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On the Random Walk Characteristics of Short- and Long-Term Interest Rates In an Efficient Market

1. INTRODUCTION

AS IS NOW WELL KNOWN, tests of market efficiency—that asset prices rapidly and fully reflect all relevant information—are inevitably tests of joint hypotheses. In each test, a particular model of market equilibrium is examined simultaneously with the question of market efficiency. Early studies of the behavior of stock prices, however, often failed to make explicit the underlying model of market equilibrium. In particular, those who studied the random walk hypothesis typically failed to make explicit their assumption that equilibrium returns are constant over time. Only under this assumption does evidence on the autocorrelation of successive one-period returns bear directly on the question of market efficiency. Recently, Phillips and Pippenger [9] have called attention to a body of evidence that suggests that interest rates may follow a random walk. They then note that this evidence "is consistent with the hypothesis that capital markets are efficient" [9, p.11]. By implication, they suggest that if interest rates do not follow a random walk, then the bond market is not efficient. Poole [11, p. 476], too, suggests that random walk behavior of interest rates is to be expected in an efficient market.

The premise of this paper is that, if the shortcomings of the early work on stock prices are to be avoided, researchers must exercise great caution in linking random

*I am indebted to Frederic Mishkin, Robert Shiller, and an anonymous referee for constructive comments on an earlier version of this paper.

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0022-2879/79/1179-0457$00.50/0
JOURNAL OF MONEY, CREDIT, AND BANKING, vol. 11, no. 4 (November 1979)

walk behavior to market efficiency. The paper demonstrates that *long-term* interest rates in an efficient market will exhibit random walk characteristics in the absence of time-varying term premiums, but cautions that this result is only approximate. There can be no presumption, however, that short-term interest rates in an efficient market will conform to the random walk model.

One of the most important themes in the term structure literature in recent years has been the role of rational expectations. In their study of the efficiency of the bond market, Phillips and Pippenger also observe: "Although the concepts of Rational Expectations and Efficient Markets seem to have much in common, the two approaches have developed entirely independently, and the relationship between them is not at all clear" [9, p. 12]. Their concern arises in the context of the Modigliani-Shiller [5] model of the term structure, which invokes the concept of rational expectations to derive a relationship between the long-term rate and a long distributed lag on current and past values of the short-term rate and the rate of inflation. This concern is misplaced, however, and stems ultimately from the failure of Phillips and Pippenger to provide a more rigorous statement of the conditions under which long-term interest rates will exhibit random walk characteristics in an efficient market. Indeed, as this paper demonstrates, it is the *approximate* nature of this result that provides the link between the model proposed by Modigliani and Shiller and the random-walk characteristics of long-term rates in an efficient market.

Finally, this paper addresses the complications arising from the need to define *empirically* the innovation in the realization of the short-term rate in order to conduct appropriate tests of market efficiency. This problem clouds the interpretation of the results recently presented by Phillips and Pippenger [10], and highlights again the potential problems of identifying random walk behavior of short-term interest rates with market efficiency.

2. RANDOM WALK CHARACTERISTICS OF INTEREST RATES IN AN EFFICIENT MARKET

Let $p_t = (p_{it}, \ldots, p_{n,t})$ be the vector of security prices at time t, ϕ_{t-1} be the complete information set available at $t - 1$, $\phi_{t-1}{}^m$ be the information set employed by the market, $f_m(p_t|\phi_{t-1}{}^m)$ be the density function for p_t assessed by the market, and $f(P_t|\phi_{t-1})$ be the true density function implied by ϕ_{t-1}. Then, following [1], the requirement for the market to be efficient can be expressed as

$$f(p_t|\phi_{t-1}) = f_m(p_t|\phi_{t-1}{}^m) \,. \tag{1}$$

In words, the joint probability distribution of security prices at time t assessed by the market must equal the true distribution of security prices. The market, in effect, must fully utilize all available information. To implement (1), however, a model of market equilibrium—which forms the basis for the setting of prices in the current period (i.e., $t - 1$)—must be specified. Researchers have generally assumed that conditions of market equilibrium can be reduced to statements about expected returns. Security prices in $t - 1$ are then set by the market so that expected returns in period t will equal their equilibrium values. For a given model of market equilibrium, equation (1) thus requires

$$E(\tilde{r}_{jt}|\phi_{t-1}) = E_m(\tilde{r}_{jt}|\phi_{t-1}{}^m) . \tag{2}$$

In words, the expected values of individual security returns as assessed by the market on the basis of information available at $t - 1$ must equal their true or equilibrium expected values. The latter, in turn, reflect risk and other factors that are of concern to market participants.

