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A model of a noncompetitive speculative market is analyzed in which privately informed traders and market makers are risk averse. Market liquidity is found to be nonmonotonic in the number of informed traders, their degree of risk aversion, and the precision of their information. It is also shown that increased liquidity trading leads to reduced price efficiency, and that, under endogenous information acquisition, market liquidity may also be nonmonotonic in the variance of liquidity trades.

Competitive noisy rational expectations models [e.g., Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), Verrecchia (1982)] have the feature that informed traders take the equilibrium price as given, even though they influence the information content of the price through their trades.[1] In these models, risk aversion prevents prices from fully revealing the private information of the informed agents. Kyle (1984, 1985) presents a noisy rational expectations model in which informed traders are risk neutral but imperfectly competitive: they strategically choose their trades realizing that their choices influence the price. In the resulting Nash equilibrium in quantities

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[1] See Hellwig (1980). Admati (1985), Pfleiderer (1984), and Grundy and McNichols (1990), among others, use competitive rational expectations models in which it is assumed that there is a continuum of traders, so that the competitive assumption holds exactly.

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(orders) among traders, the informed optimally choose to withhold some of their information from the price. In recent years, this model has become tremendously popular for addressing market microstructure issues.[2] In this article, we examine the effects of introducing risk aversion into the one-period Kyle (1984) framework.

The risk neutrality assumption of Kyle's (1984) model implies that an increase in the variance of liquidity ("noise") trades leaves the informational efficiency (informativeness) of prices unchanged because informed traders scale up their trades in response to an increase in the amount of liquidity trading [see Kyle (1984, 1985)]. Another implication of the model is that when informed traders observe strongly correlated signals, an increase in their number intensifies the Cournot-like competition between them, leading to an increase in market liquidity, where liquidity is defined as the order flow necessary to move prices a unit amount. The latter implication is exploited by Admati and Pfleiderer (1988b) to demonstrate that an increase in the amount of uninformed liquidity trading leads to an increase in the number of informed traders, which leads to improved terms of trade for liquidity traders.

Intuitively, risk-averse informed traders respond less aggressively to an increase in liquidity trading than risk-neutral ones. Thus, when information acquisition is exogenous, an increase in the variance of liquidity trades decreases price efficiency, unlike in the risk-neutral case. Also, an increase in the number of risk-averse informed traders increases the aggregate risk tolerance of these traders, which tends to raise their aggregate profits. This causes the marginal effect of an increase in the number of informed traders on market liquidity to be negative despite greater competition between these traders, provided the number of informed traders is small. Therefore, with endogenous information acquisition, increased liquidity trading, which leads to an entry of more informed traders, can lead to *decreased* market liquidity.

We also show that market liquidity may be *increasing* in the precision of private information and the risk tolerance of the informed.

[2] For example, Admati and Pfleiderer (1988b) and Foster and Viswanathan (1990) apply the model to analyze intraday and interday patterns in volume and price variability. Caballé and Krishnan (1989) present a rigorous multisecurity extension of this model. Bhushan (1991) uses the model to present a theory of diversification by uninformed traders under risk neutrality. Fishman and Hagerty apply the model to analyze the relationship between investment decisions and price efficiency (1989a) and the impact of insider trading on price efficiency (1989b). Subrahmanyam (1991) applies the model to demonstrate that the introduction of a market in a portfolio reduces the trading costs of uninformed traders who wish to trade several securities simultaneously. Chowdhry and Nanda (1991) use the model to analyze the phenomenon of a security trading in multiple markets. Kumar and Seppi (1990) analyze the optimal order placement strategy of index arbitrageurs in the context of the model.

An intuitive explanation for this finding is that the intensity of competition between the informed increases as they become more precisely informed or more risk tolerant, which leads to increased market liquidity if the number of informed traders is sufficiently large.

Our work is related to a recent article by Kyle (1989), which demonstrates the existence of a symmetric linear equilibrium with imperfect competition and risk aversion in a model in which noise traders are assumed to trade inelastically and informed and uninformed speculators submit demand curves ("limit orders") to an auctioneer, who then sets a market clearing price. Both informed and uninformed speculators behave strategically, taking into account the effect of their demands on the market clearing price. Kyle's 1984 model, which we analyze further, however, captures some specific microstructure features of the trading environment in actual financial markets by allowing for the existence of market makers, who are assumed to have exclusive access to the order flow. Our structure allows us to derive interesting comparative statics results associated with the market liquidity parameter, in addition to some results on the informational efficiency of prices. For reasons of tractability, however, we limit ourselves to the case in which informed traders observe identical signals, in contrast to Kyle (1989), who assumes that informed traders observe diverse signals.

In other related work, Admati and Pfleiderer (1988a) focus on the optimal strategy of a monopolistic seller of information in a model similar to ours. In our analysis, the focus is on the determination of market liquidity and price efficiency by strategic interaction between risk-averse market participants.

The remainder of this article is organized as follows. In Section 1, implications for market liquidity and price efficiency are derived under risk neutrality of market makers. In Section 2, the case of risk-averse market makers is discussed and some features of our model are compared to those of Kyle's 1989 model. Section 3 concludes the article.

1. The Case of a Risk-Neutral Market Maker

In our model, one risky security is traded by three types of traders: risk-averse, informed traders who possess private information about the fundamental value of the risky security, liquidity ("noise") traders whose share demands are exogenous and who trade for idiosyncratic life-cycle or liquidity reasons,[3] and a competitive market maker. The market maker, who is assumed to be risk neutral in this section, absorbs the net demand of the other traders and sets the price such

[3] Specific reasons for liquidity trading include risk-exposure adjustment, tax planning, the desire for immediate consumption, and idiosyncratic wealth shocks.

that he expects to earn zero profits conditional on observing the net order flow.[4] Trading takes place at time 0 and the security is liquidated at time 1. The liquidation value of the security at time 1 (denoted by F) is given by

$$F = \bar{F} + \delta, \tag{1}$$

$\bar{F}$ is known to all agents at time 0, and informed traders observe a noisy signal, which is a realization of the random variable $\delta + u$. All informed traders are assumed to possess constant absolute risk-aversion preferences (negative exponential utility) with a common risk-aversion coefficient A.[5] The number of informed traders is denoted by k and the total random demand of the liquidity traders is denoted by z. The random variables δ, u, and z are assumed to be mutually independent and normally distributed with mean zero. We denote $\text{var}(u) \equiv \phi$, $\text{var}(z) \equiv \sigma_z^2$, and, without loss of generality, set $\text{var}(\delta) = 1$.

