2}, \tag{15-b}$$

$$(\tau^{PW})^2 = \frac{(\pi_S^2 N \sigma_S^2 + \pi_X^2 \omega_X^2)\bar{\tau}_\theta^2}{\pi_S^2 N(N\bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \omega_X^2}. \tag{16}$$

A news watcher proceeds in a similar manner. Given any choice of N that the news watcher group happens to take, she considers the pre-posterior joint normal distribution of θ, RP, and the N signals. Then she asks herself, what her posterior knowledge will be, once having received the signal realizations $s_1, ..., s_N$ and having observed RP. For this, she can take into account that nobody else will receive better information than she does. Other investors are either price watchers and receive no signal at all, or they are news watchers and receive exact copies of her own N signals. As a consequence, prices are fully redundant for her. Prices contain no additional information beyond the knowledge that she gets out of her N newspaper copies already. A formal proof of the redundancy of RP is given in appendix C (p. 44).

Therefore, a news watcher can disregard RP for her updating and simply apply fact 1 (p. 10). As a result, her posterior belief about the dividend is that it is normally distributed with conditional mean

$$\mathbb{E}\left[\theta \,|RP; s_1, ..., s_N; \lambda, N\right] = \mu^{NW} = m_0^{NW} + m_S^{NW} \sum_{j=1}^{N} s_j \tag{17}$$

and conditional variance $\mathbb{V}\left(\theta \mid RP; s_1, ..., s_N; \lambda, N\right) = (\tau^{NW})^2$, where

$$m_0^{NW} = \frac{\sigma_S^2 \bar{\mu}_\theta}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \tag{18-a}$$

$$m_S^{NW} = \frac{\bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \tag{18-b}$$

$$(\tau^{NW})^2 = \frac{\sigma_S^2 \bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \tag{19}$$

by fact 1.

We now know the subjective posterior distributions of all investors. Investors base their portfolio decisions on these posterior distributions, and demand $x^{i,*}$ as given by (5) for $i = PW, NW$. Asset markets at Wall Street must clear. So,

$$(1-\lambda) \cdot x^{PW,*} + \lambda \cdot x^{NW,*} = \frac{x}{I},$$

where x is the realization of the uncertain asset supply X. Hence, the realization of equilibrium price must satisfy

$$\begin{aligned} RP &= \frac{1}{(1-\lambda)\frac{1-m_{RP}^{PW}}{(\tau^{PW})^2} + \lambda \frac{1}{(\tau^{NW})^2}} \\ &\quad \left((1-\lambda)\frac{m_0^{PW}}{(\tau^{PW})^2} + \lambda \frac{m_0^{NW}}{(\tau^{NW})^2} + \lambda \frac{m_S^{NW}}{(\tau^{NW})^2} \sum_{j=1}^{N} s_j - \gamma \frac{x}{I} \right) \\ &= \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \left[(1-\lambda)\frac{\pi_S(\pi_S N - 1)}{\pi_S^2 N \sigma_S^2 + \pi_X^2 \omega_X^2} + \lambda \frac{1}{\sigma_S^2}\right] N} \\ &\quad \left(\frac{\bar{\mu}_\theta}{\bar{\tau}_\theta^2} - (1-\lambda)\frac{\pi_S N(\pi_0 - \pi_X \bar{x})}{\pi_S^2 N \sigma_S^2 + \pi_X^2 \omega_X^2} + \lambda \frac{1}{\sigma_S^2} \sum_{j=1}^{N} s_j - \gamma \frac{x}{I} \right). \end{aligned} \tag{20}$$

The second step follows from (15-a) through (16) and (18-a) through (19). We can now match the coefficients π_0, π_S, and π_X in equation (13) with the according terms in (20). This yields a non-linear equation system in three equations and the three unknowns π_0, π_S, π_X. The equation system happens to have a unique closed-form solution.

Lemma 5 *There exists a unique two-group financial market equilibrium for a given share* λ *of news watchers and a given number of signals* N *under equilibrium definition 2.*

Proof. The closed-form solution of this equilibrium is derived in appendix D, p. 45. Uniqueness can be established by assuming price to be a higher-order functional of $\sum_{j=1}^{N} S_j$ and X, and leading that assumption to a contradiction. ■

This financial market equilibrium is still a partial equilibrium, given that there are λI news watchers who purchase N signals each. Our main focus lies on its implications for the incentives to acquire information and the simultaneous information market equilibrium at the news stands.

A first insight is already implicit in (13) and (20). The financial market equilibrium is unaffected by investors' individual wealth because asset demand is independent of wealth for CARA utility. Information is merely a secondary good that helps investors make better portfolio decisions. So, the demand for information within in the news watcher group is going to be unaffected by wealth as well. Therefore, since investors only differ by level of wealth due to assumptions 2, 3 and 5, whatever is optimal for one group member will be optimal for all other group members. It is thus an admissible simplification to only consider one group representative from now on.

4.2 Incentives and externalities

This subsection will take a further step towards deriving the equilibrium at news stands. Without knowing the equilibrium levels of λ^* and N^* yet, we can already establish properties that any information equilibrium must exhibit.

To choose the number of newspapers N, the representative news watcher takes a look at her *ex ante* utility. Similarly, a price watcher looks at his respective *ex ante* utility to see how the signal choice of the news watcher group affects him as an externality. Taking pre-posterior expectations of (8), the *ex ante* utility of any investor $i = PW, NW$ is

$$\mathbb{E}^i_{pre}\left[U^{i,*}\right] = -k^i \cdot e^{\gamma \frac{R}{1+R}(F^i + cN^i)} \cdot \mathbb{E}^i_{pre}\left[e^{-\frac{1}{2}\frac{(\tau^i)^2}{1+R}\left(\frac{\mu^i - RP}{(\tau^i)^2}\right)^2}\right] \qquad (21)$$

for $k^i \equiv \frac{1+R}{R}(\delta R)^{\frac{1}{1+R}} \exp\left\{-\gamma \frac{R}{1+R} W^i_0\right\} > 0$. I have used the short hand $F^i \equiv \mathbf{1}(N^i \geq 1) \cdot F$ in (21) again to indicate that only news watchers have to pay the fixed cost if they buy at least one newspaper. Since news watchers are required to buy the same amount of signals N, we can formally also define $N^i \equiv \mathbf{1}(i = NW) \cdot N$ here.

The key term in (21) is

$$\frac{\mu^i - RP}{\tau^i} = \tau^i \, \frac{\mu^i - RP}{(\tau^i)^2}.$$

We could call it the expected excess-return-standard-deviation ratio, but I will mostly refer to it as the key term. Given the closed-form financial market equilibrium of lemma 5, this term can be expressed in closed form as a function of λ, N, and parameters for all investors $i = PW, NW$. Parameters are: the interest factor R; the prior means and variances $\bar{\mu}_\theta$, $\bar{\tau}_\theta^2$, σ_S^2; $\bar{x}$, ω_X^2; the degree of risk aversion γ; the discount factor δ; and the number of investors I (initial wealth W_0^i is irrelevant due to CARA utility). The particular solutions are less important than their properties. So, the explicit terms are not reported here but in appendix D (p. 45). As will become clear shortly, what matters for information acquisition are the two pre-posterior moments of the key term. These two moments are reported in appendix E (p. 46).

We know that the subjective variance of the dividend $(\tau^i)^2$ is certain for all investors (see (16) and (19)). We also know that both the posterior mean of the dividend μ^i is a sum of normal variables (see (14) and (17)) and the opportunity cost RP is a sum of normal variables (see (13)). Since the sum of normal variables is normally distributed, all investors can apply another convenient fact of the normal distribution—fact 5 in appendix A (p. 42)—to (21) and find their *ex ante* utility to be

$$\begin{aligned} \mathbb{E}_{pre}^i\left[U^{i,*}\right] = -k^i \cdot \exp\left\{\gamma \frac{R}{1+R}(F^i + cN^i)\right\} \qquad (22) \\ \cdot \frac{1}{\sqrt{1 + \frac{(\tau^i)^2}{1+R}\mathbb{V}_{pre}^i\left(\frac{\mu^i - RP}{(\tau^i)^2}\right)}} \exp\left\{-\frac{1}{2}\frac{(\tau^i)^2}{1+R}\frac{\left(\mathbb{E}_{pre}^i\left[\frac{\mu^i - RP}{(\tau^i)^2}\right]\right)^2}{1 + \frac{(\tau^i)^2}{1+R}\mathbb{V}_{pre}^i\left(\frac{\mu^i - RP}{(\tau^i)^2}\right)}\right\}. \end{aligned}$$

Since $\mathbb{E}_{pre}^i\left[U^{i,*}\right]$ is negative for CARA utility, any change that brings (22) closer to zero is beneficial. Hence, *ex ante* utility is increasing in the pre-posterior expected excess return of the risky asset $\mathbb{E}_{pre}^i\left[\mu^i - RP\right]$, as it should be. However, the variance of the expected excess return $\mathbb{V}_{pre}^i\left(\mu^i - RP\right)$ has an ambiguous effect on *ex ante* utility. On the one hand, an investor finds a higher variance of the expected excess return bad because she is risk averse. On the other hand, she knows that a higher variance of the difference between her return and the market expectations also makes it more likely, on average, that her portfolio yields a lot. So, the double role of the asset price as opportunity cost and information provider also imposes a double role on the variance of $(\mu^i - RP)$.

The representative news watcher maximizes *ex ante* utility $\mathbb{E}^{NW}_{pre}\left[U^{NW,*}\right]$ with respect to the number of newspapers N, given a share λ of news watchers. While making her choice, she does not take into account how N affects *ex ante* utility of the $1-\lambda$ price watchers. Even though the representative news watcher has to choose a discrete number of signals, it is instructive to take the derivative of (22) with respect to N. Under certain regularity conditions, the resulting condition could even be interpreted as close to a necessary first order condition for an optimal choice of N when set to zero.[8] However, I will not use it as a first order condition. Instead, I will use it as a tool to investigate whether the derivative has a certain sign, positive or negative, in general. Then it does not matter whether N is discrete or perfectly divisible. Similarly, the derivative of $\mathbb{E}^{PW}_{pre}\left[U^{PW,*}\right]$ with respect to N can be seen as representing the externality that an additional signal inflicts on price watchers. Taking the derivative and multiplying by the positive factor $-(1+R)/\mathbb{E}^{i}_{pre}\left[U^{i,*}\right]$ yields

$$\begin{aligned} -\frac{1+R}{\mathbb{E}^{i}_{pre}\left[U^{i,*}\right]}\frac{\partial \mathbb{E}^{i}_{pre}\left[U^{i,*}\right]}{\partial N} &= -\,\gamma\, R\, c\;\mathbf{1}(i=NW) \\ &+E^{i}(\lambda,N)\cdot\left[\varepsilon^{i}_{\tau^2,N}(\lambda,N)+\varepsilon^{i}_{\mathbb{E},N}(\lambda,N)\right] \\ &+V^{i}(\lambda,N)\cdot\left[\varepsilon^{i}_{\tau^2,N}(\lambda,N)+\tfrac{1}{2}\varepsilon^{i}_{\mathbb{V},N}(\lambda,N)\right]\cdot\frac{\Delta^{i}(\lambda,N)}{1+R}. \end{aligned} \tag{23}$$

The functions $\varepsilon^{i}_{y,N}(\lambda,N)$ denote the elasticity of y with respect to N. For example, $\varepsilon^{i}_{\mathbb{E},N}$ denotes the elasticity of $\mathbb{E}^{i}_{pre}\left[(\mu^i-RP)/(\tau^i)^2\right]$ with respect to N. The definitions of the terms $E^{i}(\lambda,N), V^{i}(\lambda,N)$, and $\Delta^{i}(\lambda,N)$ are

$$E^{i}(\lambda,N) \equiv \frac{1}{N}\frac{\left(\mathbb{E}^{i}_{pre}\left[\frac{\mu^i-RP}{(\tau^i)^2}\right]\right)^2}{\frac{1}{(\tau^i)^2}+\frac{1}{1+R}\mathbb{V}^{i}_{pre}\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}, \tag{24-a}$$

$$V^{i}(\lambda,N) \equiv -\frac{1}{N}\frac{\mathbb{V}^{i}_{pre}\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}{\frac{1}{(\tau^i)^2}+\frac{1}{1+R}\mathbb{V}^{i}_{pre}\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}, \tag{24-b}$$

$$\Delta^{i}(\lambda,N) \equiv \frac{\left(\mathbb{E}^{i}_{pre}\left[\frac{\mu^i-RP}{(\tau^i)^2}\right]\right)^2-\frac{1+R}{(\tau^i)^2}-\mathbb{V}^{i}_{pre}\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}{\frac{1}{(\tau^i)^2}+\frac{1}{1+R}\mathbb{V}^{i}_{pre}\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}, \tag{24-c}$$

[8] If $\mathbb{E}^{i}_{pre}\left[U^{i,*}\right]$ is changing monotonously in N, for example, the resulting condition is fine in the following sense: The condition gives rise to a continuous function $N^*(\cdot)$ of parameters that would indicate optimal signal choices in the continuous case, and, for $N^*\in\mathbb{N}_0$, it coincides with a step function $\hat{N}^*(\cdot)$ that captures the optimal signal choices in the discrete case.

Table 1: Signs of Elasticities

	$i = PW$	compare	$i = NW$
$\varepsilon_{(\mathbb{E}^i_{pre}[\theta]-RP),N}{}^a$	< 0	$=$	< 0
$\varepsilon^i_{\tau^2,N}{}^b$	< 0	$\varepsilon^{PW}_{\tau^2,N} < \varepsilon^{NW}_{\tau^2,N}$ $\Leftrightarrow \lambda IN > \frac{\gamma\sigma^2_S\ \omega_X}{\bar{\tau}_\theta}$	< 0
$\varepsilon^i_{\tau^2,N} + \varepsilon^i_{\mathbb{E},N}{}^b$	< 0	$=$	< 0
$\varepsilon^i_{\tau^2,N} + \frac{1}{2}\varepsilon^i_{\mathbb{V},N}{}^b$	< 0	$<$	*ambiguous*

[a]This follows from (E.1) with (E.3) and (E.4) with (E.6) in appendix E, p. 46
[b]Elasticities are reported as (F.2) through (F.7) in appendix F, p. 48

respectively.