Consider now the statement that interest rates in an efficient market will follow a random walk. Specifically, this statement requires that successive changes in interest rates be drawn independently from a probability distribution with mean zero:

$$R_t = R_{t-1} + u_t . \tag{3}$$

The association of (3) with (1) requires a great deal of caution, as illustrated below.

Consider first the case of *long-term* interest rates. The spot rate on an n-period, *noncoupon* bond is period t ($R_{n,t}$) can be expressed as the geometric average of the one-period spot rate ($R_{1,t}$) and a corresponding series of one-period forward rates (${}_{t+i}f_{1,t}$):

$$R_{n,t} = [(1 + R_{1,t})(1 + {}_{t+1}f_{1,t}) \ . \ . \ . \ (1 + {}_{t+n-1}f_{1,t})]^{1/n} - 1 . \tag{4}$$

Using the arithmetic as an approximation to the geometric mean, and recognizing that ${}_{t}f_{1,t} = R_{1,t}$, one notes that the n-period bond rate evolves over time according to the following formula:

$$R_{n,t} - R_{n,t-1} = \frac{1}{n} * \left[({}_{t+n-1}f_{1,t} - R_{1,t-1}) + \sum_{i=0}^{n-2} ({}_{t+i}f_{1,t} - {}_{t+i}f_{1,t-1})\right] . \tag{5}$$

Under the pure expectations theory, each of the forward rates in (4) represents the market's expectation of the corresponding spot rate: ${}_{t+1}f_{1,t}$ represents the market's expectation at time t of the one-period spot rate in period $t + 1$ and so on. Let ϕ_t again represent the information available to the market in period t. If the pure expectations theory is valid and if the bond market is efficient, then

$$E({}_{t+i}\tilde{f}_{1,t}|\phi_{t-1}) = {}_{t+i}f_{1,t-1} \quad \text{for} \quad i = 0, \ . \ . \ . \ , n - 2 . \tag{6}$$

Equations (5) and (6) thus imply

$$E(\tilde{R}_{n,t}|\phi_{t-1}) - R_{n,t-1} = \frac{1}{n} * [E({}_{t+n-1}\tilde{f}_{1,t}|\phi_{t-1}) - R_{1,t-1}] \,. \tag{7}$$

The term on the right-hand side of (7), which represents the "non-overlapping" one-period rates, clearly approaches zero as n gets large. Long-term bond yields thus should follow (approximately) a martingale sequence[1]

$$E(\tilde{R}_{n,t}|\phi_{t-1}) \doteq R_{n,t-1} \,. \tag{8}$$

Finally, although (5) is derived under the pure expectations hypothesis, it would continue to hold (in its approximate form) in the presence of time-invariant term premiums. In this case, a constant would be added to the right-hand-side of (4), which would then disappear with the first differencing of the long-term rate in (5).

The martingale property, as noted, is only approximate. It implies that the current change in the long-term bond rate is a random variable (with mean zero) that is uncorrelated with all information available at the beginning of the period.[2] Thus, again subject to the approximation caveat, long-term interest rates will exhibit the random walk characteristics noted in (3) if the bond market is efficient *and* if term premiums, should they exist, are time-invariant. If time-varying term premiums do exist, then the change in the long-term bond rate can vary *predictably* with the *change* in the corresponding term premium without contradicting the efficient market hypothesis.