1.1 Market liquidity

As in Kyle (1984, 1985) and the subsequent literature based on his work, we characterize the unique Nash equilibrium of our model in which trading strategies are linear. Denote the net order flow by ω. Since the market maker earns zero expected profits conditional on the order flow, the price P set by the market maker satisfies

$$P = E[F \mid \omega]. \tag{2}$$

We begin by assuming that the market maker employs a linear pricing rule given by

$$P = \bar{F} + \lambda\omega, \tag{3}$$

where λ is the usual inverse measure of market depth or liquidity. In the equilibrium we describe below, the linear pricing rule is consistent with the zero-profit condition.

Denote a particular informed trader's order by x. The expected utility $E[U]$, which the informed trader obtains from trading on the signal $\delta + u$, is given by

$$E[U] = -E[\exp(-Ax(F - P)) \mid \delta + u].$$

[4] The price will satisfy the zero-profit condition if it is determined as the outcome of a Bertrand auction for the order flow between at least two market makers. In this section, the nature of the pricing is invariant to whether one market maker takes the complete order or whether many market makers split the order flow after the auction.

[5] Note that we assume that the informed traders observe identical signals and possess identical preferences. A model with diverse signals and/or diverse preferences is much more complicated and does not permit the derivation of general comparative statics results.

Let the informed trader conjecture that each of the other informed traders submits an order $\beta(\delta + u)$. *Conditional* on the informed trader's signal, his profits from trading on the information are normally distributed. From the preference assumptions and Equations (1) and (3), this implies that the objective of the trader is equivalent to maximizing the mean-variance function

$$E[x(\delta - \lambda(x + (k-1)\beta(\delta + u) + z)) \mid \delta + u] \\ -(A/2)\,\mathrm{var}[x(\delta - \lambda(x + (k-1)\beta(\delta + u) + z)) \mid \delta + u],$$

with x as the choice variable.

Differentiating with respect to x and using standard formulas for the moments of a conditional normal distribution, we have

$$x = \frac{\delta + u}{(1+\phi)(2\lambda + A\lambda^2\sigma_z^2) + A\phi} - \frac{(k-1)\beta\lambda(1+\phi)(\delta + u)}{(1+\phi)(2\lambda + A\lambda^2\sigma_z^2) + A\phi}. \tag{4}$$

In this equation, the second term on the RHS is the amount by which informed traders scale down their orders because of the presence of other informed traders in the market (when $k = 1$, this term disappears). Thus, for a given λ, either a decrease in the risk-aversion coefficient A, or a decrease in the signal noise ϕ has two opposing effects: (i) it increases the trading aggressiveness of informed traders in their interaction with the *market maker* (represented by the first term), and (ii) it also increases the amount by which a particular informed trader scales down his order because of the presence of other informed traders (represented by the second term).

We obtain the unique linear (Nash) equilibrium by setting $x = \beta(\delta + u)$ in Equation (4) and solving for β.[6] We then have the following.

Lemma 1. *For a given market depth parameter λ, the order submitted by each informed trader is $\beta(\delta + u)$, where*

$$\beta = \frac{1}{(1+\phi)\lambda(k+1) + A\left[\phi + \lambda^2\sigma_z^2(1+\phi)\right]}. \tag{5}$$

The first term in the denominator of (5) is the inverse of the coefficient of the signal in each informed trader's optimal order when informed traders are risk neutral. From this term, we see that a decrease in λ,

[6] It is straightforward to show that, given the linear pricing rule of the market maker, the symmetric equilibrium is the unique Nash equilibrium.

k, or ϕ causes risk-neutral informed traders to trade more aggressively, in keeping with intuition. The second term of the denominator reflects the fact that when informed traders are risk averse, their equilibrium trades are smaller than when they are risk neutral for a given λ. The amount by which the trades are smaller depends on the degree of risk aversion of informed traders, on the variance of the noise in their signals, and on the variance of the component of the price which involves liquidity trades.

It follows from the zero-profit condition and normality of the order flow that λ is the coefficient of the OLS regression of liquidation value on the order flow, so that[7]

$$\lambda = \mathrm{cov}(\delta, \omega)/\mathrm{var}(\omega). \tag{6}$$

Note that the order flow $\omega = k\beta(\delta + u) + z$. Substituting for β from (5), we have Lemma 2.

Lemma 2. *(i) When informed traders are risk neutral, the equilibrium value of λ is given by*

$$\lambda = \frac{1}{k+1}\sqrt{\frac{k}{\sigma_z^2(1+\phi)}}. \tag{7}$$

(ii) When informed traders are assumed to be risk averse with common risk-aversion coefficient A, the equilibrium value of λ satisfies the quintic equation

$$\lambda[k^2(1+\phi) + \alpha^2\sigma_z^2] = k\alpha, \tag{8}$$

where

$$\alpha \equiv \beta^{-1} = A[\phi + \lambda^2(1+\phi)\sigma_z^2] + (k+1)\lambda(1+\phi).$$

Part (i) of Lemma 2 shows that under risk neutrality of informed traders, the equilibrium value of λ is decreasing in the number of informed traders (because of competition) and the variance of liquidity trading, and increasing in the precision of information. The comparative statics associated with λ for the risk-averse case are derived in the Appendix and are presented in the following proposition. Following the proposition, we provide a discussion that illustrates the intuition behind our results.

[7] Note that if the market maker uses a linear rule, it is optimal for the informed traders to use linear strategies (because their objectives are quadratic in their orders) and, conversely, if the informed traders are restricted to linear strategies, it is optimal for the market maker to use a linear pricing rule (because the order flow is normally distributed). Thus, the mutual consistency of linear pricing by the market maker and linear order placement strategies by the informed traders, a characteristic of Kyle's (1984, 1985) model, extends to the case in which the informed possess CARA preferences.