We can interpret the derivative (23) as follows. γRc is the marginal utility loss from an additional signal as it reduces wealth. The second term on the right hand side of (23) reflects the marginal utility change that stems from a change in $\tau^i\,\mathbb{E}^i_{pre}\left[(\mu^i - RP)/(\tau^i)^2\right] = \mathbb{E}^i_{pre}\left[(\mu^i - RP)/\tau^i\right]$. Similarly, the third term captures the change of the variance $(\tau^i)^2\,\mathbb{V}^i_{pre}\left((\mu^i - RP)/(\tau^i)^2\right) = \mathbb{V}^i_{pre}\left((\mu^i - RP)/\tau^i\right)$ and its impact on utility. The factor $\Delta^i(\lambda, N)$ is proportional to

$$\left(\mathbb{E}^i_{pre}\left[\frac{\mu^i - RP}{\tau^i}\right]\right)^2 - (1+R) - \mathbb{V}^i_{pre}\left(\frac{\mu^i - RP}{\tau^i}\right)$$

and can thus be positive or negative. It reflects the ambiguous effect that an increase in the variance has on *ex ante* utility.

Table 1 displays signs of elasticities. They indicate how more newspapers affect important variables that enter investors' *ex ante* utility. Just as under fully revealing prices before, the pre-posterior expected excess return $(\mathbb{E}^i_{e.a}\left[\theta\right] - RP)$ is falling in N for both groups of investors. Price watchers and news watchers even perceive the relative strength of this effect as the same (first row of table 1). More information brings expected return and opportunity cost closer to each other as individual beliefs become more similar to market beliefs. This is bad (as row three confirms). However, more information also reduces the dividend's pre-posterior variance for both types of investors (second row). This can be good or bad for utility. Moreover, the portfolio variance, that is the pre-posterior variance of the key term, falls

Table 2: SIGNS OF UTILITY EFFECTS

	$i = PW$	comp.	$i = NW$
$E^i \cdot (\varepsilon^i_{\tau^2,N} + \varepsilon^i_{\mathbb{E},N})^a$	< 0	$>$	< 0
$V^i \cdot (\varepsilon^i_{\tau^2,N} + \frac{1}{2}\varepsilon^i_{\mathbb{V},N})^a$	> 0	-	*ambiguous*
$\Delta^i/(1+R)^{a,b}$	$< 0 \quad \Leftrightarrow$ $\bar{x}^2 < (\bar{x}_c^{\Delta,PW})^2$	$<$	$< 0 \quad \Leftrightarrow$ $\bar{x}^2 < (\bar{x}_c^{\Delta,NW})^2$

[a]For derivations see appendix F, p. 47
[b]Definitions of the threshold values $\bar{x}_c^{\Delta,i}$ are given in (F.14) and (F.15), p. 52

for price watchers, but it may rise of fall for news watchers (last row). To make more definitive statements we need to look at the complete terms in condition (23).

Table 2 gives an overview of the signs of major terms in condition (23). The first row is no surprise any longer: More information has a negative impact on utility because it reduces the pre-posterior expectation of the key term $\mathbb{E}^i_{pre}\left[(\mu^i - RP)/\tau^i\right]$. Both price and news watchers agree that they dislike this. Price watchers perceive this negative effect as less pronounced in absolute terms. They only put a little more weight on the price when extracting information, and a little less weight on their priors. This brings the expected value $\mathbb{E}^{PW}_{pre}\left[\mu^{PW}\right]$ a little closer to the price, but not too much. News watchers, however, do feel the reduction in $\mathbb{E}^{NW}_{pre}\left[(\mu^{NW} - RP)/\tau^{NW}\right]$ from both sides. First, the signal realizations enter $\mathbb{E}^{NW}_{pre}\left[\mu^{NW}\right]$ directly and news watchers start putting more weight on the signal realizations, and less on their priors. Since some other investors also receive the same signals, this brings $\mathbb{E}^{NW}_{pre}\left[\mu^{NW}\right]$ closer to RP. At the same time, price watchers start updating their believes, and the price RP also starts moving closer to $\mathbb{E}^{NW}_{pre}\left[\mu^{NW}\right]$. For news watchers, the excess return is narrowed with "double" speed, so to say.

Overall, the impact of an additional newspaper on utility is ambiguous for news watchers. The reason is that the effect of an additional signal on the variance $\mathbb{V}^{NW}_{pre}\left((\mu^{NW} - RP)/\tau^{NW}\right)$ is indeterminate (second row in table 2). So, there is hope that news watchers are going to acquire information in equilibrium, but they might also prefer no newspaper at all.

Things are more immediate for price watchers. If the stock market is a "small market," that is if the expected supply of risky assets is smaller than a cutoff value so that $\bar{x}^2 < \left[\bar{x}_c^{\Delta,PW}(\lambda,N)\right]^2$, then $\Delta^{PW} < 0$. As a consequence, any signal to news watchers must have the character of a pure negative externality for price watchers if it falls below the threshold $\bar{x}_c^{\Delta,PW}(\lambda,N)$ in absolute value (second and third row in table 2). What if markets are large in size so that $\Delta^{PW} \geq 0$? Can it happen that this effect becomes so strong that the entire condition (23) turns positive for price watchers? As it turns out, the answer is no. The positive effect of more information through the variance can never outweigh the negative effect through a diminished excess return. So, in the present model, more information always inflicts a strictly negative externality on price watchers.

Theorem 6 *Any signal to news watchers inflicts a negative externality on price watchers in a two-group rational expectations equilibrium (definition 2).*

Proof. In appendix G, p. 53. ■

This is a strong result. One might imagine that, when markets are very large in size and the noise in price matters little, price watchers could extract extremely much information from price, and strongly benefit from the variance-lowering effect. This is not the case in the current framework. The utility-reducing effect of a shrinking excess return is always stronger.[9]

When taking her decision about newspaper acquisition, the news watcher representative does not care about this externality of her decision. She exclusively considers her private incentives. And her incentives happen to coincide with all other news watchers' incentives because the only difference among them is their initial wealth W_0^i, and that does not matter as condition (23) shows. Evaluating condition (23) is difficult in general since the effect of an additional newspaper on news watchers' pre-posterior variance $\mathbb{V}_{pre}^{NW}\left((\mu^{NW} - RP)/\tau^{NW}\right)$ is ambiguous (table 2). It is therefore instructive to see how condition (23) behaves in the limits.

Imagine the extreme case that the club of news watchers has attracted every single investor. Then the incentive to acquire newspapers becomes

$$\lim_{\lambda \to 1} -\frac{1+R}{\mathbb{E}_{pre}^{NW}\left[U^{NW,*}\right]} \frac{\partial \mathbb{E}_{pre}^{NW}\left[U^{NW,*}\right]}{\partial N} = -\gamma\, R\, c - \frac{(1+R)\gamma^2\sigma_S^2\bar{\tau}_\theta^4}{\sigma_S^2 + N\bar{\tau}_\theta^2} \cdot \frac{I^2(1+R)(\sigma_S^2 + N\bar{\tau}_\theta^4)(\bar{x}^2 + \omega_X^2) + \gamma^2\sigma_S^2\bar{\tau}_\theta^2\omega_X^4}{\left[I^2(1+R)(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\bar{\tau}_\theta^2\omega_X^2\right]^2} < 0. \tag{25}$$

[9]This raises the question whether price watchers should rather stop watching. Would it be rational to stay ignorant? To answer this question, we have to alter the equilibrium concept because news watchers rationally anticipate that price watchers prefer to ignore the information in price. This changes how price responds to more information. The resulting equilibrium remains to be worked out.

In other words, there is a strong disincentive to receive information even if newspapers are for free. This establishes

Theorem 7 *There must be at least one price watcher in a two-group rational expectations equilibrium (definition 2).*

In any rational expectations equilibrium (definition 1),

- *either at least one investor must receive less signals than any other investor (when signals are sold in perfect copies),*
- *or at least one investor must acquire a signal that no other investor has received.*

Proof. Suppose every investor became a news watcher in a two-group rational expectations equilibrium (definition 2), then news watchers would want to acquire no signal by (25), a contradiction. The limit (25) itself follows from (F.9), (F.11), and (F.13) in appendix F (p. 49).

Now consider the more general case of definition 1. If signals are sold in perfect copies as supposed in this section, then a symmetric equilibrium under definition 1 in which all investors receive the same number of signals coincides with an equilibrium that involves $\lambda = 1$ under definition 2. However, this kind of equilibrium does not exist. So, at least one investor must receive less of the same signals or at least one investor must receive a signal that nobody else received in any rational expectations equilibrium. ∎

In other words, a symmetric equilibrium cannot exist in which all investors would receive a positive number of copies of the same newspapers. It doesn't even exist if newspapers are free of charge. The statement shows that our definition of a two-group equilibrium has not been so restrictive after all. There must be at least two groups of differently informed investors in any rational expectations equilibrium. They may not choose to receive the same copies, though. More generally, theorem 7 further supports the insight that investors dislike agreement and do not want information to become too common.

The most important question is, however, what happens in the other extreme. What does condition (23) look like when there is no news watcher yet? The potential news watcher representative asks herself whether she should start a news watcher club with one member—herself. As it turns out,

$$\lim_{\lambda\to 0} -\frac{1+R}{\mathbb{E}^{NW}_{pre}\left[U^{NW,*}\right]}\frac{\partial \mathbb{E}^{NW}_{pre}\left[U^{NW,*}\right]}{\partial N} \quad = \quad -\gamma\, R\, c \tag{26}$$

$$+\frac{I^2(1+R)\sigma_S^2\bar{\tau}_\theta^2}{2(\sigma_S^2+N\bar{\tau}_\theta^2)\left[I^2\left[(1+R)\sigma_S^2+N\bar{\tau}_\theta^2\right]+\gamma^2\bar{\tau}_\theta^2\omega_X^2(\sigma_S^2+N\bar{\tau}_\theta^2)\right]^2}$$

$$\cdot\left(I^2\left[(1+R)\sigma_S^2+N\bar{\tau}_\theta^2\right]+\gamma^2\bar{\tau}_\theta^2(\sigma_S^2+N\bar{\tau}_\theta^2)(\omega_X^2-\bar{x}^2)\right).$$

Even if she were to become the solely informed investor in the market, a representative news watcher may prefer to remain dumb. Apart from the uninteresting case of a prohibitively high newspaper price c, this is likely to occur if $\bar{x}^2$ is relatively high compared to ω_X^2. Then the last factor in (26) can turn negative. In other words, nobody may want to become informed, if markets are large! Why? In large markets, given a level of noise in the price ω_X^2, the asset prices is very informative for price watchers. News watchers know that price watchers will start putting a lot of weight on the observed price and little weight on their priors. As a result, news watchers must rationally anticipate that the expected excess return $\mathbb{E}^{NW}_{pre}\left[\mu^{NW}-RP\right]$ is falling quite strongly with every newspaper as opportunity cost RP moves closer to $\mathbb{E}^{NW}_{pre}\left[\mu^{NW}\right]$ while price watchers are updating their beliefs. So, market size and informativeness of the price are closely linked for the incentives to acquire information.

Formally, condition (26) implies that, in the limit where $\lambda = 0$, the threshold of market size is given by

$$\lim_{\lambda\to 0} -\frac{1+R}{\mathbb{E}^{NW}_{pre}\left[U^{NW,*}\right]}\frac{\partial \mathbb{E}^{NW}_{pre}\left[U^{NW,*}\right]}{\partial N} < 0 \quad \Leftrightarrow \quad \bar{x}^2 > \left[\bar{x}^{news}_{c,\lambda=0}(N;c)\right]^2,$$

with a cutoff value

$$\bar{x}^{news}_{c,\lambda=0}(N;c)^2 \equiv \frac{I^2\left((1+R)\sigma_S^2N\bar{\tau}_\theta^2\right)+\gamma^2\bar{\tau}_\theta^2\omega_X^2(\sigma_S^2+N\bar{\tau}_\theta^2)}{I^2\gamma^2\bar{\tau}_\theta^2(\sigma_S^2+N\bar{\tau}_\theta^2)} \tag{27}$$

$$\cdot\left(1-\frac{2R\gamma(\sigma_S^2+N\bar{\tau}_\theta^2)\left(I^2\left((1+R)\sigma_S^2N\bar{\tau}_\theta^2\right)+\gamma^2\bar{\tau}_\theta^2\omega_X^2(\sigma_S^2+N\bar{\tau}_\theta^2)\right)}{I^2(1+R)\sigma_S^2\bar{\tau}_\theta^2}\,c\right).$$

Theorem 8 † *Suppose the incentive to acquire a signal is strongest when* $\lambda = 0$*, then the following is true.*

†As for now, theorem 8 is presented in a fairly weak form which I hope to strengthen. The theorem is stated under the assumption that condition (23) is maximal at $\lambda = 0$. I conjecture, however, that this is generally the case, and no assumption in fact. One can show that condition (23) is strictly falling in λ at $\lambda = 0$. In addition, as the limit at $\lambda = 1$ has shown, condition (23) must ultimately drop below zero. Yet, a proof that condition (23) is maximal at $\lambda = 0$ remains to be given.

There are only price watchers in a two-group rational expectations equilibrium (definition 2) if risky asset supply exceeds a threshold such that $\bar{x}^2 \geq \left[\bar{x}^{news}_{c,\lambda=0,N=0}(c)\right]^2$.

Proof. Under the assumption made, condition (23) takes its maximum at $\lambda = 0$ (on the interval $\lambda \in [0,1]$). So, if (23) is smaller than zero at $\lambda = 0$, it cannot exceed zero for any other value of λ, given N. Thus, no news watcher would want to buy a signal under a sufficient condition that forces (26) below zero, and there will only be price watchers in a two-group rational expectations equilibrium (definition 2).