The random walk characteristics of long-term rates in an efficient market are grounded ultimately in the well-documented role of expectations as a key determinant of the term structure, together with empirical studies [3, 7] that suggest that term premiums may well be time-invariant. By contrast, the proposition that short-term rates follow a random walk in an efficient market can be obtained *only by direct assumption*. If 90-day Treasury bills are equated with the one-period interest rate, then their nominal return is also their expected or equilibrium return. If and only if the equilibrium return on Treasury bills follows a random walk will the one-period rate in an efficient market follow a random walk. There is, however, no

[1]This result requires only that the short-term rate not be "too" nonstationary, so that its expected value n periods in the future is not dramatically different from its latest value. As noted by Sargent [12], the fact that empirical term structures are reasonably flat suggests that such nonstationarity is not likely to invalidate the approximation in (8).

In a comment on an earlier draft of this paper, Robert Shiller notes that the variance of the long-term rate in an efficient market (and in the absence of time-varying term premiums) may under certain circumstances approach zero as term to maturity increases. Such would be the case, for example, if the short-term rate was simply white noise around some fixed mean. In this case, the martingale approximation cited in (8) would continue to hold, but the result would be uninteresting in view of the degenerate nature of the distribution of the long-term rate. In fact, many observers have noted the perhaps surprising volatility of long-term interest rates (and hence of holding-period returns on long-term bonds), a result that motivates Shiller [13] to address the question of whether long-term rates are too volatile to be consistent with market efficiency.

[2]Note that (8) is less restrictive than the random walk model, since it does not require that successive changes in long-term interest rates be independently and identically distributed over time.

requirement of efficient market theory that the equilibrium return—and hence the 90-day bill rate—behave in such a fashion.[3]

3. THE MARTINGALE PROPERTY OF LONG-TERM INTEREST RATES: MORE ON THE APPROXIMATION

As (5) indicates, interest rates on long-term *noncoupon* bonds will clearly exhibit random walk characteristics in an efficient market, again in the absence of time-varying term premiums. For *coupon* bonds, whose yields are generally employed in empirical research, the result is less clear. For such bonds, as Modigliani and Shiller [5] note, an exponentially weighted average should probably replace the arithmetic average employed in (5). If the exponentially weighted average is used, and if λ equals $1/(1 + \bar{R})$ where $\bar{R}$ is the representative or normal one-period rate, then (7) is replaced by

$$\begin{aligned} E(\tilde{R}_{n,t}|\phi_{t-1}) - R_{n,t-1} = \frac{(1-\lambda)}{(1-\lambda^n)}[(1-\lambda)_t f_{1,t-1} + \lambda(1-\lambda)_{t+1} f_{1,t-1} \\ + \cdots + \lambda^{n-2}(1-\lambda)_{t+n-2} f_{1,t-1} \\ + \lambda^{n-1}E({}_{t+n-1}\tilde{f}_{1,t}|\phi_{t-1}) - R_{1,t-1}] \,. \end{aligned} \tag{9}$$

Although the size of this term is not immediately obvious (more on this below), it does suggest that the *anticipated* component of the change in the long-term bond rate could be significant.

Equations (7) and (9) thus provide alternative expressions for the ex ante or anticipated component of the change in the long-term rate. In an efficient market, the covariance between the anticipated and unanticipated changes must be zero. As a result, the variance of the observed *change* in the long-term rate can be decomposed into the sum of two orthogonal components, the variances of the anticipated change and the variance of the unanticipated change, where the latter corresponds to the receipt of new information. For *coupon* bonds, as implied by (9), the variance of the anticipated component might be significant. This point merits emphasis since a necessary (but not sufficient) condition for the change in the long-term rate to bear a systematic relation to a prior information set is for this term to be significant.