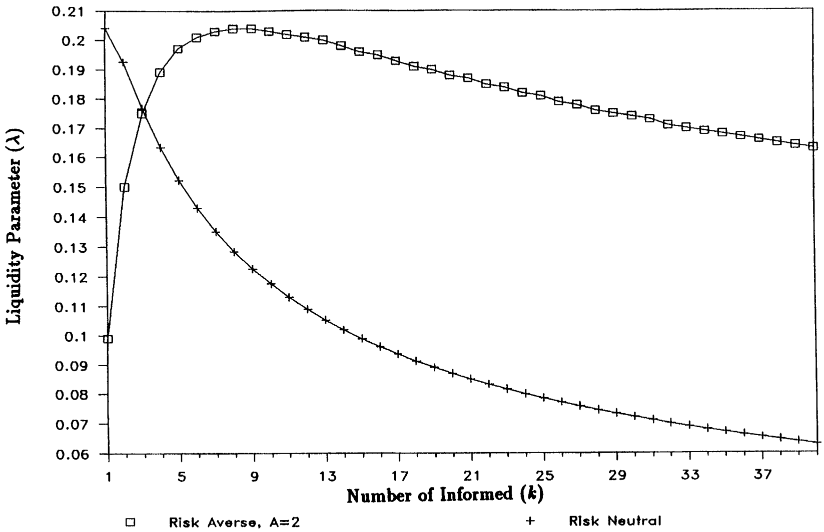

Figure 1
Equilibrium liquidity parameter λ as a function of the number of informed traders *k*, under risk neutrality and risk aversion of informed traders
It is assumed that the variance of liquidity trades $\sigma_z^2 = 2$ and that the variance of the noise in the informed traders' signals $\phi = 2$.

Proposition 1. *(i) The quintic equation (8) has a unique positive real root.*

(ii) λ is either a unimodal function of k, or is monotonically decreasing in k. (For $A = 0$, $d\lambda/dk < 0$.)

(iii) λ is either a unimodal function of A, or is monotonically decreasing in A. (For $k = 1$, $d\lambda/dA < 0$.)

(iv) λ is either a unimodal function of ϕ, or is monotonically decreasing in ϕ. (For $k = 1$ and/or $A = 0$, $d\lambda/d\phi < 0$.)

(v) $d\lambda/d\sigma_z^2 < 0$.

In Figure 1, the equilibrium λ is plotted as a function of the number of informed traders k for the risk-averse and the risk-neutral cases. In a market with risk-neutral informed traders, λ monotonically decreases in k because competition between these traders intensifies as k increases. [This result is also discussed by Admati and Pfleiderer (1988b).] However, in the market in which informed traders are risk averse, λ does not possess this property but is a unimodal function of k, as stated in Proposition 1 (ii). This finding can be explained as follows.

Let $t \equiv k\beta$ denote the coefficient of the signal in the total trading position of the informed traders. Then, from (6), λ satisfies the equation

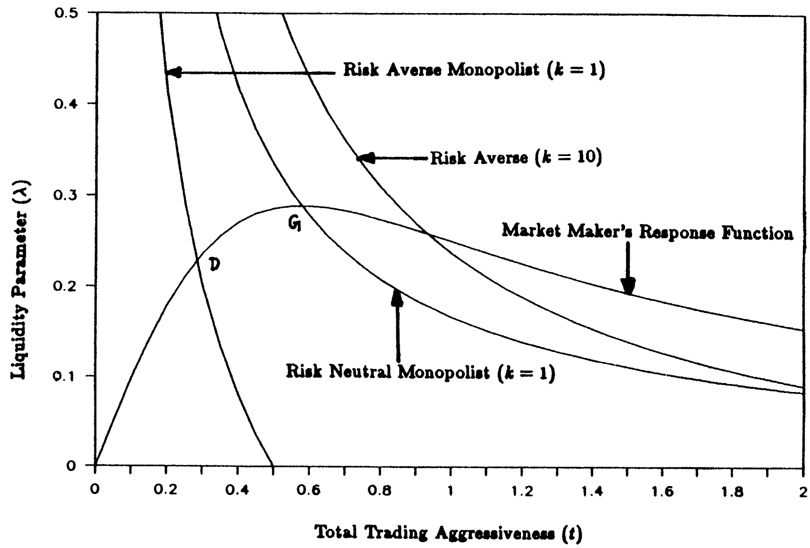

Figure 2
Determination of equilibrium under risk neutrality and risk aversion of informed traders, with a risk-neutral market maker
It is assumed that the variance of liquidity trades $\sigma_z^2 = 1$, the variance of signal noise $\phi = 2$, and that the risk-aversion coefficient $A = 1$. Point G represents the equilibrium corresponding to a risk-neutral monopolistic informed trader, while point D represents the equilibrium corresponding to the case of a risk-averse monopolist with a risk aversion coefficient $A = 1$.

$$\lambda = \frac{t}{t^2(1 + \phi) + \sigma_z^2}. \tag{9}$$

We refer to the expression in the RHS of the above equation as the "market maker's response function." Note that this function is decreasing in σ_z^2 and ϕ (which is intuitive) but unimodal in t. The last result obtains because an increase in the measure of trading aggressiveness t increases both the covariance of the order flow with the security's liquidation value and the variance of the order flow. From (9), the covariance is linear in t while the variance is quadratic in t. Thus, for small t, an increase in t implies a larger λ; whereas for large t, the opposite is true.

In Figure 2, the market maker's response function and the informed traders' optimal strategies [obtained from (5)] are plotted in λ-t space. From this figure, it can be seen that the equilibrium λ corresponding to the risk-neutral monopolist is at the point G, the peak of the response function. To see that this result is not specific to the parameter values considered in the figure, first differentiate the RHS of (9) with respect to t and set the result equal to zero, obtaining $t = [\sigma_z^2/(1 + \phi)]^{1/2}$ (a maximum). Now note that this value of t is identical to the actual equilibrium value of t obtained from (5) and (7), with $k = 1$ and $A = 0$. Thus, the equilibrium λ-t pair for the case of a risk-neutral monopolist always lies at a point where the market maker's response function is at a maximum with respect to t.[8] Now, as k increases, t increases (because of competition), so that in the risk-neutral case, an increase in the number of informed traders causes the equilibrium point to move down the market maker's response function to the right of the maximum.