The limit (26) follows from (F.9), (F.11), and (F.13) in appendix F (p. 49). It is linear in $\bar{x}^2$. Setting it equal to zero, and solving out for $\bar{x}^2$ yields the threshold level (27). Condition (27) is sufficient for no signal acquisition to occur under the assumption made. It is not a necessary condition due to the indivisibility of signals. ■

Since investors can go long or short in the risky asset, this result depends on market size in absolute value (the square of $\bar{x}$). Information is the more valuable for news watchers the smaller markets are. The reason is that "size of markets" is just the flip side of "informative prices." News watchers do not want price watchers to free-ride on their newspaper acquisitions because that reduces the expected excess return of the risky asset. So, the larger markets, the more informative prices are for price watchers, and the stronger the negative effect of price watchers' updating on news watchers utility. The close relation between market size and the informativeness of price would not change if the noise in the price system came from another source than asset supply. Looking (far back) at the structure of equilibrium price in (13), we could also have added Gaussian white noise to the price, and results would have carried over.

Table 3 reports some further noteworthy limits. When the price system becomes extremely informative as $\omega_X \to 0$, news watchers perceive the negative impact on the excess return more strongly than the positive impact on the variance of the excess return and prefer to be price watchers. This is nothing but the extreme case of the preceding section 3 where prices were fully revealing. On the other extreme, when the price system ceases to be informative as $\omega_X \to \infty$, news watchers must not fear a negative impact in excess return any more. However, the potentially positive effect on the variance of the excess return turns negative because, if ω_X^2 is large, RP gets more noisy with more newspapers. When investors become extremely risk averse ($\gamma \to \infty$), they loose their interest in risky assets and consequently their interest in information. In all these cases, news watchers would not even want to accept a signal for free.

Table 3: Incentives for News Watchers in the Limit

	$E^{NW}(\varepsilon^{NW}_{\tau^2,N} + \varepsilon^{NW}_{\mathbb{E},N})$	$V^{NW}(\varepsilon^{NW}_{\tau^2,N} + \frac{1}{2}\varepsilon^{NW}_{\mathbb{V},N})\frac{\Delta^{NW}}{1+R}$
$\lim_{\omega_X\to 0}{}^a$	$-\frac{\gamma^2\sigma_S^2\bar{\tau}_\theta^4}{I^2(\sigma_S^2+N\bar{\tau}_\theta^2)^2}\bar{x}^2$	0
$\lim_{\omega_X\to\infty}{}^a$	0	$-\frac{(1+R)\lambda\bar{\tau}_\theta^2}{\sigma_S^2+\lambda N\bar{\tau}_\theta^2}$
$\lim_{\gamma\to 0}{}^a$	0	0
$\lim_{\gamma\to\infty}{}^a$	$-\frac{(1+R)\lambda\bar{\tau}_\theta^2}{(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\omega_X^2}\bar{x}^2$	$\frac{(1+R)\lambda\bar{\tau}_\theta^2}{(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\omega_X^2}(\bar{x}^2-\omega_X^2)$
$\lim_{1/\sigma_S\to 0}{}^a$	0	0
$\lim_{1/\sigma_S\to\infty}{}^a$	0	0

[a]Limits follow from (F.9) and (F.11) in appendix F, p. 49

When investors become risk neutral ($\gamma \to 0$), they do not mind receiving signals for free, but they would never want to pay for it—as lemma 1 has established before in general. Similarly, when a signal is absolutely imprecise ($1/\sigma_S^2 \to 0$) investors are indifferent about receiving it or not: It does neither harm nor good, but never pay for it. Finally, when signals become absolutely precise and reveal the realization of θ itself as $1/\sigma_S^2 \to \infty$, news watchers would accept it but not pay for it. Such an infinitely precise signal turns the previously risky asset into a second, riskless bond and mandates that RP equal θ/R. In this extreme case, the two assets become perfect substitutes.

4.3 The information market equilibrium

The previous subsection has characterized properties of an equilibrium at the news stands. It remains to derive this information equilibrium itself. Suppose again that we can treat the number of signals N as if it were close to perfectly divisible.[10] Then, the news watcher representative can decide the equilibrium amount of information by looking at condition (23) and setting

[10]See footnote 8 (p. 25).

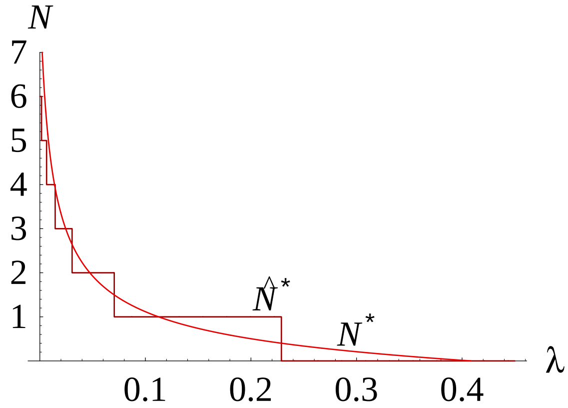

Figure 2: **Optimal Choice of** N

it to zero

$$
\begin{aligned}
-\frac{1+R}{\mathbb{E}^{NW}_{pre}\left[U^{NW,*}\right]}\frac{\partial \mathbb{E}^{NW}_{pre}\left[U^{NW,*}\right]}{\partial N} &= -\gamma R c \\
&+\frac{1}{N}\,E^{NW}(\lambda,N)\left[\varepsilon^{NW}_{\tau^2,N}(\lambda,N)+\varepsilon^{NW}_{\mathbb{E},N}(\lambda,N)\right] \\
&+\frac{1}{N}\,V^{NW}(\lambda,N)\left[\varepsilon^{NW}_{\tau^2,N}(\lambda,N)+\tfrac{1}{2}\varepsilon^{NW}_{\mathbb{V},N}(\lambda,N)\right]\frac{\Delta^{NW}(\lambda,N)}{1+R}=0.
\end{aligned} \tag{28}
$$

The news watcher representative chooses N given the share λ of members in the news watcher club (definition 2). So, the above condition implies an equilibrium amount of signals $N^*(\lambda; c)$. Unfortunately, the acquisition rule $N^*(\lambda; c)$ has no closed form (but can be shown to be a polynomial of ninth degree). Things are getting easier, however, if we look at them graphically.

The falling curve in figure 2 is a plot of condition (28).[11] It shows combinations of N and λ for which (28) is satisfied. Or, put in economic terms, this curve shows the optimal choice of N^* if it were continuous. The curve shifts to the Southwest when the cost of a signal c increases as can be seen from (28). Since signal choice has to be concrete, however, the optimal choice of N^* given λ is a step function $\hat{N}^*(\lambda; c)$. Figure 2 also depicts this proper "newspaper acquisition curve." The two curves show that condition (28) does a pretty good job for a large number of signals, but is not so helpful when N gets small.

Any equilibrium must occur along the "newspaper acquisition curve" $\hat{N}^*(\lambda; c)$. Given news watchers' anticipated choice of $\hat{N}^*(\lambda; c)$, each investor

[11] The underlying parameter values are $I = 100$; $c = .005$, $F = 10c$; $R = 1.1$, $\bar{\mu}_\theta = 1.3$; $\bar{x} = 1$; $\gamma = 1$, $\sigma_S = 1$, $\bar{\tau}_\theta = 1$, $\omega_X = 100$; $\delta = .9$, $W = 1$.

decides whether to become a price watcher or a news watcher. In equilibrium, every news watcher must find it preferable to remain a news watcher. Her *ex ante* utility must be weakly higher than a price watcher's *ex ante* utility. Formally, $\mathbb{E}_{pre}^{NW}\left[U^{NW,*}(N,\lambda;c,F)\right] \geq \mathbb{E}_{pre}^{PW}\left[U^{PW,*}(N,\lambda)\right]$. Similarly, every price watcher must find it preferable not to become a news watcher. This implies $\mathbb{E}_{pre}^{PW}\left[U^{PW,*}(N,\lambda)\right] \geq \mathbb{E}_{pre}^{NW}\left[U^{NW,*}(N,\lambda;c,F)\right]$. As a result,

$$\mathbb{E}_{pre}^{NW}\left[U^{NW,*}(N,\lambda;c,F)\right] - \mathbb{E}_{pre}^{PW}\left[U^{PW,*}(N,\lambda)\right] = 0 \tag{29}$$

must hold in equilibrium. Given news watchers' signal choice N, this condition implies an equilibrium share of news watchers $\lambda^*(N;c,F)$. It also implies that the initial wealth of investors within each group must be the same if the same fixed information cost F applies to everyone. To keep things interesting, make a final

Assumption 12 (Same wealth) *Initial wealth is identical, $W_0^i = W_0$, across all investors $i = 1,...,I$.*

This assumption would not be needed if we allowed for more than only two groups of investors. Then, however, no closed-form financial market equilibrium would exist.

In equilibrium, every news watcher receives N^* signals for c dollars each and pays the fixed cost F. So, a news watcher's *ex ante* utility depends on both c and F. Thus, condition (29) also depends on c and F implicitly. As a consequence, it is not of much concern that both N and $\lambda \equiv I^{NW}/I$ are not perfectly divisible. Either the newspaper price c or the fixed information cost F, or both, adjust to clear the markets accordingly.

Figure 3 shows contour plots of condition (29) for various levels of the fixed cost F.[12] These "indifference contours" need not satisfy a functional relationship between N and λ. In fact, they are mostly correspondences. By varying F we can find a combination of N and λ that lies on the "newspaper acquisition curve" *and* on an according "indifference contour." This is one equilibrium. By varying F further, we can find several additional combinations of N and λ that lie on the "newspaper acquisition curve" at some other point. So, the information equilibrium need not be unique. For many different levels of F one may find an equilibrium pair $(N^*(c,F), \lambda^*(c,F))$ that makes this particular F an equilibrium price together with some unit price c that is implicit in both the "newspaper acquisition curve" and the "indifference contour."

Lemma 9 *Countably many two-group rational expectations equilibria (definition 2) may exist.*

[12] Parameter values are the same as in figure 2. In addition, $W = 1$. See footnote 11 (p. 33).

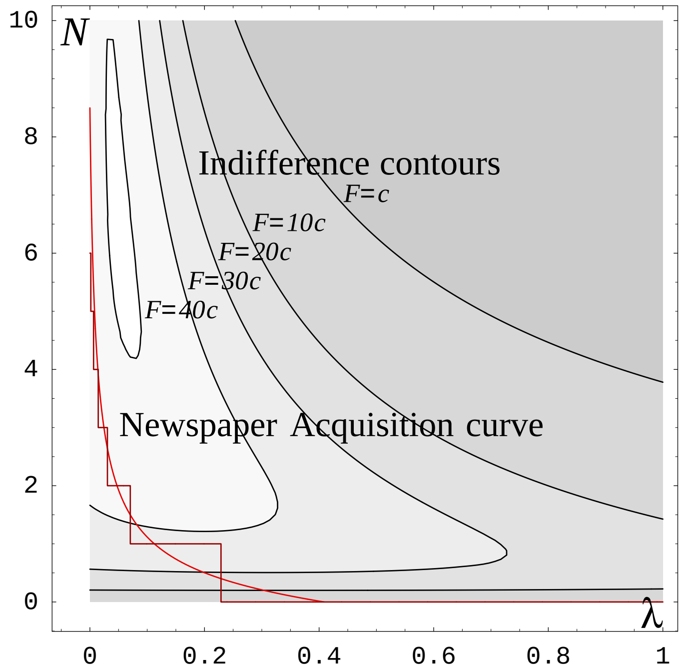

Figure 3: **Equilibrium Combinations of N and λ**

Proof. The number of equilibria must be countable because N is an integer. Parameters permitting, we can construct examples as in figure 3 in which multiple equilibria can be found by varying the fixed information cost F. ■

So, the equilibrium at the news stands at 9am need not be unique, whereas the partial equilibrium at Wall Street at 10am will be unique *given* N^* and λ^*. The number of signals N has to be discrete. This makes it hard to derive general conditions under which there are at least two equilibria.

The previous argument also suggests that the fixed information cost F will take a strictly positive value in many equilibria. In fact, it must always be strictly positive. Recall that price watchers suffer a negative externality and are strictly worse off than news watchers if the fixed information cost F is zero. Consequently, no information equilibrium with a positive amount of information can exist for $F = 0$ as long as at least one investor has an incentive to become a news watcher.

Theorem 10 *An equilibrium (definition 2) with at least one news watcher requires a fixed information cost F that is strictly positive.*

Proof. Suppose there is at least one news watcher, then $N^* \geq 1$. In addition, by theorem 7 there must be at least one price watcher, $\lambda^* < 1$. Further suppose that $F = 0$. Since a news watcher is free to choose N, it must be the case that $\mathbb{E}_{pre}^{NW}\left[U^{NW,*}(N^*)\right] \geq \mathbb{E}_{pre}^{NW}\left[U^{NW,*}(N=0)\right]$ by revealed preference. By theorem 6 price watchers face a negative externality so that they suffer a utility loss $\mathbb{E}_{pre}^{PW}\left[U^{PW,*}(N \geq 1)\right] < \mathbb{E}_{pre}^{PW}\left[U^{PW,*}(N=0)\right]$. Since, $\mathbb{E}_{pre}^{NW}\left[U^{NW,*}(N=0)\right] = \mathbb{E}_{pre}^{PW}\left[U^{PW,*}(N=0)\right]$ we can infer that, for small markets, $\mathbb{E}_{pre}^{NW}\left[U^{NW,*}(N \geq 1)\right] > \mathbb{E}_{pre}^{PW}\left[U^{PW,*}(N \geq 1)\right]$. So, in equilibrium a strictly positive fixed information cost F must bring news watchers' utility down to price watchers' utility. ■

In the present framework, information has to be priced with a two-part tariff. Otherwise no equilibrium at the news stands would exist as long as at least one investor has an incentive to become a news watcher. Since we know from theorem 7 that there must be at least two groups of differently informed individuals in general (equilibrium definition 1), the present theorem 10 also hints at the general case. If information inflicts a negative externality on at least one investor, at least the best informed group of investors must pay a strictly positive fixed cost to access information. Otherwise no equilibrium exists. Even "large" markets may require that the fixed information cost is strictly positive in equilibrium, but they need not. In general, the utility difference (29) is a complicated function of λ, N, c, and F.[13]

4.4 Informational efficiency

Are the equilibria at news stands informationally efficient? That is, would a benevolent social planner allocate signals to investors in the same manner? A benevolent social planner maximizes $\sum_{i=1}^{I} \mathbb{E}_{pre}^{i}\left[U^{i,**}\right]$ with respect to $\{N^1, ..., N^I\}$. Since there is no closed-form solution to the financial market equilibrium in general, it is difficult to characterize the unconstrained social optimum. However, we can investigate the welfare properties of two-group equilibria (definition 2) in the current context. A benevolent social planner can dictate the news watcher group to buy N^{**} signals for each member, charging every news watcher the marginal cost c of signal provision. To keep matters simple, suppose c is precisely the marginal cost of the newspaper copy and does not include the production of the newspaper content, for instance. Then a social planner will find a charge of c for each copy the right price, and we can focus on further welfare aspects of the equilibrium.