In spite of the fact that the exponentially weighted formula is only an approximation, it is useful to obtain a rough estimate of the magnitude and importance

[3]If the term premium on long-term bonds is time invariant, then the relationship between the ex ante return on short- and long-term bonds can be written as follows: $R_{1,t} + K = R_{n,t} + g_t^*$, where g_t^* is the ex ante or expected capital gain or loss on holding long-term bonds for one period. The fact that $R_{1,t}$ or the ex ante return on the one-period bond *may* have considerable variance in an efficient market is *not*, of course, inconsistent with the claim that the long-term bond rate follows (approximately) a martingale sequence. The crucial point is that the ex ante change in the long-term rate that produces the ex ante capital gain or loss necessary to equilibrate the expected holding-period returns is very small, or approximately equal to zero.

of the anticipated change so identified. Rough calculations with Canadian data[4] indicate that, at least for the period 1961.I–1976.IV, the proportion of the change in long-term Canada yields that has been anticipated is of relatively minor importance. For the 64 observations in the sample, the absolute value of the anticipated change in the long-term Canada rate is estimated to be less than 5 basis points for two-thirds of the observations, and never exceeds 11 basis points. By contrast, over one-third of the actual changes in this rate exceed 30 basis points in absolute value, and ranged in value as high as 127 basis points. As a result, only 1.75 percent of the variance of the change in the long-term rate could be assigned to the anticipated component. The vast majority—98.25 percent—of the variance of the long-term rate thus represents, under the joint hypothesis being investigated, the receipt of new information.

Following the important contribution of Modigliani and Shiller [5], students of the term structure have placed great emphasis on the concept of rational expectations formation. As noted in the introduction to this paper, Phillips and Pippenger suggest that the efficient market and rational expectations literatures have developed independently and that the relationship between them is not at all clear. Their concern relates to the fact that Modigliani and Shiller, after invoking the concept of rational expectations formation, link the long-term rate to a long distributed lag on current and past values of the short-term rate and the rate of inflation. In first difference form, the Modigliani-Shiller model indicates that the change in the long rate is related to the current *and prior* changes in the respective variables. This result provides a sharp contrast to the random walk model proposed by Phillips and Pippenger, associated by these authors with an efficient market, which requires that changes in the long rate be uncorrelated with *all* prior information.

In fact, the apparent confusion of Phillips and Pippenger relates ultimately to their failure to provide a more rigorous statement of the conditions under which long-term interest rates will exhibit random walk characteristics in an efficient market. Indeed, it is the *approximate* nature of this result that provides the link between the model proposed by Modigliani and Shiller—which draws explicitly on the concept of rational expectations—and the random-walk characteristics of long-term rates in an efficient market. In particular, there is no reason a priori that the "approximation" term on the RHS of (9) cannot bear a significant relationship to prior changes in the short rate and the rate of inflation or, indeed, to any predetermined variables. If, as *presumed* by Modigliani and Shiller, the short rate

[4]The first step in seeking to approximate the RHS of (9) is to eliminate the presence of the (presumed) time-invariant term premium. This can be accomplished by setting this premium equal to the average spread between the long- and the short-term rate, or between long-term Government of Canada bonds and 90-day Treasury bills in my example. The next and most difficult step is to approximate the forward rates that appear in (9). To this end, it was assumed that the difference between the long-term rate (purged of the term premium) and the Treasury bill rate implied a monotonically rising or declining yield curve. By assigning a specific maturity of 10 years or 40 quarters to the long-term rates, this difference can then be allocated in equal increments to the succession of forward rates. A difference, for example, of 200 basis points between the long and short rates is (roughly) consistent with the final forward rate on the RHS of (9) exceeding the initial rate by 400 basis points. Finally, the average or representative short-term rate is set to equal to 5, implying a value for λ of approximately 0.95.

has a representation in terms of the histories of both the short rate and the rate of inflation, then—via the "chain principle" of forecasting—predicted values of the short rate will have such a representation as well. The forward rates on the RHS of (9)—and thus the "approximation" term itself—will then bear a systematic relationship to the histories of these two variables if expectations are rationally formed on the basis of the hypothesized information set. In fact, however, this term would appear to be of sufficiently minor importance that the actual change in the long-term rate, again under the joint hypothesis, is likely to be dominated by the receipt of new information and thus to bear no discernible relation to the history of these two variables or, indeed, to any set of predetermined variables.