The risk-averse monopolist, however, trades less aggressively than his risk-neutral counterpart; therefore, the equilibrium corresponding to the risk-averse monopolist is to the left of point G (point D in Figure 2). In this case, as k increases, the equilibrium first moves up and then down the market maker's response function. Thus, λ is a unimodal function of k when informed traders are risk averse.

Similarly, one can explain the unimodality of λ in A by first noting that as A decreases, the measure of aggregate trading aggressiveness t increases. As A is decreased starting from an equilibrium point on the upward sloping part of the market maker's response function, the equilibrium first moves up the response function and then down it. For $k = 1$, a decrease in A causes the equilibrium to approach the peak of the response function from below, but the equilibrium always remains to the left of the peak, implying that λ decreases in A for $k = 1$.

Note that changes in k or A only affect t for a given λ; they do not affect the market maker's response function. However, since changes in ϕ affect both t and the market maker's response function simultaneously, it is expositionally convenient to illustrate the comparative statics associated with ϕ using Figure 3, which presents the *equilibrium* λ as a function of k for three values of ϕ. The figure demonstrates that in the market where the informed possess perfect information, λ declines sharply (after reaching its peak) as k increases; in fact, at certain values of k, it declines below the λ in the markets where the information is imprecise (specifically, at $k = 13$ for $\phi = 1$ and at $k = 21$ for $\phi = 2$). Thus, for a sufficiently large k, λ *decreases* in the precision of private information. The intuition for this result is that increasing the precision of information intensifies competition between risk-averse informed traders, which can decrease λ if the number of informed traders is large.

[8] I thank the referee for pointing out an interesting coincidence [also mentioned in Admati and Pfleiderer (1988b)]: namely, that if the risk-neutral monopolist behaves as a Stackelberg leader, taking into account the effect of his trades on λ, the equilibrium obtained is exactly the same as that under our Nash assumption. To see this, observe that if there is only one risk-neutral informed trader in the market, his expected profits equal the expected losses of the noise traders and are thus given by $E[(P - F)z] = \lambda\sigma_z^2$. Since the informed trader chooses his strategy parameter $t = \beta$ to maximize expected profits, he chooses that value at which λ as a function of t is maximized, which again results in the equilibrium occurring at the peak of the response function.

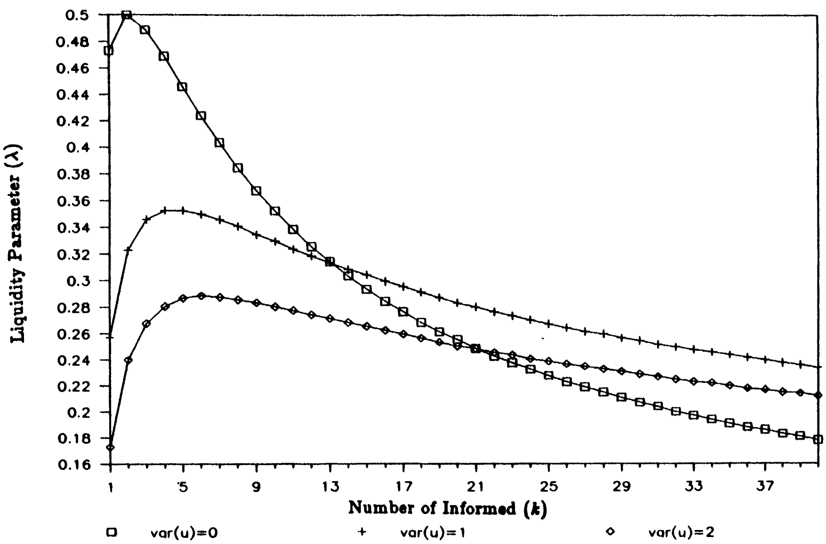

Figure 3
Equilibrium liquidity parameter λ as a function of the number of informed traders *k*, for different values of the variance of the noise in the informed traders' signals, ϕ
It is assumed that the variance of liquidity trades $\sigma_z^2 = 2$ and that the risk-aversion coefficient $A = 2$.

Another way of stating this result is that if informed traders are risk averse, increasing the degree of informational asymmetry between the informed and the uninformed can actually *improve* the terms of trade for the uninformed. From the discussion above, it can be inferred that the tendency for this counterintuitive result to obtain is strong if the number of informed traders is large and the increase in the precision of private information is sufficiently small.

From Proposition 1 (v), however, an increase in the variance of liquidity trades σ_z^2 causes λ to decrease in both the risk-neutral and the risk-averse cases by decreasing the adverse selection faced by the market maker.

1.2 Price efficiency

In this subsection, implications of our model are analyzed for the degree to which prices reveal private information. We define the informational efficiency (informativeness) of prices (denoted by Q) to be the posterior precision of the innovation δ conditional on the price (or the order flow); that is, $Q = [\text{var}(\delta \mid P)]^{-1} = [\text{var}(\delta \mid \omega)]^{-1}$. Using properties of the normal distribution and Equations (3) and (5), it is straightforward to show that

$$Q = 1 + \frac{1}{\phi + (\sigma_z^2/t^2)}. \tag{10}$$

This is not a closed-form expression since t is determined endogenously. Note, however, that an increase in trading aggressiveness (measured by an increase in t) causes more private information to be revealed in the price and thus increases Q. It is possible to obtain a closed-form expression for Q for the case in which informed traders are risk neutral. [This exercise was first performed in Kyle (1984, 1985).] Noting that $t = k/[(k + 1)\lambda(1 + \phi)]$ under risk neutrality of informed traders and substituting for λ from (7) we have

$$Q_{rn} = 1 + \frac{k}{1 + \phi + k\phi}, \tag{11}$$

where Q_{rn} denotes price efficiency for the risk-neutral case. Equation (11) reflects the fact that an increase in the variance of liquidity trading causes informed traders to scale up their trading in such a way that price efficiency is unchanged.