So, the social planner will maximize $(1-\lambda)\mathbb{E}_{pre}^{PW}\left[U^{PW,**}\right] + \lambda\mathbb{E}_{pre}^{NW}\left[U^{NW,**}\right]$ given c, where $U^{i,**}$ denotes posterior indirect utility after the social planner

[13]The present model still contains Grossman and Stiglitz' (1980) version as a special case. There are some noteworthy differences in the equilibria that result, however, on which I comment in appendix H (p. H).

has interfered at the news stands.[14] Treating signals as if they were perfectly divisible, we can differentiate this weighted average with respect to N, given λ, and find

$$\begin{aligned}
&-\frac{1+R}{\mathbb{E}_{pre}^{NW}\left[U^{NW,**}\right]}\frac{\partial}{\partial N}\left((1-\lambda)\mathbb{E}_{pre}^{PW}\left[U^{PW,**}\right]+\lambda\mathbb{E}_{pre}^{NW}\left[U^{NW,**}\right]\right)= \\
&= -\lambda\,\gamma\,R\,c \\
&\quad+\lambda\left(E^{NW}\left[\varepsilon_{\tau^2,N}^{NW}+\varepsilon_{\mathbb{E},N}^{NW}\right]+V^{NW}\left[\varepsilon_{\tau^2,N}^{NW}+\tfrac{1}{2}\varepsilon_{\mathbb{V},N}^{NW}\right]\tfrac{\Delta^{NW}}{1+R}\right) \\
&\quad+(1-\lambda)\frac{\mathbb{E}_{pre}^{PW}\left[U^{PW,**}\right]}{\mathbb{E}_{pre}^{NW}\left[U^{NW,**}\right]} \\
&\qquad\cdot\left(E^{PW}\left[\varepsilon_{\tau^2,N}^{PW}+\varepsilon_{\mathbb{E},N}^{PW}\right]+V^{PW}\left[\varepsilon_{\tau^2,N}^{PW}+\tfrac{1}{2}\varepsilon_{\mathbb{V},N}^{PW}\right]\tfrac{\Delta^{PW}}{1+R}\right). \qquad (30)
\end{aligned}$$

From the preceding analysis we know that, in a market equilibrium, at least one investor must be a price watcher. A social planner clearly agrees. For $\lambda = 1$, the last term in condition (30) vanishes. Moreover, the second term on the right hand side turns negative: If the news watcher group included every single investor then any newspaper would strictly reduce news watchers' *ex ante* utility (theorem 7). So, it cannot be socially desirable that all investors read the same N newspapers, even if newspapers are for free.

We know that any signal inflicts a negative externality on price watchers (from theorem 6). So, the last term in condition (30) is always negative. In addition, we know that when markets are large in size, news watchers do not even have an incentive to buy a newspaper if the group has only one member (theorem 8). Again, a social planner agrees.

Theorem 11 *An informationally efficient allocation of signals*

- *has to be asymmetric so that at least one investor receives either less or different signals if the allocation involves a positive number of signals;*
- *will involve zero information if risky asset supply exceeds a threshold such that* $\bar{x}^2 \geq \left[\bar{x}_{c,\lambda=0,N=0}^{news}(c)\right]^2$ *(provided the incentive to acquire a signal is strongest when* $\lambda = 0$*).*

Proof. By an extension of theorems 7, 6 and 8, and condition (30). The threshold for market size $\left[\bar{x}_{c,\lambda=0,N=0}^{news}(c)\right]^2$ is given in (27). ■

Loosely speaking, a social planner agrees with the market outcomes in the extremes. However, this is not so in general. Since every signal causes

[14] The social planner cannot transfer knowledge between investors so that *ex ante* utility is taken with respect to investors' individual pre-posterior beliefs.

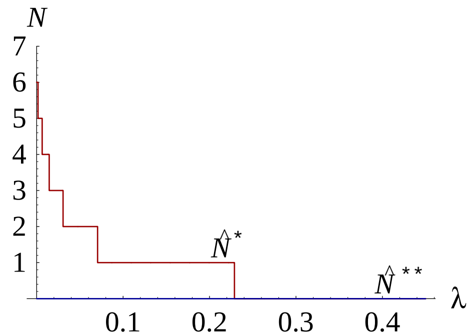

Figure 4: **Informationally Efficient Choice of N**

a negative externality to price watchers, a social planner would tell news watchers to acquire less signals for every given share λ of news watchers. Suppose signals were perfectly divisible. Then the social planner strictly prefers a signal allocation in which less signals than in the market equilibrium are given to news watchers. Since signals have to be purchased in integer numbers, however, the social planner might settle with the market outcome for ranges of equilibria. Market outcomes could be informationally efficient by coincidence, so to say.

Claim 12 *When there is at least one news watcher in equilibrium (definition 2), markets provide inefficiently much information in the following sense. A benevolent social planner would, for any given λ^*, allocate strictly less signals if signals were perfectly divisible.*

Proof. By theorem 6 and (30). ■

Figure 4 depicts the "newspaper acquisition" curves for a news watcher representative and for a social planner under the same parameter as used in the preceding figures.[15] The social planner would rather prefer to implement no information at all instead of having a group of news watchers read perfect copies of the same newspapers. I still have to analyze whether this only occurs for the presently chosen parameter values or whether it is generally the case. An implication of this finding may be: A positive amount of information can only be informationally efficient if news watchers receive different information. A social planner likes investors to hold heterogeneous beliefs. We can conclude, however, that the market outcome in two-group rational expectations equilibria tends to be informationally inefficient. Too many signals are likely to be purchased in equilibrium.

[15] See footnote 11 (p. 33).

5 Conclusion

The present paper has investigated investors' incentives to acquire information about a risky asset and compared their choice to a social planner's preferred allocation of information. It has thus given a formal notion to informational efficiency in financial markets. Gaussian random variables and CARA utility were used to obtain closed-form solutions where possible. Beyond previous work, the model allowed investors to choose a finite number of information sources.

As the framework has clarified, information is not a good (or bad) in its own right. It is only valuable inasmuch investors anticipate to act upon it. Therefore, risk averse investors never want to buy information. In addition, information can have features of a bad in many circumstances. If prices are *fully revealing*, for instance, every investor perceives more information as detrimental. Since their information enters price and gets fully revealed to all other investors, the "market information" gets closer to each investor's own beliefs. In other words, each investor's dividend expectation and the opportunity cost of buying the asset are moving towards each other. The reason is that asset price precisely reflects average expectations in the market. This reduces the expected return from holding a risky asset, which is to say that each investor must expect to receive a lower average payoff the more information she buys. The negative effect of this is so strong that it always outweighs the benefits of information under fully revealing prices. As a consequence, no investor wants to buy information when prices are fully revealing.

When prices are noisy and only *partly informative* about other investors' information, this effect remains present. Information acquisition by the informed investors always inflicts a strict negative externality on the less informed investors who do not purchase own information but merely observe the price realization. For the less informed, the beneficial effect of more precise information never outweighs the loss from a reduced expected return. When markets are not too large, so that prices do not become too informative, there is a group of investors who prefer to buy information. It can never be the case that this group includes all investors if signals are sold in perfect copies. Yet, some fraction of investors may choose to become informed. More information lowers the expected variance of their portfolio, which raises their *ex ante* utility as they are risk averse.

A benevolent social planner agrees that markets should never make everybody equally well informed. Whenever there are some informed investors in equilibrium, markets are informationally inefficient because they involve *more* information acquisition than a social planner would implement. Informed investors do not account for the negative externality that they inflict on the less informed. The presence of a negative externality raises the question whether less informed investors should stop extracting information from price, and only rely on their own priors. This is to be investigated in future versions of the paper.

The framework has assumed that investors are price takers when they make their portfolio decision but that they are fully aware of how their acting upon information affects equilibrium price. This is a useful assumption for tractability, but may seem problematic from a theoretical point of view. In fact, Hellwig (1980) has called investors of this kind "schizophrenic." Would the results fade if the price taking assumption were dropped? While a theoretical model remains to be worked out, main results seem likely to carry over. Information has only value to investors if they will act upon it. Thus, if they buy information at all, investors must rationally anticipate that they will adjust their portfolio positions accordingly once they receive the signal realizations. But then, even if they strategically alter their demand to affect price possibly little, their demand will still affect price a little. As a result, all other investors rationally extract the information contained in the price move, adjust their own beliefs, and bring market expectations and price closer to the informed investor's expected payoff. Thus, investors still face a negative effect—just as it was the case when the investor behaved as a price taker.

The present framework has shown that well-defined simultaneous information market and financial market equilibria do exist. As opposed to earlier work, investors can rationally choose the amount of information. In particular, Grossman and Stiglitz' (1976, 1980) paradox that no equilibrium existed when prices were fully revealing fails in the present more general framework. Based on their "no equilibrium conjecture" for fully revealing prices and further results, Grossman and Stiglitz (1980) stated: "The assumptions that all markets, including that for information, are always in equilibrium and always perfectly arbitraged are inconsistent when arbitrage is costly." Given the equilibria in the present more general framework, neither the special case of fully revealing prices nor the more general equilibria can serve as examples of costly arbitrage. The results do not lend support to their statement from a purely theoretical point of view. Yet, there may be good behavioral reasons why arbitrage opportunities prevail in financial markets. The relationship between the present perfectly rational framework and behaviorally inspired models remains an open field to be explored.

Appendix

A Properties of the normal distribution

A rational (Bayesian) investor updates her beliefs using the conditional normal distribution of the dividend given the signal and price realizations. Since signals and price are not conditionally independent, rational investors will make use of the following fact in general.

Fact 2 *Consider a multivariate normal p.d.f.* $f\left((\theta;\mathbf{z}^T)\,|\mu,\mathbf{\Sigma}\right)$ *with* $\mathbf{Z}=(Z_1,...,Z_K)^T$, $\mu\equiv(\bar{\mu}_\theta;\mathbb{E}[Z_1],...,\mathbb{E}[Z_K])^T$ *and*

$$\mathbf{\Sigma}\equiv\begin{pmatrix}\bar{\tau}_\theta^2 & \mathbb{C}\mathtt{ov}(\theta.\mathbf{Z})^T\\ \mathbb{C}\mathtt{ov}(\theta.\mathbf{Z}) & \mathbb{C}\mathtt{ov}\left(\mathbf{Z}.\mathbf{Z}^T\right)\end{pmatrix}.$$

Then the conditional p.d.f. of θ*, given a vector* $\mathbf{z}$ *of realizations of* $\mathbf{Z}$ *is normal with*

$$f\Big(\theta\;\Big|\;\bar{\mu}_\theta+\mathbb{C}\mathtt{ov}(\theta.\mathbf{Z})^T\,\mathbb{C}\mathtt{ov}\left(\mathbf{Z}.\mathbf{Z}^T\right)^{-1}(\mathbf{z}-\mathbb{E}[z]),\;\left[\bar{\tau}_\theta^2-\mathbb{C}\mathtt{ov}(\theta.\mathbf{Z})^T\,\mathbb{C}\mathtt{ov}\left(\mathbf{Z}.\mathbf{Z}^T\right)^{-1}\mathbb{C}\mathtt{ov}(\theta.\mathbf{Z})\right]^{-1}\Big).$$

Proof. See Raiffa and Schlaifer (1961, 8.2.1). ■

Fact 1 (p. 10) is a special case of fact 2 when all signals are conditionally independent.

Apart from this property, three further characteristics of the normal distribution are of use in the present framework.

Fact 3 *For a normally distributed random variable* $z\sim\mathcal{N}(\mu,\sigma^2)$ *and an arbitrary constant* A*, the expected value of* $e^{-A\cdot z}$ *is*

$$\mathbb{E}\left[e^{-A\cdot z}\,|\mu,\sigma\right]=\exp\left\{-A\mu+\frac{A^2}{2}\sigma^2\right\}$$

Fact 4 *For a normally distributed random variable* $z\sim\mathcal{N}(\mu,\sigma^2)$ *and an arbitrary constant* A*, the expected value of* $z\cdot e^{-A\cdot z}$ *is*

$$\mathbb{E}\left[ze^{-A\cdot z}\,|\mu,\sigma\right]=\left(\mu-A\sigma^2\right)\exp\left\{-A\mu+\frac{A^2}{2}\sigma^2\right\}.$$

Proof. Although fact 3 is a well-known property, I will prove it again here since fact 4 follows as a corollary. Note that

$$-\frac{1}{2}\left(\frac{z-(\mu-A\sigma^2)}{\sigma}\right)^2 = -A(z-\mu) - \frac{A^2\sigma^2}{2} - \frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2.$$

Thus,

$$\begin{aligned}\mathbb{E}\left[e^{-Az}\right] &= \int_{-\infty}^{\infty} e^{-Az}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2}\mathrm{d}z \\ &= e^{-A\mu+\frac{A^2}{2}\sigma^2}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{z-(\mu-A\sigma^2)}{\sigma}\right)^2}\mathrm{d}z = e^{-A\mu+\frac{A^2}{2}\sigma^2}.\end{aligned}$$

This proves fact 3. Similarly,

$$\begin{aligned}\mathbb{E}\left[ze^{-Az}\right] &= e^{-A\mu+\frac{A^2}{2}\sigma^2}\int_{-\infty}^{\infty} z\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}\left(\frac{z-(\mu-A\sigma^2)}{\sigma}\right)^2}\mathrm{d}z \\ &= e^{-A\mu+\frac{A^2}{2}\sigma^2}\left[\mu-A\sigma^2\right],\end{aligned}$$

and fact 4 follows. ■

Finally, the following fact is useful to derive *ex ante* utility in the case of partly informative prices.