4. DEFINING THE INNOVATION IN THE SHORT-TERM RATE IN EMPIRICAL TESTS OF MARKET EFFICIENCY

In [10], Phillips and Pippenger seek to provide further evidence on the question of whether long-term interest rates follow a random walk. In order to test whether the lagged long-term rate embodies all relevant information as to its future course, Phillips and Pippenger estimate the following equation:[5]

$$R_{n,t} = \alpha_0 + \alpha_1 R_{n,t-1} + B_0 (R_{1,t} - R_{1,t-1}) + \sum_{i=1}^{17} B_i (R_{1,t-i} - R_{1,t-i-1}) + \cdots + u_t . \tag{10}$$

The authors find (Table 2) that several of the individual coefficients of the lagged short-term rates are significant in determining the current yields on both corporate and government bonds, and that the *set* of lagged short-term rates is significant at the 1 percent level in the corporate bond equation. The results, in the authors' minds, are somewhat disconcerting, since they appear to suggest that economic agents do not process information efficiently in the bond market.

In fact, this conclusion does not necessarily follow. The difficulty lies in the failure of the authors to recognize the problem of *empirically* defining the innovation or new information contained in the realization of the short-term rate. Let ${}_tR_{1,t-1}{}^*$ denote the efficient forecast, made in period $t - 1$, of the short-term rate in period t. As is well known, the "chain rule" of forecasting implies that the change in period t in each of the forward rates applicable to the n-period bond rate is proportional to the current forecasting error, or $R_{1,t} - {}_tR_{1,t-1}{}^*$.[6] The martingale

[5]The equations estimated by Phillips and Pippenger also include (1) the change in the volatility of short-term rates, a proxy for the term premium, and (2) the current change in the rate of inflation. Neither is relevant to the interpretative problem cited in the text and thus both are ignored in the subsequent discussion.

[6]In other words, ${}_{t+i}f_{1,t} - {}_{t+i}f_{1,t-1} = b_i(R_{1,t} - {}_tR_{1,t-1}{}^*)$ for the $n - 1$ forward rates ($i = 0$ to $n - 2$) in the second term on the RHS of (5). The error-learning coefficient (B_0) in (11) is an unweighted average of these individual (b_i) error-learning or revision coefficients. These latter coefficients, in turn, are derived from the stochastic properties of the one-period interest rate. See, for example, [12].

model of equation (8), ignoring the approximation caveat, thus requires that the current change in the long-term rate be related to the realization of the short-term rate as follows:

$$R_{n,t} - R_{n,t-1} = B_0(R_{1,t} - {}_tR_{t-1}^*) + u_t \,. \tag{11}$$

Only if the short-term rate follows a random walk, in which case ${}_tR_{t-1}^*$ is simply equal to $R_{1,t-1}$, does the current change in the short-term rate correspond to the innovation in the short-term rate. In this case, regressing the current change in the long-term rate on the current and prior changes in the short-term rate will result in insignificant coefficients on the lagged terms if the bond market is efficient. If, on the other hand, the short-term rate varies systematically with certain of its prior values, then such a regression need *not* produce insignificant coefficients for the lagged changes in the short rate if the market is efficient. Suppose, for example, that the time series properties of the short-term rate are such that it evolves according to the following scheme:[7]

$$R_{1,t} = R_{1,t-1} + \sum_{i=1}^{17} \alpha_i(R_{1,t-i} - R_{1,t-i-1}) + \varepsilon_t \,. \tag{12}$$

Equation (11) would then require

$$\begin{aligned} R_{n,t} - R_{n,t-1} &= B_0(R_{1,t} - {}_tR_{1,t-1}^*) + u_t \\ &= B_0(R_{1,t} - \left[R_{1,t-1} + \sum_{i=1}^{17} \alpha_i(R_{1,t-i} - R_{1,t-i-1})\right] + u_t \\ &= B_0(R_{1,t} - R_{1,t-1}) - \sum_{i=1}^{17} B_0\alpha_i(R_{1,t-i} - R_{1,t-i-1}) + u_t \,. \end{aligned} \tag{13}$$