When informed traders are risk averse, however, we cannot obtain a closed-form expression for Q. In this case Q is obtained from (10) after substituting for the equilibrium value of t (which, in turn, is obtained from the equilibrium value of λ). The following proposition related to the comparative statics associated with Q can be derived (the proof is in the Appendix).

Proposition 2. *(i) When informed traders are risk averse, an increase in the variance of liquidity trading decreases price efficiency.*

(ii) Price efficiency Q is decreasing in the risk-aversion coefficient A.

The intuition for (i) of Proposition 2 is that risk-averse traders respond less aggressively to an increase in the variance of liquidity trades than risk-neutral ones, and that this leads to reduced price efficiency. Also, increasing the degree of risk aversion reduces price efficiency because more risk-averse informed traders trade less aggressively than less risk-averse ones.

The number of informed traders, k, is endogenized in the next subsection, demonstrating that if informed traders are risk averse, an increase in the variance of liquidity trading may lead to an *increase* in λ.

1.3 Endogenous information acquisition

In this subsection, we assume that each informed trader pays a fixed cost c to acquire his signal. Each informed trader decides to acquire the signal and trade if his unconditional expected utility from trading exceeds the utility from not trading. For convenience, we normalize the expected utility from not trading to -1 (which is equivalent to normalizing initial wealth to zero). The *unconditional* expected utility from trading on information is related to the moment-generating function of the product of two normally distributed random variables—namely, the signal and the difference between the security's liquidation value and its price. The following lemma (which is proved in the Appendix) gives an expression for each informed trader's unconditional expected utility from trading on the signal $\delta + u$.[9]

Lemma 3. *The unconditional expected utility v which a trader obtains from trading on the signal $\delta + u$ is given by*

$$v = -1/\sqrt{[A\beta(1 - k\beta\lambda(1 + \phi)) + 1]^2 - A^2\beta^2(1 + \phi)(1 - k\beta\lambda)}. \tag{12}$$

Observe that β and λ are both endogenously determined and therefore (12) is analytically complicated. For this subsection, therefore, our focus will be on a numerical example rather than on a formal derivation of comparative statics. The equilibrium number of informed traders is given by the largest nonnegative integer k for which $v \geq -\exp(-Ac)$. In other words, the equilibrium number of informed traders is the greatest value of k such that the certainty equivalent of profits per trader is at least as large as the cost of acquiring information.

Consider now the effect of increasing σ_z^2 on λ when the number of informed traders is endogenous. As σ_z^2 increases, the expected utility from trading on information increases for a given number of informed traders, and this leads to an increase in the equilibrium number of informed traders. If informed traders are risk neutral, the increase in the number of informed traders decreases λ because of increased competition between the traders. If informed traders are risk averse, however, the relationship between λ and k is nonmonotonic, and this may lead to a nonmonotonicity between λ and the variance of liquidity trades σ_z^2, as we demonstrate in the following example.

Numerical example. We assume that $A = 2$ and $\phi = 2$. Table 1 gives the equilibrium λ's and certainty equivalents per trader as a function of k for $\sigma_z^2 = 2$ and 3. Note that λ is increasing in k (in the range of k specified) and that the certainty equivalent is decreasing in k. Let $\lambda_k(\sigma_z^2)$ denote λ corresponding to a particular number of informed traders k and a particular level of liquidity trading σ_z^2. From Table 1, it is seen that $\lambda_2(3) > \lambda_1(2)$. The certainty equivalent corresponding

[9] The result in this lemma is also stated in Admati and Pfleiderer (1988a) [see equation (5) of their paper].

Table 1
Liquidity parameter λ and the certainty equivalent per trader versus the number of informed traders k

$A = 2, \phi = 2$

$\sigma_z^2 = 2$			$\sigma_z^2 = 3$		
k	λ	Cert. equiv.	k	λ	Cert. equiv.
1	0.099	0.088	1	0.070	0.092
2	0.150	0.071	2	0.111	0.077
3	0.175	0.058	3	0.134	0.064

to $k = 2$ and $\sigma_z^2 = 2$ is 0.071, while the certainty equivalent corresponding to $k = 2$ and $\sigma_z^2 = 3$ is 0.077. If

$$0.071 < c < 0.077,$$

then the equilibrium number of informed traders corresponding to $\sigma_z^2 = 2$ is 1, while the equilibrium number corresponding to $\sigma_z^2 = 3$ is 2. This implies that the equilibrium λ corresponding to $\sigma_z^2 = 2$ is 0.099, while that corresponding to $\sigma_z^2 = 3$ is 0.111. From this discussion one immediately observes that when informed traders are risk averse, the relationship between the measure of market liquidity λ and the level of liquidity trading is not necessarily monotonic.

The proposition stated below follows directly from the above example.

Proposition 3. *When informed traders are risk averse and information acquisition is endogenous, then there exists a nonempty set of exogenous parameter values such that increasing the variance of liquidity trading in the market increases λ, the inverse measure of market liquidity.*

In the next section, we briefly discuss the implications of a model with a risk-averse market maker.

2. Risk Aversion of Market Makers

Consider a market that is identical to that of the previous section, except in the respect that market makers possess CARA utility with a risk-aversion coefficient A_m. We assume that a single market maker takes the entire order flow and impose the condition that he earns the "autarky" utility (the utility he would obtain by not making the market), which is normalized to zero for convenience.[10] The expected

[10] For justification of the zero-expected-utility-gain assumption by Bertrand competition between market makers, it is necessary to make the assumption that a *single* market maker absorbs the entire order flow (unlike the case in which market makers are risk neutral). Suppose instead that risk-

utility $E[U_m]$, which this market maker obtains by "making the market," can be written in the mean-variance fashion

$$E[U_m] = E[\omega(P - F) \mid \omega] - (A_m/2)\text{var}[\omega(P - F) \mid \omega]. \qquad (13)$$

It is easy to show that the unique linear equilibrium is characterized by the market maker using a linear rule and the informed following symmetric linear strategies. Substituting for the linear pricing rule $P = \bar{F} + \lambda\omega$ in Equation (13) and setting the RHS of this equation to zero yield

$$\lambda = v + (A_m/2)\text{var}[\delta \mid \omega], \qquad (14)$$

where v is the regression coefficient in the forecast of δ on ω.[11] From (9) and (10), we have (with $t \equiv k\beta$) the following lemma.