Fact 5 *For a normally distributed random variable* $z \sim \mathcal{N}(\mu, \sigma^2)$ *and three arbitrary constants* A, B, D*, the expected value of* $e^{-\frac{A}{2}(B+Dz)^2}$ *is*

$$\mathbb{E}\left[e^{-\frac{A}{2}(B+Dz)^2}|\mu,\sigma\right] = \frac{1}{\sqrt{1+A\,D^2\sigma^2}}\exp\left\{-\frac{A}{2}\frac{(B+D\mu)^2}{1+A\,D^2\sigma^2}\right\}.$$

Proof. To derive this fact, consider the expectations of $e^{-A_1z-A_2z^2}$ for two arbitrary constants A_1, A_2. Note that

$$\begin{aligned}&-\frac{1}{2}\left(\frac{z-\left[\mu-(1+2\frac{A_2}{A_1}\frac{\mu-A_1\sigma^2}{1+2A_2\sigma^2})A_1\sigma^2\right]}{\frac{\sigma}{\sqrt{1+2A_2\sigma^2}}}\right)^2 \\ &= -z(A_1+A_2z) + \frac{\mu(A_1+A_2\mu)-\frac{c_1^2}{2}\sigma^2}{1+A_2\sigma^2} - \frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2.\end{aligned}$$

Thus,

$$
\begin{aligned}
\mathbb{E}\left[e^{-A_1 z - A_2 z^2}\right] &= \int_{-\infty}^{\infty} e^{-A_1 z - A_2 z^2} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{z-\mu}{\sigma}\right)^2} \,\mathrm{d}z \\
&= \frac{e^{-\frac{\mu(A_1+A_2\mu)-\frac{A_1^2}{2}\sigma^2}{1+A_2\sigma^2}}}{\sqrt{1+2A_2\sigma^2}} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\frac{\sigma}{\sqrt{1+2A_2\sigma^2}}} e^{-\frac{1}{2}\left(\frac{z-\left[\mu-(1+2\frac{A_2}{A_1}\frac{\mu-A_1\sigma^2}{1+2A_2\sigma^2})A_1\sigma^2\right]}{\frac{\sigma}{\sqrt{1+2A_2\sigma^2}}}\right)^2} \,\mathrm{d}z \\
&= \frac{1}{\sqrt{1+2A_2\sigma^2}} \exp\left\{-\frac{\mu(A_1+A_2\mu)-\frac{(A_1)^2}{2}\sigma^2}{1+A_2\sigma^2}\right\}. \qquad \text{(A.1)}
\end{aligned}
$$

To arrive at fact 5, observe that

$$\mathbb{E}\left[e^{-\frac{A}{2}(B+Dz)^2}\right] = e^{-\frac{A}{2}B^2}\mathbb{E}\left[e^{-\frac{A}{2}(2BDz+D^2z^2)}\right].$$

Then defining $A_1 \equiv \frac{A}{2}2BD$ and $A_2 \equiv \frac{A}{2}D^2$, multiplying (A.1) by $e^{-\frac{A}{2}B^2}$, and collecting terms yields fact 5. ■

B Posterior indirect expected utility

Using $H^i \equiv \exp\left(-\gamma\left[(1+R)b^i + Px^i - W_0^i + F^i + cN^i\right]\right)$ and solving out for b^i yields demand for the bond

$$b^{i,*} = \frac{1}{1+R}\left(W_0^i - F^i - cN^i - Px^{i,*} - \frac{1}{\gamma}\ln H^{i,*}\right). \qquad \text{(B.1)}$$

For each unit of the risky asset, bond demand is adjusted by a factor of $P/(1+R)$ to achieve tomorrow's desired consumption level.

To derive indirect utility (7) in the text, note that (1) simplifies to

$$U^i = -e^{-\gamma(W_0^i - F^i - cN^i)}e^{\gamma(b^i+Px^i)} - \delta e^{-\gamma R b^i}\mathbb{E}^i\left[e^{-\gamma x^i\theta}\right] \qquad \text{(B.2)}$$

for CARA utility. By (B.1) (which holds for CARA utility irrespective of the risky asset's distribution), we can write

$$b^{i,*} + Px^{i,*} = \frac{1}{1+R}\left(W_0^i - F^i - cN^i - \frac{1}{\gamma}\ln H^{i,*} + RPx^{i,*}\right),$$

where $H^{i,*}$ is certain and implicitly given by the first order condition (4-a). Using the above fact and (B.1) in (B.2) yields

$$
\begin{aligned}
U^{i,*} &= -e^{-\gamma\frac{R}{1+R}(W_0^i - F^i - cN^i)}e^{-\frac{1}{1+R}\ln H^{i,*}}e^{\gamma\frac{R}{1+R}Px^{i,*}}(1+\delta e^{\ln H^{i,*}}e^{-\gamma x^{i,*}\theta}) \\
&= -\exp\left\{-\gamma\frac{R}{1+R}(W_0^i - F^i - cN^i)\right\}\left(\frac{e^{\gamma RPx^{i,*}}}{H^{i,*}}\right)^{\frac{1}{1+R}}\left(1+\frac{1}{R}\right).
\end{aligned}
$$

The second step follows by using the first order condition (4-a) to substitute for $H^{i,*}$. This establishes (7) in the text.

C News watchers' pre-posterior distribution

Take a news watcher's point of view. Given any choice of N, the pre-posterior joint normal distribution of θ, N signals, and RP, that is the pre-posterior distribution of $(\theta; S_1, ..., S_N; RP)^T$ has a vector of means

$$\bar{\mu}^{NW} = (\bar{\mu}_\theta; \bar{\mu}_\theta, ..., \bar{\mu}_\theta; \pi_0 + \pi_S N \bar{\mu}_\theta - \pi_X \bar{x})^T$$

and an $(N+2) \times (N+2)$ variance-covariance matrix

$$\bar{\mathbf{\Sigma}}^{NW} = \begin{pmatrix} \bar{\tau}_\theta^2 & \bar{\tau}_\theta^2 \cdot \iota_N^T & \pi_S N \bar{\tau}_\theta^2 \\ \bar{\tau}_\theta^2 \cdot \iota_N & \mathbb{C}\mathtt{ov}(\mathbf{S}.\mathbf{S}^T)_N & \pi_S(N\bar{\tau}_\theta^2 + \sigma_S^2) \cdot \iota_N \\ \pi_S N \bar{\tau}_\theta^2 & \pi_S(N\bar{\tau}_\theta^2 + \sigma_S^2) \cdot \iota_N^T & \pi_S^2 N(N\bar{\tau}_\theta^2 + \sigma_S^2) + \pi_X^2 \omega_X^2 \end{pmatrix}.$$

$\mathbf{S} = (S_1, ..., S_N)^T$ is the vector of N signals, ι_N denotes an N vector of ones, and

$$\mathbb{C}\mathtt{ov}(\mathbf{S}.\mathbf{S}^T)_N = \begin{pmatrix} \bar{\tau}_\theta^2 + \sigma_S^2 & \bar{\tau}_\theta^2 & \cdots & \bar{\tau}_\theta^2 \\ \bar{\tau}_\theta^2 & \bar{\tau}_\theta^2 + \sigma_S^2 & & \bar{\tau}_\theta^2 \\ \vdots & & \ddots & \vdots \\ \bar{\tau}_\theta^2 & \cdots & & \bar{\tau}_\theta^2 + \sigma_S^2 \end{pmatrix}.$$

After observing signal realizations $(s_1, ..., s_N)$ and RP, news watchers apply fact 2 to this pre-posterior joint normal distribution and obtain a posterior normal distribution of the dividend with conditional mean

$$\mathbb{E}\left[\theta \,|RP; s_1, ..., s_N; \lambda, N\right] = \mu^{NW} = m_0^{NW} + m_S^{NW} \sum_{j=1}^{N} s_j + m_{RP}^{NW} RP \quad \text{(C.1)}$$

and conditional variance $\mathbb{V}(\theta \mid RP; s_1, ..., s_N; \lambda, N) = (\tau^{NW})^2$, where

$$m_0^{NW} = \frac{\sigma_S^2 \bar{\mu}_\theta}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \quad \text{(C.2-a)}$$

$$m_S^{NW} = \frac{\bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \quad \text{(C.2-b)}$$

$$m_{RP}^{NW} = 0, \quad \text{(C.2-c)}$$

$$(\tau^{NW})^2 = \frac{\sigma_S^2 \bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N}. \quad \text{(C.3)}$$

This is precisely what has been stated in fact 1 (p. 10) before.

D Two-group financial market equilibrium

A two-group financial market equilibrium is given by matching the coefficients π_0, π_S, π_X in equation (13) with the according terms in (20). Defining

$$u \equiv \frac{1}{\bar{\tau}_\theta^2} + \left[(1-\lambda)\frac{\pi_S(\pi_S N - 1)}{\pi_S^2 N\sigma_S^2 + \pi_X^2\omega_X^2} + \lambda\frac{1}{\sigma_S^2}\right] N \tag{D.1}$$

and matching coefficients π_0, π_S, π_X yields

$$\pi_0 = \frac{1}{u}\left(\frac{\bar{\mu}_\theta}{\bar{\tau}_\theta^2} - (1-\lambda)\frac{\pi_S N(\pi_0 - \pi_X\bar{x})}{\pi_S^2 N\sigma_S^2 + \pi_X^2\omega_X^2}\right), \tag{D.2}$$

$$\pi_S = \frac{1}{u}\frac{\lambda}{\sigma_S^2}, \tag{D.3}$$

$$\pi_X = \frac{1}{u}\frac{\gamma}{I}. \tag{D.4}$$

Plugging (D.3) and (D.4) into (D.1) and simplifying shows that (D.1) is a linear equation indeed. In general, if there are I^{NW} (groups) of investors who acquire a strictly positive number of signals, u is a polynomial of order $1 + 2I^{NW}$ (see appendix I). Here, however, u has the unique solution

$$u = \frac{1}{\bar{\tau}_\theta^2} + \left(\frac{1}{\sigma_S^2} - \frac{1}{\bar{\tau}_\theta^2}\frac{(1-\lambda)I^2}{\lambda I \cdot NI + \gamma^2\sigma_S^2\omega_X^2}\right)\lambda N.$$

Hence,

$$\pi_0 = \frac{\left[(\lambda I)^2 N + \gamma^2\sigma_S^2\omega_X^2\right]\sigma_S^2 \cdot \bar{\mu}_\theta + (\lambda I)(1-\lambda)N\sigma_S^2\bar{\tau}_\theta^2 \cdot \gamma\bar{x}}{(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)}, \tag{D.5}$$

$$\pi_S = \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \left(\frac{1}{\sigma_S^2} - \frac{1}{\bar{\tau}_\theta^2}\frac{(1-\lambda)I^2}{\lambda I \cdot NI + \gamma^2\sigma_S^2\omega_X^2}\right)\lambda N}\frac{\lambda}{\sigma_S^2}, \tag{D.6}$$

$$\pi_X = \frac{1}{\frac{1}{\bar{\tau}_\theta^2} + \left(\frac{1}{\sigma_S^2} - \frac{1}{\bar{\tau}_\theta^2}\frac{(1-\lambda)I^2}{\lambda I \cdot NI + \gamma^2\sigma_S^2\omega_X^2}\right)\lambda N}\frac{\gamma}{I}. \tag{D.7}$$

The key term for both investors is $(\mu^i - RP)/\tau^i = \tau^i(\mu^i - RP)/(\tau^i)^2$. To solve for the according price watcher term, first plug (D.5) through (D.7) into m_0^{PW} (15-a), m_{RP}^{PW} (15-b), and $(\tau^{PW})^2$ (16). This yields $(\tau^{PW})^2$. Then plug the solutions for m_0^{PW}, m_{RP}^{PW}, and $(\tau^{PW})^2$ along with the solution for the opportunity cost RP (13) into $(\mu^{PW} - RP)/(\tau^{PW})^2$. Collecting terms and simplifying yields

$$\frac{\mu^{PW} - RP}{(\tau^{PW})^2} = \frac{\gamma}{I}\frac{1}{(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)} \cdot \left((\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2)\cdot\bar{x} - \lambda I\gamma\sigma_S^2\omega_X^2 \cdot \sum_{j=1}^{N}(S_j - \bar{\mu}_\theta) + \gamma^2\sigma_S^4\omega_X^2 \cdot X\right) \tag{D.8}$$

and

$$(\tau^{PW})^2 = \frac{\left[(\lambda I)^2 N\sigma_S^2 + \gamma^2\sigma_S^4\omega_X^2\right]\bar{\tau}_\theta^2}{(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2} \tag{D.9}$$

for price watchers. Similarly, using (D.5) through (D.7) for news watchers yields

$$\begin{aligned}\frac{\mu^{NW} - RP}{(\tau^{NW})^2} &= \frac{\gamma}{I}\frac{1}{(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)} \\ &\cdot\Bigg(-\lambda I(1-\lambda)IN(\sigma_S^2 + N\bar{\tau}_\theta^2)\cdot\bar{x} + (1-\lambda)I\gamma\sigma_S^2\omega_X^2\cdot\sum_{j=1}^{N}(S_j - \bar{\mu}_\theta) \\ &+ \left(I^2\lambda N + \gamma^2\sigma_S^4\omega_X^2\right)(\sigma_S^2 + N\bar{\tau}_\theta^2)\cdot X\Bigg)\end{aligned} \tag{D.10}$$

by (18-a), (18-b), and (19) along with (13), while

$$(\tau^{NW})^2 = \frac{\sigma_S^2\bar{\tau}_\theta^2}{\sigma_S^2 + \bar{\tau}_\theta^2 N} \tag{D.11}$$

as given in (19).