Estimation of equation (13) would yield significant coefficients for the set of lagged changes in the short-term rate, although the significance of such coefficients would *not* indicate that the market fails to process information efficiently. As both Mishkin [4, p. 727] and Pesando [8, p. 1064] note, an appropriate test for market efficiency under such circumstances is to run the regression *without* the current change in the short-term rate. In this case, since only prior information is contained in the set of explanatory variables, all coefficients must be zero if the market is efficient (and, of course, if term premiums are time invariant).[8] The important lesson is that their

[7]This example, in essence, is the one employed in [4] to make a similar point.
[8]A number of other limitations cast considerable doubt on the usefulness of the regression results presented by Phillips and Pippenger. Contrary to the spirit of the random walk model, they fail to

inappropriate claim that short-term rates will exhibit random walk characteristics in an efficient market fails to alert Phillips and Pippenger to the interpretative problems noted above, and results in their failure to conduct the further tests necessary to determine if the bond market is efficiently processing information.

To sum up, the current change in the long-term bond rate in an efficient market may be correlated with prior changes (for example) in the short-term rate by virtue of (1) the approximate nature of the martingale result (equation (8)) and (2) the problem of empirically defining the innovation in the realization of the short rate. The latter source of correlation is ruled out by market efficiency if only prior changes in the short rate are included in the regression, whereas the former is not. Since the martingale approximation appears to be very close, however, only the correlation arising from the problem of defining the innovation in the short rate is likely to pose problems in empirical research. Finally, the importance of these two sources of correlation is linked ultimately to the stochastic properties of the short-term rate. The importance of these properties in determining the evolution of the long-term rate is, of course, the major message conveyed by Modigliani and Shiller in their initial contribution.[9]

4. SUMMARY AND CONCLUDING OBSERVATIONS

Under the joint hypothesis that (1) the bond market is efficient and (2) term premiums—if they exist—are time invariant, long-term interest rates will *approximately* follow a martingale sequence and thus will exhibit random walk characteristics. Recent evidence presented by Pesando [8] and Mishkin [4] provides support for this joint hypothesis. By contrast, the claim that short-term rates will follow a random walk in an efficient market obtains only from the direct assumption that the equilibrium return on one-period rates follows a random walk.

If long-term interest rates do follow (approximately) a martingale, then the *change* in the long-term rate should be the dependent variable in equations designed to isolate (for example) the innovation in monetary policy and/or to search for the existence of time-varying term premiums. Alternately, equations designed to explain the *level* of these rates are likely to be characterized by a high degree of

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constrain the coefficient of the lagged long-term rate to unity. They then employ the Cochrane-Orcutt procedure to handle the possible problem of serial correlation in an equation with a lagged dependent variable. Although this procedure is correct from a technical viewpoint, the authors fail to recognize that serial correlation may provide de facto evidence of market inefficiency. A more appropriate procedure would be to use the change in the long-term rate as the dependent variable, and then recognize that market efficiency requires (1) that all coefficients of predetermined variables in regressions that contain *only* predetermined variables must not differ significantly from zero and (2) that such regressions be devoid of serial correlation.

[9]In an earlier paper [8], the present author neglected to emphasize the importance of this point. In n. 9, the results for the regression with *only* predetermined variables are noted. The lack of significance of any of the predetermined variables provides unequivocal support for the hypothesis that the market is efficient. The results for the regressions that include the current change in the short rate are presented in Table 1 in the text. The lack of significance of any of the lagged coefficients in these regressions, in turn, is consistent with the low degree of correlation among successive changes in the short-term rate, a result alluded to in n. 5.

positive serial correlation.[10] From this perspective, the serial correlation that characterizes (for example) recent studies of the impact of price expectations on the *level* of long-term yields in both Canada [6] and the United States [2] is not surprising.

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[10]Ignoring the approximation term, the martingale model implies that the "true" specification for the explanation of the level of the long-term interest rate is $R_{n,t} = R_{n,t-1} + u_t$, where u_t is a serially uncorrelated disturbance term. Alternative models of the determinants of $R_{n,t}$ that *exclude* its lagged value omit a variable with a serial correlation parameter of unity, and thus are likely to be characterized by a high degree of positive serial correlation.