Lemma 4. *The response function of a risk-averse market maker is given by*

$$\lambda = \frac{t + (A_m/2)(t^2\phi + \sigma_z^2)}{t^2(1 + \phi) + \sigma_z^2}. \qquad (15)$$

Observe that even if there is no informed trading ($t = 0$), λ is positive because the market maker demands compensation for bearing the risk associated with the liquidity trades. Also, unlike the case of a risk-neutral market maker, the response function may be either increasing or decreasing in σ_z^2, depending on the sizes of t and A_m, because an increase in σ_z^2 has two opposing effects on λ: (i) it increases the risk borne by the market maker, and (ii) it provides him additional compensation for losses to informed traders. The determination of λ in this case is illustrated in Figure 4, which plots the monopolist informed trader's optimal strategy coefficient and the market maker's response function for the cases in which they are risk averse and risk neutral. From the figure, one observes that risk aversion causes the market maker's response function to shift up and the equilibrium λ to be higher. It is evident that the nonmonotonicity results [(ii)–(iv)] of Proposition 1 continue to obtain.

Purely for expositional simplicity, we assume for the remainder of

averse market makers with identical preferences can bid for portions of the order flow and that informed traders conjecture that the pricing will be linear. In this case, Bertrand competition causes the price to equal the marginal value of an additional share to a market maker when all market makers hold the equally divided order size. All market makers then gain a surplus on all shares except the marginal one, and formal characterization of the pricing is much more difficult. I am grateful to the referee for pointing out the contrasting nature of pricing (under risk aversion of market makers) in the two cases.

[11] Note that under risk aversion of market makers, prices lose their "unbiasedness" property (i.e., the price is no longer given by the expected value conditional on public information).

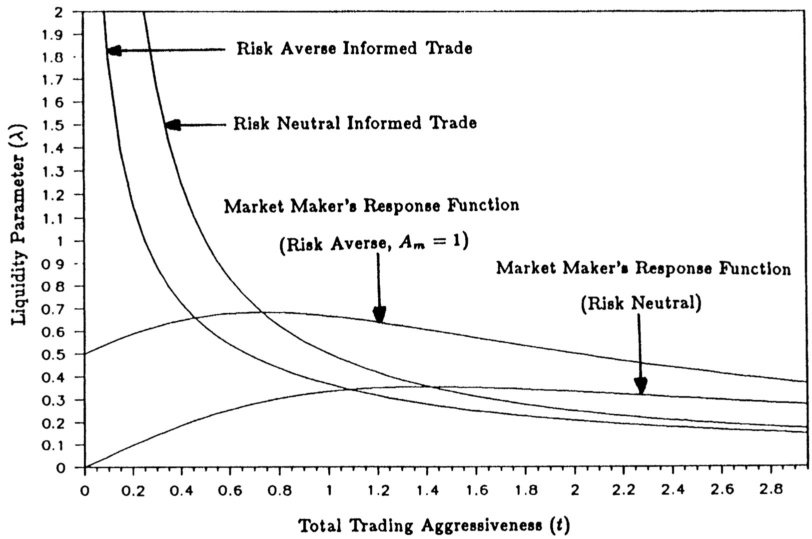

Figure 4
Determination of equilibrium under risk aversion of both informed traders and market makers
It is assumed that the variance of liquidity trades $\sigma_z^2 = 2$, the variance of signal noise $\phi = 1$, the number of informed traders $k = 1$, and that the risk-aversion coefficient of the informed traders $A = 1$.

this section that $\phi = 0$. We first present a closed-form solution for λ for the case of *risk-neutral* informed traders in the following proposition, the proof of which is straightforward and is therefore omitted.[12]

Proposition 4. *For the case of $\phi = 0$, a risk-averse market maker, and risk-neutral informed traders, the equilibrium value of λ is given by*

$$\lambda = \frac{A_m}{4} + \sqrt{\left(\frac{A_m}{4}\right)^2 + \frac{k}{(k+1)^2\sigma_z^2}}. \tag{16}$$

From (16), it can be seen that λ declines in σ_z^2. This property holds for risk-averse informed traders as well, and is described in the following proposition (which is proved in the Appendix).

Proposition 5. *$d\lambda/\sigma_z^2 < 0$ for the case of a risk-averse market maker and risk-averse informed traders.*

[12] It can be verified that for the case of $\phi > 0$ and risk-neutral informed traders, λ is the solution to a cubic equation with a unique positive root. While it is possible in principle to analyze the closed-form solution for this case, the solution is extremely complicated and does not promise any gain in intuition.

To understand this result, first observe from (15) that if for any given t, the market maker's response function declines in σ_z^2, then this property must hold for any smaller t as well. Also, from (16), the risk-*neutral* informed traders' optimal strategy curve always intersects the risk-averse market maker's response function (in λ-t space) at a point where the market maker's response is declining in σ_z^2. Since making the informed traders risk averse causes t to decrease, the intersection of the risk-*averse* informed traders' strategy curve and the risk-averse market maker's response function also occurs at a point where the market maker's response function declines in σ_z^2. This property of the model causes λ to decline in σ_z^2 for the case in which both market makers and informed traders are risk averse.

The discussion above implies that the comparative statics associated with λ are not affected by risk aversion of market makers. Turning to the results on price efficiency, note that in the case in which both market makers and informed traders are risk neutral, λ is exactly proportional to σ_z^{-1} [see (7)]. This leads to the invariance result that, under risk neutrality, changes in σ_z^2 have no effect on the efficiency of prices. However, by examining expression (16), we see that in the case of a risk-averse market maker, λ is not exactly proportional to σ_z^{-1}, so that an increase in σ_z causes λ to decrease less than proportionately. Noting that $\beta = 1/(k + 1)\lambda$ under risk neutrality of informed traders and calculating Q from (10) and (16), it is a straightforward but tedious exercise to show the following.