E Moments of key term

In subsequent analysis, the pre-posterior moments of the key term $\tau^i(\mu^i - RP)/(\tau^i)^2$ will be of most interest. Since $(\tau^i)^2$ is certain, and both μ^i and RP are normally distributed from a pre-posterior perspective, $(\mu^i - RP)/(\tau^i)^2$ is normally distributed. To derive the moments, start with price watchers. Taking expectations and the variance of (D.8), one finds

$$\mathbb{E}^{PW}_{pre}\left[\frac{\mu^{PW} - RP}{(\tau^{PW})^2}\right] = \frac{\gamma}{I}\frac{\left[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2\right]\bar{x}}{(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)}, \tag{E.1}$$

$$\mathbb{V}^{PW}_{pre}\left(\frac{\mu^{PW} - RP}{(\tau^{PW})^2}\right) = \frac{\gamma^2}{I^2}\frac{\left[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2\right]\gamma^2\sigma_S^4\omega_X^4}{\left[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\right]^2} \tag{E.2}$$

for price watchers. Their $(\tau^{PW})^2$ is

$$(\tau^{PW})^2 = \frac{\left[(\lambda I)^2 N\sigma_S^2 + \gamma^2\sigma_S^4\omega_X^2\right]\bar{\tau}_\theta^2}{(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2} \tag{E.3}$$

by (D.9).

Similarly, taking expectations and the variance of (D.10) for news watchers, one finds

$$\mathbb{E}_{pre}^{NW}\left[\frac{\mu^{NW}-RP}{(\tau^{NW})^2}\right]=\frac{\gamma}{I}\frac{\left[(\lambda I)^2N+\gamma^2\sigma_S^4\omega_X^2\right](\sigma_S^2+N\bar{\tau}_\theta^2)\,\bar{x}}{(\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)}, \quad (E.4)$$

$$\mathbb{V}_{pre}^{NW}\left(\frac{\mu^{NW}-RP}{(\tau^{NW})^2}\right)=\frac{\gamma^2}{I^2}\frac{\omega_X^2(\sigma_S^2+N\bar{\tau}_\theta^2)}{\left[(\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2}$$
$$\cdot\Bigg(\lambda^2I^4N^2(\sigma_S^2+N\bar{\tau}_\theta^2)+I^2N\gamma^2\sigma_S^2\omega_X^2\left((1+\lambda^2)\sigma_S^2+2\lambda N\bar{\tau}_\theta^2\right)$$
$$+\gamma^4\sigma_S^4\omega_X^4(\sigma_S^2+N\bar{\tau}_\theta^2)\Bigg). \quad (E.5)$$

Their $(\tau^{NW})^2$ is

$$(\tau^{NW})^2=\frac{\sigma_S^2\bar{\tau}_\theta^2}{\sigma_S^2+N\bar{\tau}_\theta^2} \quad (E.6)$$

by (D.11).

If $N=0$, news watchers' terms (E.4), (E.5), and (D.11) coincide with the respective terms for price watchers (E.1), (E.2), and (D.9), as it should be.

F Two-group information market equilibrium

Written out, the derivative of *ex ante* utility (22) with respect to N, given λ, is

$$\frac{1+R}{\mathbb{E}_{pre}^i\left[U^{i,*}\right]}\frac{\partial\mathbb{E}_{pre}^i\left[U^{i,*}\right]}{\partial N^i}=-\gamma\,R\,c$$
$$+\frac{1}{N}\;\frac{\left(\mathbb{E}_{pre}^i\left[\frac{\mu^i-RP}{(\tau^i)^2}\right]\right)^2}{\frac{1}{(\tau^i)^2}+\frac{1}{1+R}\mathbb{V}_{pre}^i\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}\;\left(\varepsilon_{\tau^2,N}^i+\varepsilon_{\mathbb{E},N}^i\right) \quad (F.1)$$
$$-\frac{1}{N}\;\frac{\mathbb{V}_{pre}^i\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}{\frac{1}{(\tau^i)^2}+\frac{1}{1+R}\mathbb{V}_{pre}^i\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}\;\left(\varepsilon_{\tau^2,N}^i+\tfrac{1}{2}\varepsilon_{\mathbb{V},N}^i\right)$$
$$\cdot\frac{1}{1+R}\;\frac{\left(\mathbb{E}_{pre}^i\left[\frac{\mu^i-RP}{(\tau^i)^2}\right]\right)^2-\frac{1+R}{(\tau^i)^2}-\mathbb{V}_{pre}^i\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)}{\frac{1}{(\tau^i)^2}+\frac{1}{1+R}\mathbb{V}_{pre}^i\left(\frac{\mu^i-RP}{(\tau^i)^2}\right)},$$

where $\varepsilon^i_{y,N}$ denotes the elasticity of y with respect to N. In the text, this condition has been given a nicer look be defining E^i, V^i, and Δ^i accordingly.

The terms in condition (F.1) can all be evaluated in closed-form using the moments of the key term $\tau^i(\mu^i - RP)/(\tau^i)^2$ as derived in the previous appendix E. Again, start with price watchers. Differentiating the moments (E.1) and (E.2) and $(\tau^{PW})^2$ with respect to N yields the elasticities

$$\begin{aligned}
\varepsilon^{PW}_{\mathbb{E},N} &= \frac{\gamma^2\sigma_S^2\bar{\tau}_\theta^2\omega_X^2\,\lambda N}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2]} \\
&\quad \cdot \frac{(\lambda I)^2 N^2\bar{\tau}_\theta^2 - \gamma^2\sigma_S^4\omega_X^2}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]}, \qquad \text{(F.2)} \\
\varepsilon^{PW}_{\mathbb{V},N} &= -\frac{\lambda N}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2]} \\
&\quad \frac{1}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]} \\
&\quad \cdot \Bigg(\lambda^3 I^4 N(\sigma_S^2 + N\bar{\tau}_\theta^2)(\sigma_S^2 + 2N\bar{\tau}_\theta^2) \\
&\quad + \gamma^2\sigma_S^4\omega_X^2\lambda I^2\left(\sigma_S^2 + (2+\lambda)N\bar{\tau}_\theta^2\right) + 2\gamma^4\sigma_S^6\bar{\tau}_\theta^2\omega_X^4 \Bigg), \qquad \text{(F.3)} \\
\varepsilon^{PW}_{\tau^2,N} &= -\frac{(\lambda I)^2 N^2\bar{\tau}_\theta^2\,[(\lambda I)^2 N + 2\gamma^2\sigma_S^2\omega_X^2]}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2]\,[(\lambda I)^2 N + \gamma^2\sigma_S^2\omega_X^2]}. \qquad \text{(F.4)}
\end{aligned}$$

Differentiating news watchers' moments (E.4) and (E.5) and $(\tau^{NW})^2$ with respect to N yields the elasticities

$$\begin{aligned}
\varepsilon^{NW}_{\mathbb{E},N} &= -\frac{\gamma^2\sigma_S^2\bar{\tau}_\theta^2\omega_X^2\,(1-\lambda)N}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2]} \\
&\quad \cdot \frac{(\lambda I)^2 N^2\bar{\tau}_\theta^2 - \gamma^2\sigma_S^4\omega_X^2}{[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]}, \qquad \text{(F.5)} \\
\varepsilon^{NW}_{\mathbb{V},N} &= -\frac{1}{v}\,\frac{\gamma^2\sigma_S^4\omega_X^2\,(1-\lambda)N}{(\sigma_S^2 + N\bar{\tau}_\theta^2)\,[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)]} \\
&\quad \cdot \Bigg((\lambda I)^2(1+\lambda)I^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2)(\sigma_S^2 + 2N\bar{\tau}_\theta^2) \\
&\quad + \gamma^2\sigma_S^2\omega_X^2 I^2\left[(1+\lambda)\sigma_S^4 + (2 + \lambda(5+\lambda))\,N\sigma_S^2\bar{\tau}_\theta^2 + 6\lambda N^2\bar{\tau}_\theta^4)\right] \\
&\quad + 2\gamma^4\sigma_S^4\bar{\tau}_\theta^2\omega_X^4(\sigma_S^2 + N\bar{\tau}_\theta^2) \Bigg), \qquad \text{(F.6)} \\
\varepsilon^{NW}_{\tau^2,N} &= -\frac{\bar{\tau}_\theta^2 N}{\sigma_S^2 + \bar{\tau}_\theta^2 N}, \qquad \text{(F.7)}
\end{aligned}$$

where

$$v \equiv [\lambda^2 I^3 N^2(\sigma_S^2 + N\bar{\tau}_\theta^2) + I^2 N\gamma^2\sigma_S^2\omega_X^2\left((1+\lambda^2)\sigma_S^2 + 2\lambda N\bar{\tau}_\theta^2)\right) + \gamma^4\sigma_S^4\omega_X^4(\sigma_S^2 + N\bar{\tau}_\theta^2)].$$

With all these results at hand, we can evaluate (F.1). Take the E^i-terms first. For a price watcher[16]

$$\begin{aligned}
&\frac{1}{N}\,E^{PW}\cdot\left[\varepsilon^{PW}_{\tau^2,N} + \varepsilon^{PW}_{\mathbb{E},N}\right] = \\
&\quad -(1+R)\,\lambda\,\bar{x}^2\gamma^2\sigma_S^2\bar{\tau}_\theta^4\left(I^2N\lambda^2(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2\right) \\
&\qquad \cdot(\lambda^3 I^4 N^2 + 2\lambda I^2 N\gamma^2\sigma_S^2\omega_X^2 + \gamma^4\sigma_S^4\omega_X^4) \Big/ \\
&\quad \Bigg(\left[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\right] \\
&\qquad \cdot\Big[(1+R)\lambda^4 I^6 N^2(\sigma_S^2 + N\bar{\tau}_\theta^2)^2 + 2(1+R)\lambda^2 I^4 N\gamma^2\sigma_S^2\omega_X^2 \\
&\qquad\quad \cdot(\sigma_S^2 + N\bar{\tau}_\theta^2)(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2) + I^2\gamma^4\sigma_S^4\omega_X^4\Big((1+R)\sigma_S^4 + \gamma^6\sigma_S^8\bar{\tau}_\theta^2\omega_X^6 \\
&\qquad\qquad +\lambda N\sigma_S^2\bar{\tau}_\theta^2(2(1+R)+\lambda) + (1+R)\lambda^2N^2\bar{\tau}_\theta^4\Big)\Big]\Bigg) < 0,
\end{aligned} \tag{F.8}$$

while for a news watcher

$$\begin{aligned}
&\frac{1}{N}\,E^{NW}\cdot\left[\varepsilon^{NW}_{\tau^2,N} + \varepsilon^{NW}_{\mathbb{E},N}\right] = \\
&\quad -(1+R)\,\lambda\,\bar{x}^2\gamma^2\sigma_S^2\bar{\tau}_\theta^4(\sigma_S^2 + N\bar{\tau}_\theta^2)(I^2N\lambda^2 + \gamma^2\sigma_S^2\omega_X^2) \\
&\qquad \cdot(\lambda^3 I^4 N^2 + 2\lambda I^2 N\gamma^2\sigma_S^2\omega_X^2 + \gamma^4\sigma_S^4\omega_X^4) \Big/ \\
&\quad \Bigg(\left[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\right] \\
&\qquad \cdot\Big[(1+R)\lambda^4 I^6 N^2(\sigma_S^2 + N\bar{\tau}_\theta^2)^2 \\
&\qquad\quad +\lambda^2 I^4 N\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + N\bar{\tau}_\theta^2)\left[2(1+R)\sigma_S^2 + N\left(1 + 2(1+R)\lambda\right)\right] \\
&\qquad\quad +I^2\gamma^4\sigma_S^4\omega_X^2\Big((1+R)\sigma_S^4 + N\sigma_S^2\bar{\tau}_\theta^2\left(1 + \lambda(2(1+R)+\lambda)\right) \\
&\qquad\qquad +\lambda N^2\bar{\tau}_\theta^4(2+\lambda(1+R))\Big) + \gamma^6\sigma_S^6\bar{\tau}_\theta^2\omega_X^2(\sigma_S^2 + N\bar{\tau}_\theta^2)\Big]\Bigg) < 0.
\end{aligned} \tag{F.9}$$

[16]The following terms have been calculated and simplified using Mathematica 4. The according notebook file can be found online at http://socrates.berkeley.edu/~muendler/.