Proposition 6. *When market makers are risk averse and informed traders are risk neutral, an increase in the variance of liquidity trades reduces price efficiency.*

Thus, the tendency for price efficiency to decrease in the variance of liquidity trades is reinforced in the case of a risk-averse market maker.

Comparison with Kyle (1989)

In both Kyle's 1989 model and the present model, traders who behave strategically are risk averse. The main difference between the two models is the market mechanism: In Kyle (1989), informed and uninformed speculators and noise traders submit demand functions to an auctioneer; whereas in this article, market orders are submitted by informed speculators and noise traders to a market maker. It is of interest to compare some features of Kyle's (1989) article to those of our article.

In the major part of Kyle's (1989) model, a finite number of uninformed speculators earn positive surpluses from trading.[13] The analog

[13] If the number of uninformed speculators is unboundedly large, they behave as a risk-neutral, competitive group, and therefore earn zero surplus.

of this class of traders in the present article is the market maker who takes the order flow. The market maker, however, earns a zero surplus as a result of Bertrand-like competition. Also, in Kyle's (1989) model, informed traders observe diverse signals; whereas here, all informed traders observe the same piece of information.

The focus in Kyle (1989) is on the comparative statics properties of the price efficiency parameter. In contrast, we focus on both the market liquidity and the price efficiency parameters. With regard to price efficiency, Proposition 2 of our article is analogous to theorem 8.2 in Kyle (1989), which is derived under the assumption that the number of uninformed speculators is unboundedly large. (This assumption is equivalent to our assumption that the market maker is risk neutral and earns zero expected profits.) The proposition and the theorem indicate that, in both models, increasing the degree of risk aversion of the informed traders or the variance of liquidity trades decreases price efficiency. The invariance result under risk neutrality also obtains in Kyle's (1989) framework—see the discussion following his theorem 8.2.

Section 10 of Kyle (1989) presents additional results on informational efficiency under endogenous information acquisition. One noteworthy result is that when information acquisition is endogenous and the supply of potential informed traders is perfectly elastic, an increase in the amount of noise trading increases priced efficiency, though the opposite comparative statics result obtains when the number of informed traders is exogenously specified. It would be interesting to examine the properties of our model in this regard. Unfortunately, our structure precludes obtaining general comparative statics results under endogenous information acquisition because in this case, the conditions for equilibrium are extremely complicated.

3. Concluding Remarks

We have presented an adverse selection model of market microstructure in which agents are assumed to be risk averse. Several implications of this model differ from those of the models of Kyle (1984) and Admati and Pfleiderer (1988b), in which informed traders and market makers are assumed to be risk neutral. For example, when informed traders are risk neutral and observe strongly correlated signals, an increase in their number decreases their aggregate expected profits because of increased competition, resulting in a more liquid market. Under risk aversion of the informed, we demonstrate that an increase in their number may increase their aggregate expected profits, causing market liquidity to decrease. If information acquisition is endogenous, increased liquidity trading may also decrease the liquidity of the market by causing the entry of more informed traders.

We also show that increasing the precision of private information may improve terms of trade (market liquidity) because risk-averse traders who are precisely informed compete more aggressively than those who are imprecisely informed. This result suggests an interesting comparison between having private information enter the price through the trades of insiders versus the trades of security analysts. Under the plausible assumption that the insiders' information is more precise than that of security analysts, our analysis suggests that allowing insiders to trade may result in a *more liquid* market than having security analysts. The tendency for this result to obtain is stronger the smaller the difference in precision between the two groups of informed traders.

Another implication of our model is that price efficiency may be decreasing in the amount of liquidity trading in the market. While we have not proved this result for the case of endogenous information acquisition, it is worth observing that information acquisition is not entirely endogenous in security markets because of the presence of insiders, who are typically endowed with information which is oftentimes very precise relative to that of the general public. The presence of such insiders may completely deter other traders from acquiring costly information and trading [see Fishman and Hagerty (1989b)].

Finally, we note that this article has not explored dynamic issues such as those analyzed in Kyle (1985), Admati and Pfleiderer (1988b), and Foster and Viswanathan (1990). Risk aversion causes the multiperiod strategies of the informed to become complicated, and this model cannot be readily extended to a dynamic framework. [This is true even if information lasts only one period, as in Admati and Pfleiderer (1988b), unless one makes the assumptions that market makers are risk neutral and no trader can become informed in more than one period.] For example, it is difficult to model preferences to account for intertemporal hedging by agents. Developing multiperiod models with risk aversion and strategic informed trading is clearly an important area for future research.

Appendix

Proof of Proposition 1

Define $\alpha \equiv \beta^{-1}$. Now $\lambda = \text{cov}(\delta, \omega)/\text{var}(\omega)$, and

$$\alpha = A\lambda^2\sigma_z^2(1 + \phi) + \lambda(k + 1)(1 + \phi) + A\phi.$$

Thus, λ must satisfy the equations

$$\lambda = f_1(\alpha) = \frac{k\alpha}{k^2(1 + \phi) + \alpha^2\sigma_z^2} \tag{A1}$$

and

$$A\lambda^2\sigma_z^2(1 + \phi) + \lambda(k + 1)(1 + \phi) + A\phi - \alpha = 0. \tag{A2}$$

The above quadratic equation has only one positive root (for a given α) and it is given by

$$\begin{aligned}\lambda = f_2(\alpha) = \Big[&-(k + 1)(1 + \phi) \\ &+ \sqrt{(k + 1)^2(1 + \phi)^2 + 4(\alpha - A\phi)A\sigma_z^2(1 + \phi)}\Big] \\ &\times \left[2A\sigma_z^2(1 + \phi)\right]^{-1}.\end{aligned} \tag{A3}$$

Thus, λ and α are determined by the point at which the graphs of these functions intersect in λ-α space. Note that f_1 is unimodal in α and that $f_1 = 0$ when $\alpha = 0$. On the other hand, $f_2 < 0$ when $\alpha = 0$, assuming that f_2 evaluates to a real number at $\alpha = 0$. Also, f_2 is monotonically increasing in α. To prove that these functions have a unique point of intersection in the first quadrant, it is sufficient to prove that (for given parameter values) whenever they intersect, $df_2/d\alpha > df_1/d\alpha$. This will be proved later. (The curve f_1 can intersect f_2 at more than one point if and only if the difference in the slopes reverses in sign at intersection points.) The equilibrium analysis performed below assumes that the relations (A1) and (A2) hold. A "root" in the following analysis refers to a real root.