Now consider the V^i-terms. For a price watcher

$$\frac{1}{N}\, V^{PW} \cdot \left[\varepsilon^{PW}_{\tau^2,N} + \tfrac{1}{2}\varepsilon^{PW}_{\mathbb{V},N}\right] =$$

$$\Bigg(\lambda\left((\lambda I)^2 N\gamma^4\sigma_S^4\omega_X^2(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^6\sigma_S^8\omega_X^6\right)$$

$$\Big[\lambda^5 I^6 N^2(\sigma_S^2 + N\bar{\tau}_\theta^2)(\sigma_S^2 + 4N\bar{\tau}_\theta^2) + \lambda^3 I^4 N\gamma^2\sigma_S^2\omega_X^2$$

$$\cdot\Big(2\sigma_S^2 + N(11+\lambda)\sigma_S^2\bar{\tau}_\theta^2 + 2N^2(3+\lambda)\bar{\tau}_\theta^4\Big) + \lambda I^2\gamma^4\sigma_S^4\omega_X^4$$

$$\cdot\Big(\sigma_S^4 + 3N\,(2+\lambda)\,\sigma_S^2\bar{\tau}_\theta^2 + 4\lambda N^2\bar{\tau}_\theta^4\Big) + 2\gamma^6\sigma_S^8\bar{\tau}_\theta^2\omega_X^6\Big]\Bigg) \Bigg/$$

$$\Bigg(2\left((\lambda I)^2 N + \gamma^2\sigma_S^2\omega_X^2\right)\left((\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2\right)$$

$$\left((\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\right)$$

$$\left[\lambda^2 I^3 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + I\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\right]^2$$

$$\Bigg[\frac{1}{\bar{\tau}_\theta^2} + \frac{(\lambda I)^2 N^2}{(\lambda I)^2 N^2\sigma_S^2 + \gamma^2\sigma_S^4\omega_X^2}$$

$$+\left((\lambda I)^2 N\gamma^4\sigma_S^4\omega_X^4(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^6\sigma_S^8\omega_X^6\right)\Big/\Big[(1+R)$$

$$\cdot\Big(\lambda^2 I^3 N(\sigma_S^2 + N\bar{\tau}_\theta^2) I\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\Big)^2\Big]\Bigg]\Bigg) > 0, \quad \text{(F.10)}$$

and for a news watcher

$$\frac{1}{N}\, V^{NW} \cdot \left[\varepsilon^{NW}_{\tau^2,N} + \tfrac{1}{2}\varepsilon^{NW}_{\mathbb{V},N}\right] =$$

$$\Bigg((1+R)\gamma^2\sigma_S^2\bar{\tau}_\theta^2)\omega_X^2$$

$$\Big[2\lambda^4 I^6 N^3\bar{\tau}_\theta^2(\sigma_S^2 + N\bar{\tau}_\theta^2)^2 - \lambda^2 I^4 N\gamma^2\sigma_S^2(\sigma_S^2 + N\bar{\tau}_\theta^2)$$

$$\cdot\Big((1-\lambda^2)\sigma_S^4 - 2N(1+2\lambda^2)\sigma_S^2\bar{\tau}_\theta^2 - 6\lambda N^2\bar{\tau}_\theta^4\Big) - I^2\gamma^4\sigma_S^4\omega_X^4$$

$$\cdot\Big((1-\lambda^2)\sigma_S^6 + \lambda N\left(3 - \lambda(8+\lambda)\right)\sigma_S^4\bar{\tau}_\theta^2 - 2\lambda^2 N^2(5+\lambda)\sigma^2\bar{\tau}_\theta^4$$

$$-6\lambda^2 N^3\bar{\tau}_\theta^6\Big) + 2\lambda\gamma^6\sigma_S^6\bar{\tau}_\theta^2\omega_X^6(\sigma_S^2 + N\bar{\tau}_\theta^2)^2\Big]\Bigg) \Bigg/$$

$$
\begin{aligned}
&\Bigg(2(\sigma_S^2+N\bar{\tau}_\theta^2)\left((\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right)\\
&\quad\Big[(1+R)\lambda^4I^6N^2(\sigma_S^2+N\bar{\tau}_\theta^2)^2\\
&\qquad+\lambda^2I^4N\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+N\bar{\tau}_\theta^2)\left(2(1+R)\sigma_S^2N(1+2(1+R)\lambda)\right)\\
&\qquad+I^2\gamma^4\sigma_S^2\omega_X^4\Big((1+R)\sigma^4+N\sigma_S^2\bar{\tau}_\theta^2\left(1+\lambda(2(1+R)+\lambda)\right)\\
&\qquad\quad+\lambda N^2\bar{\tau}_\theta^2(2+(1+R)\lambda)\Big)+\gamma^6\sigma_S^6\bar{\tau}_\theta^2\omega_X^6(\sigma_S^2+N\bar{\tau}_\theta^2)\Big]\Bigg). \qquad \text{(F.11)}
\end{aligned}
$$

Finally, take the Δ^i terms. For a price watcher

$$
\begin{aligned}
&\frac{1}{1+R}\Delta^{PW}=\\
&-\Bigg(R+\Big[\left((\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^4\omega_X^2\right)\\
&\qquad\Big(\lambda^2I^4N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^4\sigma_S^4\bar{\tau}_\theta^2\omega_X^2(\omega_X^2-\bar{x}^2)\\
&\qquad\quad+I^2\gamma^2\sigma_S^2\left(\sigma_S^2\omega_X^2+\lambda N\bar{\tau}_\theta^2(2\omega_X^2-\lambda\bar{x}^2)\right)\Big)\Big]\Big/\\
&\qquad\left[\lambda^2I^3N(\sigma_S^2+N\bar{\tau}_\theta^2)+I\gamma\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2\Bigg)\Bigg/\\
&\quad\left(1+R+\frac{\gamma^4\sigma_S^6\bar{\tau}_\theta^2\omega_X^2\left((\lambda I)^2N+\gamma^2\sigma^2\omega_X^2\right)}{\left[\lambda^2I^3N(\sigma_S^2+N\bar{\tau}_\theta^2)+I\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2}\right), \qquad \text{(F.12)}
\end{aligned}
$$

while for a news watcher

$$
\begin{aligned}
&\frac{1}{1+R}\Delta^{NW}=\\
&-\Bigg(R+\Big[(\sigma_S^2+N\bar{\tau}_\theta^2)\left((\lambda I)^2N+\gamma^2\sigma_S^2\omega_X^2\right)\Big(\lambda^2I^4N(\sigma_S^2+N\bar{\tau}_\theta^2)\\
&\qquad+\gamma^4\sigma_S^4\bar{\tau}_\theta^2\omega_X^2(\omega_X^2-\bar{x}^2)+I^2\gamma^2\sigma_S^2\left(\sigma_S^2\omega_X^2+\lambda N\bar{\tau}_\theta^2(2\omega_X^2-\lambda\bar{x}^2)\right)\Big)\Big]\Big/\\
&\quad\left[\lambda^2I^3N(\sigma_S^2+N\bar{\tau}_\theta^2)+I\gamma\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2\Bigg)\Bigg/
\end{aligned}
$$

$$\left(R+\left[(\sigma_S^2+N\bar{\tau}_\theta^2)\left((\lambda I)^2N+\gamma^2\sigma_S^2\omega_X^2\right)\left(\lambda^2I^4N(\sigma_S^2+N\bar{\tau}_\theta^2)\right.\right.\right.$$
$$\left.\left.+I^2\gamma^2\sigma_S^2\omega_X^2\left(\sigma_S^2+2\lambda N\bar{\tau}_\theta^2\right)+\gamma^4\sigma_S^4\bar{\tau}_\theta^2\omega_X^4\right)\right]\Big/$$
$$\left.\left[\lambda^2I^3N(\sigma_S^2+N\bar{\tau}_\theta^2)+I\gamma\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2\right). \tag{F.13}$$

The relationships in the last row of table 2 (p. 27) follow from

$$(\tau^{PW})^2\left(\mathbb{E}_{pre}^{PW}\left[\frac{\mu^{PW}-RP}{(\tau^{PW})^2}\right]\right)^2-(\tau^{PW})^2\mathbb{V}_{pre}^{PW}\left[\frac{\mu^{PW}-RP}{(\tau^{PW})^2}\right]<1+R$$
$$\Leftrightarrow\quad\frac{\bar{x}^2}{I^2}<\frac{1+R}{\gamma^2\sigma_S^2\bar{\tau}_\theta^2}\frac{\left[(\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2}{\left((\lambda I)^2N+\gamma^2\sigma_S^2\omega_X^2\right)\left((\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^4\omega_X^2\right)}$$
$$+\frac{\omega_X^2}{I^2}\frac{\gamma^2\sigma_S^4\omega_X^2}{(\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^4\omega_X^2}\ \equiv\frac{(\bar{x}_c^{\Delta,PW})^2}{I^2} \tag{F.14}$$

by (E.1) and (E.2), and from

$$(\tau^{NW})^2\left(\mathbb{E}_{pre}^{NW}\left[\frac{\mu^{NW}-RP}{(\tau^{NW})^2}\right]\right)^2-(\tau^{NW})^2\mathbb{V}_{pre}^{NW}\left[\frac{\mu^{NW}-RP}{(\tau^{NW})^2}\right]<1+R$$
$$\Leftrightarrow\quad\frac{\bar{x}^2}{I^2}<\frac{1+R}{\gamma^2\sigma_S^2\bar{\tau}_\theta^2}\frac{\left[(\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2}{\left((\lambda I)^2N+\gamma^2\sigma_S^2\omega_X^2\right)^2(\sigma_S^2+N\bar{\tau}_\theta^2)}$$
$$+\frac{\omega_X^2}{I^2}\left(1+\frac{\left((\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2\left[(1+\lambda)\sigma_S^2+2\lambda N\bar{\tau}_\theta^2\right]\right)}{(\sigma_S^2+N\bar{\tau}_\theta^2)\left((\lambda I)^2N+\gamma^2\sigma_S^2\omega_X^2\right)^2}\right.$$
$$\left.\cdot(1-\lambda)I^2N\right)\ \equiv\frac{(\bar{x}_c^{\Delta,NW})^2}{I^2} \tag{F.15}$$

by (E.4) and (E.5).

Similarly, the relationships in the middle column can be inferred from

$$E^{NW}\cdot\left[\varepsilon_{\tau^2,N}^{NW}+\varepsilon_{\mathbb{E},N}^{NW}\right]-E^{PW}\cdot\left[\varepsilon_{\tau^2,N}^{PW}+\varepsilon_{\mathbb{E},N}^{PW}\right]=$$
$$-\left(R(1+R)\lambda I^2N\bar{x}^2\gamma^4\sigma_S^4\bar{\tau}_\theta^6\omega_X^2\right.$$
$$\cdot\left((\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right)$$
$$\left.\cdot\left(\lambda^3I^4N^2+2\lambda I^2N\gamma^2\sigma_S^2\omega_X^2\gamma^4\sigma_S^4\omega_X^4\right)\right)\Big/$$

$$
\begin{aligned}
&\Bigg(\Big[(1+R)\lambda^4 I^6 N^2(\sigma_S^2+N\bar{\tau}_\theta^2)^2 \\
&\quad +2(1+R)\lambda^2 I^4 N\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+N\bar{\tau}_\theta^2)(\sigma_S^2+\lambda N\bar{\tau}_\theta^2) \\
&\quad +I^2\gamma^4\sigma_S^4\omega_X^4\Big((1+R)\sigma_S^4+\lambda N\left(2(1+R)+\lambda\right)\sigma_S^2\bar{\tau}_\theta^2 \\
&\qquad +(1+R)\lambda^2N^2\bar{\tau}_\theta^4\Big)+\gamma^6\sigma_S^8\bar{\tau}_\theta^2\omega_X^6\Big] \\
&\cdot\Big[(1+R)\lambda^4 I^6 N^2(\sigma_S^2+N\bar{\tau}_\theta^2)^2+\lambda^2 I^4 N\gamma^2\sigma_S^2\omega_X^2 \\
&\quad (\sigma_S^2+N\bar{\tau}_\theta^2)\left(2(1+R)\sigma_S^2+N\left(1+2(1+R)\lambda\right)\bar{\tau}_\theta^2\right) \\
&\quad +I^2\gamma^4\sigma_S^4\omega_X^4\Big((1+R)\sigma_S^4+N\left(1+\lambda\left(2(1+R)+\lambda\right)\right)\sigma_S^2\bar{\tau}_\theta^2 \\
&\qquad +N^2\lambda\left(2+(1+R)\lambda\right)\bar{\tau}_\theta^2\Big)+\gamma^6\sigma_S^6\bar{\tau}_\theta^2\omega_X^6(\sigma_S^2+N\bar{\tau}_\theta^2)\Big]\Bigg)<0,
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{1}{1+R}\left(\Delta^{NW}-\Delta^{PW}\right)= \\
&\Bigg(RI^2N\bar{x}^2\gamma^4\sigma_S^4\bar{\tau}_\theta^2\omega_X^2\left((\lambda I)^2N+\gamma^2\sigma_S^2\omega_X^2\right) \\
&\quad \left[(\lambda I)^2N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\right]^2\Bigg)\Bigg/ \\
&\Bigg(\Big[(1+R)\lambda^4 I^6 N^2(\sigma_S^2+N\bar{\tau}_\theta^2)^2+2(1+R)\lambda^2 I^4 N\gamma^2\sigma_S^2\omega_X^2 \\
&\quad \cdot(\sigma_S^2+N\bar{\tau}_\theta^2)(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)+I^2\gamma^4\sigma_S^4\omega_X^4\Big((1+R)\sigma_S^4 \\
&\qquad +\lambda N\left(2(1+R)+\lambda\right)\sigma_S^2\bar{\tau}_\theta^2+(1+R)\lambda^2N^2\bar{\tau}_\theta^4\Big)+\gamma^6\sigma_S^8\bar{\tau}_\theta^2\omega_X^6\Big] \\
&\quad \cdot\Big[(1+R)\lambda^4 I^6 N^2(\sigma_S^2+N\bar{\tau}_\theta^2)^2+\lambda^2 I^4 N\gamma^2\sigma_S^2\omega_X^2 \\
&\qquad \cdot(\sigma_S^2+N\bar{\tau}_\theta^2)^2\left(2(1+R)\sigma^2+N\left(1+2(1+R)\lambda\right)\bar{\tau}_\theta^2\right) \\
&\quad +I^2\gamma^4\sigma_S^4\omega_X^4\Big((1+R)\sigma_S^4+N\left(1+\lambda\left(2(1+R)+\lambda\right)\right)\sigma_S^2\bar{\tau}_\theta^2 \\
&\qquad +\lambda N^2\left(2+(1+R)\lambda\right)\bar{\tau}_\theta^4\Big)+\gamma^6\sigma_S^6\bar{\tau}_\theta^2\omega_X^6(\sigma_S^2+N\bar{\tau}_\theta^2)\Big]\Bigg)>0.
\end{aligned}
$$

G Negative externality: Proof of theorem 6

To show that any signal to news watchers inflicts a negative externality on price watchers, it suffices to look at condition (23) if it does not change sign for any N. I will prove that this condition always has a negative sign.

First note that Δ^{PW} (F.12) can only turn positive for sufficiently large $\bar{x}^2$, where the threshold value for $\bar{x}^2$ is given by (F.14). Thus, no other parameter of the model can make condition (23) positive. So, it will suffice to show that condition (23) is strictly negative for any (weakly positive) $\bar{x}^2$. Note that condition (23) is linear in $\bar{x}^2$. Defining $A \equiv \mathbb{E}_{pre}^{PW}\left[(\mu^{PW} - RP)/\tau^{PW}\right]^2/\bar{x}^2$, $B \equiv -\mathbb{V}_{pre}^{PW}\left((\mu^{PW} - RP)/\tau^{PW}\right)$, $D_1 \equiv (\varepsilon_{\tau^2,N}^{PW} + \varepsilon_{\mathbb{E},N}^{PW})/N$, $D_2 \equiv (\varepsilon_{\tau^2,N}^{PW} + \frac{1}{2}\varepsilon_{\mathbb{V},N}^{PW})/N$, $G^{-1} \equiv (\tau^{PW})^2/(1+R)$, $K^{-1} \equiv 1 - B/(1+R)$, we can rewrite condition (23) as

$$AK(D_1 + BD_2\tfrac{K}{1+R})\bar{x}^2 + BD_2\tfrac{K^2}{1+R}(B+G). \tag{G.1}$$

We are interested in the signs of the two terms in (G.1). By their definition, $B < 0$, $K > 0$, $G < 0$. In addition, $D_2 < 0$ by (F.2) and (F.4) (see third row in table 1, p. 26). Thus, the second term in (G.1) is strictly negative, $BD_2\frac{K^2}{1+R}(B+G) < 0$. The other term is harder to evaluate, however, since $D_1 < 0$ by the sum of (F.2) and (F.4) (see third row in table 1 again). To find its sign, we can proceed in the following manner.

Setting the rewritten condition (G.1) equal to zero and solving out for $\bar{x}^2$, we find

$$(\bar{x}_0^{neg.ext.})^2 = -\frac{BD_2\frac{K^2}{1+R}(B+G)}{AK(D_1 + BD_2\frac{K}{1+R})}.$$

Thus, the sign of $AK(D_1 + BD_2\frac{K}{1+R})$ is the same as that of $(\bar{x}_0^{neg.ext.})^2$. By the equilibrium values (E.1) through (E.3) and (F.2) through (F.4) we find

$$\begin{aligned}
&(\bar{x}_0^{neg.ext.})^2 = \\
&-\Bigg(\gamma^2\sigma_S^2\omega_X^4 \\
&\quad\Big[\lambda^5 I^6 N^2(\sigma_S^2 + N\bar{\tau}_\theta^2)(\sigma_S^2 + 4N\bar{\tau}_\theta^2) + \lambda^3 I^4 N \\
&\qquad\cdot\gamma^2\sigma_S^2\omega_X^2\left(2\sigma_S^4 + N(11+\lambda)\sigma + S^2\bar{\tau}_\theta^2 + 2N^2(3+\lambda)\bar{\tau}_\theta^4\right) \\
&\qquad+\lambda I^2\gamma^4\sigma_S^4\omega_X^4\cdot\left(\sigma_S^4 + 3N(2+\lambda)\sigma_S^2\bar{\tau}_\theta^2 + 4\lambda N^2\bar{\tau}_\theta^4\right) \\
&\qquad+2\gamma^6\sigma_S^8\bar{\tau}_\theta^2\omega_X^6\Big] \\
&\quad\Big[(1+R)\lambda^6 I^8 N^3(\sigma_S^2 + N\bar{\tau}_\theta^2)^3 + (1+R)\lambda^4 I^6 N^2 \\
&\qquad\cdot\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + N\bar{\tau}_\theta^2)^2(3\sigma_S^2 + 2\lambda N\bar{\tau}_\theta^2) + \lambda^2 I^4 N \\
&\qquad\gamma^4\sigma_S^4\omega_X^4\Big(3(1+R)\sigma_S^6 + (1+R)N(3+4\lambda)\sigma_S^4\bar{\tau}_\theta^2 + \lambda N \\
&\qquad\quad\cdot\sigma_S^2\bar{\tau}_\theta^2\left((1+R)N(4+\lambda) + \lambda\sigma_S^2\right) + (1+R)\lambda^2 N^3\bar{\tau}_\theta^2\Big)
\end{aligned}$$

$$
\begin{aligned}
&+I^2\gamma^6\sigma_S^8\omega_X^6\Big((1+R)\sigma_S^4+(1+R)N^2\lambda^2\bar{\tau}_\theta^2 \\
&\quad +2\lambda N\sigma_S^2\bar{\tau}_\theta^2(1+R+\lambda\bar{\tau}_\theta^2)\Big)+\gamma^8\sigma_S^1 2\bar{\tau}_\theta^4\omega_X^8\Big]\Big)\Big/ \\
&\Big(I^2\bar{\tau}_\theta^4(\lambda^2 I^2 N+\gamma^2\sigma_S^2\omega_X^2) \\
&\quad\big((\lambda I)^2 N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^4\omega_X^2\big) \\
&\quad\big((\lambda I)^2 N(\sigma_S^2+N\bar{\tau}_\theta^2)+\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+\lambda N\bar{\tau}_\theta^2)\big) \\
&\quad\Big[2(1+R)\lambda^7 I^8 N^4(\sigma_S^2+N\bar{\tau}_\theta^2)^2 \\
&\quad +2(1+R)\lambda^5 I^6 N^3\gamma^2\sigma_S^2\omega_X^2(\sigma_S^2+N\bar{\tau}_\theta^2) \\
&\quad\quad \big(4\sigma_S^2+N(2+\lambda)\bar{\tau}_\theta^2\big)+\lambda^3 I^4 N^2\gamma^4\sigma_S^4\omega_X^4 \\
&\quad\quad \cdot\Big(\left(10+(2-\lambda)\lambda+2R(5+\lambda)\right)\sigma_S^4+2N\sigma_S^2\bar{\tau}_\theta^2 \\
&\quad\quad\quad \cdot\left(4+(5-\lambda)\lambda+R(4+5\lambda)\right)\sigma_S^2\bar{\tau}_\theta^2+6(1+R)\lambda N^2\bar{\tau}_\theta^4\Big) \\
&\quad +2\lambda I^2 N\gamma^6\sigma_S^6\omega_X^6\Big(\left(2+(2-\lambda)\lambda+2R(1+\lambda)\right)\sigma_S^4 \\
&\quad\quad +\lambda N\left(4-\lambda+R(4+\lambda)\right)\sigma_S^2\bar{\tau}_\theta^2+(1+R)\lambda^2 N^2\bar{\tau}_\theta^2\Big) \\
&\quad +\gamma^8\sigma_S^{10}\left((2(1+R)-\lambda)\sigma_S^2+2R\lambda N\bar{\tau}_\theta^2\right)\Big]\Big) < 0. \qquad \text{(G.2)}
\end{aligned}
$$

So, this zero-point of condition (23) would lie in the strictly negative range of $\bar{x}^2$ if that existed. Therefore, $AK(D_1+BD_2\frac{K}{1+R})<0$ so that all terms in condition (23) are strictly negative, which concludes the proof. ■

H Grossman and Stiglitz' (1980) version

We can compare the two-group equilibrium in section 4 to the equilibrium in Grossman and Stiglitz' (1980) model. Since Grossman and Stiglitz assume, too, that news watchers get perfect copies of the newspapers, their model remains a special case of the two-group model in section 4. There is a continuum of investors in Grossman and Stiglitz' world and investors may either watch the price or receive exactly one signal in addition. Thus, by setting $I=1$ and $N=1$ throughout the present model, Grossman and Stiglitz' version results. Accordingly, the cost of becoming a news watcher can be redefined as $F'=F+c$ here. Figure 5 depicts the equilibrium share of news

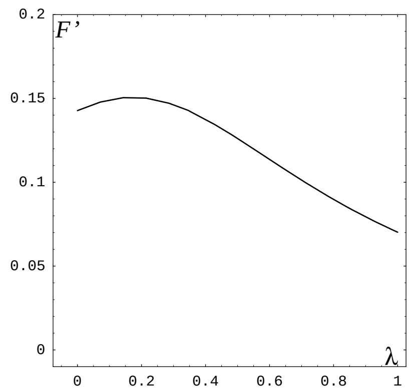

Figure 5: **Equilibria in Grossman and Stiglitz' (1980) Model**

watchers $\lambda^*(F')$ as a function of the fixed information cost F'.[17] Just as in the present framework, multiple equilibria may arise in Grossman and Stiglitz' model, too. For high values of F, there are two possible equilibrium levels of λ. In addition, we can implement any equilibrium share of news watchers λ by varying F' in the example of figure 5. This stands in contrast to theorem 7 which states that, as soon as news watchers can choose the number of newspapers, at least one investor must obtain less (or different) information in equilibrium. The reason for this difference is that the implicit equilibrium definition in Grossman and Stiglitz' variant of the model has suppressed the optimizing behavior of investors. It is merely a fixed point that equalizes *ex ante* utility. In the present framework, however, news watchers may have second thoughts once they have become news watchers. They can pay the fixed cost of joining the news watchers group, but then discover that their representative actually prefers to order zero news papers for everyone. This possibility for a second thought makes all the difference. As so often in game theoretically oriented models, the outcome depends on the structure of the game.

Apart from the "no equilibrium conjecture" for fully revealing prices, discussed at large in the previous section 3, Grossman and Stiglitz (1980) introduced several further conjectures. Many of them concern the informativeness of the equilibrium price. Formally, the informativeness of a signal is its precision, which, in turn, is defined as the inverse of its pre-posterior

[17]Levels of F' on the vertical axis are expressed as shares of wealth. Parameter values are the same as in figure 2, except for $I = 1$, for $W = 1$, which takes the same value as in figure 3, and for F, which is endogenous here. See footnote 11 (p. 33).

variance. So, to investigate the informativeness of price, consider its variance

$$\begin{aligned}\mathbb{V}_{pre}\,(RP) &= \pi_S^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \pi_X^2\omega_X^2 \\ &= \frac{\bar{\tau}_\theta^4\left[(\lambda I)^2 N + \gamma^2\sigma_S^2\omega_X^2\right]^2\left[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2\right]}{I^2\left[(\lambda I)^2 N(\sigma_S^2 + N\bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda N\bar{\tau}_\theta^2)\right]^2}.\end{aligned}$$

This follows from (13) in the text and lemma 5. For $N = I = 1$, the precision of RP is

$$\mathbb{V}_{pre}\,(RP)^{-1}\Big|_{N=I=1} = \frac{\left[\lambda^2(\sigma_S^2 + \bar{\tau}_\theta^2) + \gamma^2\sigma_S^2\omega_X^2(\sigma_S^2 + \lambda\bar{\tau}_\theta^2)\right]^2}{\bar{\tau}_\theta^4\left[\lambda^2 + \gamma^2\sigma_S^2\omega_X^2\right]^2\left[\lambda^2(\sigma_S^2 + \bar{\tau}_\theta^2) + \gamma^2\sigma_S^4\omega_X^2\right]}. \qquad \text{(H.1)}$$

It is not difficult to show that the precision of the price can be rising or falling in λ, and rising or falling in the precision of news watchers' signals σ_S^2—by taking the respective derivatives and playing with parameter values.[18] This result weakens Grossman and Stiglitz' (1980) conjectures 1 and 4 which asserted monotonous changes. The difference between the two models arises because Grossman and Stiglitz assume in their derivation that the realization of the equilibrium price depends on the expected value of the risky asset supply (see (A10) in Grossman and Stiglitz 1980), while the realization of equilibrium price in the present model is dependent on the realization of asset supply, and not its expectation (see (20)). The latter is consistent with our typical notion of a Walrasian equilibrium.

I General model

To be provided.

[18]For example, use the values underlying figure 2 (as in footnote 11) to evaluate the according elasticities and then use $\gamma = .1, \omega_X = 1, \lambda = .9$.

References

Admati, Anat R. (1991): "The Informational Role of Prices: A Review Essay," *Journal of Monetary Economics*, 28(2), 347–61

Allen, Beth E. (1981): "Generic Existence of Completely Revealing Equilibria for Economies with Uncertainty when Prices Convey Information," *Econometrica*, 49(5), 1173–99

Banerjee, Abhijit V. (1992): "A Simple Model of Herd Behavior," *Quarterly Journal of Economics*, 107(3), 797–817

Barlevy, Gadi, and Veronesi, Pietro (2000): "Information Acquisition in Financial Markets," *Review of Economic Studies*, 67(1), 79–90

Bikhchandani, Sushil, Hirshleifer, David, and Welch, Ivo (1992): "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades," *Journal of Political Economy*, 100(52), 992–1026

Burguet, Roberto, and Vives, Xavier (2000): "Social Learning and Costly Information Acquisition," *Economic Theory*, 15(1), 185–205

Dutta, Jayasri, and Morris, Stephen (1997): "The Revelation of Information and Self-Fulfilling Beliefs," *Journal of Economic Theory*, 73(1), 231–44

Grossman, Sanford J., and Stiglitz, Joseph E. (1976): "Information and Competitive Price Systems," *American Economic Review*, 66(2), 246–53

——— (1980): "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70(3), 393–408

Hellwig, Martin F. (1980): "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, 22(3), 477–98

Jordan, James S. (1982): "The Generic Existence of Rational Expectations Equilibrium in the Higher Dimensional Case," *Journal of Economic Theory*, 26(2), 224–43

Kyle, Albert S. (1989): "Informed Speculation with Imperfect Competition," *Review of Economic Studies*, 56(3), 317–55

Lee, In Ho (1998): "Market Crashes and Informational Avalanches," *Review of Economic Studies*, 65(4), 741–59

Li, Lode, McKelvey, Richard D., and Page, Talbot (1987): "Optimal Research for Cournot Oligopolists," *Journal of Economic Theory*, 42(1), 140–66

Pietra, Tito, and Siconolfi, Paolo (1998): "Fully Revealing Equilibria in Sequential Economies with Asset Markets," *Journal of Mathematical Economics*, 29(3), 211–23

RAHI, ROHIT (1995): "Partially Revealing Rational Expectations Equilibria with Nominal Assets," *Journal of Mathematical Economics*, 24(2), 137–46

RAIFFA, HOWARD, and SCHLAIFER, ROBERT (1961): *Applied Statistical Decision Theory*, Studies in managerial economics. Graduate School of Business Administration, Harvard University, Boston, Massachusetts

RAITH, MICHAEL (1996): "A General Model of Information Sharing in Oligopoly," *Journal of Economic Theory*, 71(1), 260–88

ROMER, DAVID (1993): "Rational Asset-Price Movements Without News," *American Economic Review*, 83(5), 1112–30

SAMUELSON, PAUL A. (1954): "The Pure Theory of Public Expenditure," *Review of Economics and Statistics*, 36(4), 387–89