Let $Y \equiv [k^2(1 + \phi) + \alpha^2\sigma_z^2]\lambda - k\alpha$. Then

$$\frac{d\lambda}{d\sigma_z^2} = \frac{-\partial Y/\partial\sigma_z^2}{\partial Y/\partial\lambda}.$$

We will first prove that $\partial Y/\partial\lambda > 0$. Let $\theta \equiv (\alpha - A\phi)$. Then

$$\begin{aligned}\lambda\frac{\partial Y}{\partial\lambda} &= kA\phi + \lambda\sigma_z^2\big[2(\theta + A\phi)(\theta + A\lambda^2\sigma_z^2(1 + \phi)) \\ &\qquad - kA\lambda(1 + \phi)\big] \\ &= kA\phi + \lambda\sigma_z^2\big[(\alpha^2 - \alpha A\phi) + (\alpha^2 - \alpha A\phi - kA\lambda(1 + \phi))\big] \\ &\quad + 2A\lambda^2\sigma_z^2(1 + \phi)(\theta + A\phi).\end{aligned} \tag{A4}$$

Note that $\alpha^2\sigma_z^2 = kA\phi/\lambda + kA\lambda\sigma_z^2(1 + \phi) + k(1 + \phi)$. Thus,

$$\alpha^2 = kA\lambda(1 + \phi) + \frac{kA\phi}{\lambda\sigma_z^2} + \frac{k(1 + \phi)}{\sigma_z^2}. \tag{A5}$$

Substituting for α^2 from above into the RHS of (A4), we have

$$\lambda\frac{\partial Y}{\partial\lambda} = kA\phi + \lambda\sigma_z^2\left[\alpha^2 - \alpha A\phi + \frac{kA\phi}{\lambda\sigma_z^2} + \frac{k(1+\phi)}{\sigma_z^2} - \alpha A\phi\right]$$
$$+ 2A\lambda^2\sigma_z^2(1+\phi)(\theta + A\phi)$$
$$= kA\phi + \lambda\sigma_z^2\left[\alpha^2 - \alpha A\phi + \frac{A\phi}{\lambda\sigma_z^2}(k - \lambda\sigma_z^2\alpha) + \frac{k(1+\phi)}{\sigma_z^2}\right]$$
$$+ 2A\lambda^2\sigma_z^2(1+\phi)(\theta + A\phi).$$

Note that $\alpha > A\phi$. We now show that $k > \lambda\sigma_z^2\alpha$ and thus that $\partial Y/\partial\lambda > 0$.

Now, from (A1), α must solve the quadratic equation

$$\alpha^2\sigma_z^2 + k^2(1+\phi) - k\alpha/\lambda = 0.$$

The sum of the two (positive) roots of this equation is $k/(\sigma_z^2\lambda)$, proving that

$$\alpha < k/\lambda\sigma_z^2, \tag{A6}$$

so that $k > \lambda\sigma_z^2\alpha$. Thus, $\partial Y/\partial\lambda > 0$.

We next prove that $\partial Y/\partial\sigma_z^2 > 0$ (and thus that $d\lambda/d\sigma_z^2 < 0$). Denote x_y as the partial derivative of the variable x with respect to the variable y. Now

$$\frac{\partial Y}{\partial\sigma_z^2} = \lambda\alpha^2 + 2\lambda\alpha\sigma_z^2\alpha_{\sigma_z^2} - k\alpha_{\sigma_z^2}$$
$$= \lambda\left[\alpha^2 - A\lambda k(1+\phi)\right] + 2\lambda\alpha\sigma_z^2(A\lambda^2(1+\phi)).$$

Since $\alpha^2 > A\lambda k(1+\phi)$ [from (A5)], $\partial Y/\partial\sigma_z^2 > 0$. Thus, $d\lambda/d\sigma_z^2 < 0$.

We now prove that $df_2/d\alpha > df_1/d\alpha$. We have proved that $\partial Y/\partial\lambda > 0$, which is equivalent to proving the validity of the condition

$$\left[k^2(1+\phi) + \alpha^2\sigma_z^2\right]^2 > k\alpha_\lambda\left[k^2(1+\phi) - \alpha^2\sigma_z^2\right]. \tag{A7}$$

Now

$$\frac{df_1}{d\alpha} = \frac{k\left[k^2(1+\phi) - \alpha^2\sigma_z^2\right]}{\left[k^2(1+\phi) + \alpha^2\sigma_z^2\right]^2}$$

and $df_2/d\alpha = 1/\alpha_\lambda$. Thus, $df_2/d\alpha > df_1/d\alpha$ is equivalent to the condition (A7), proving that indeed $df_2/d\alpha > df_1/d\alpha$ at the point(s) of intersection of these curves. Thus, the quintic equation (8) has a unique root.

To prove the other comparative statics results, our approach will be first to prove that the second derivatives are negative at all points where the first derivative with respect to the relevant variable is zero. Note that a function with this property must be unimodal or monotonically increasing or decreasing. (In other words, a function is multimodal if and only if it has at least two *different types* of extrema.)

Now, $d\lambda/dk = -(\partial Y/\partial k)/(\partial Y/\partial \lambda)$. Thus, since $\partial Y/\partial \lambda > 0$, if $d\lambda/dk = 0$, then $\partial Y/\partial k = 0$. Also,

$$\left.\frac{d^2\lambda}{dk^2}\right|_{d\lambda/dk=0} = \frac{d(-\partial Y/\partial k)/dk}{\partial Y/\partial \lambda}.$$

It is straightforward to show that $d(-\partial Y/\partial k)/dk = -2\lambda^2(k+1)(1+\phi)^2 < 0$. Since $\partial Y/\partial \lambda > 0$, $d^2\lambda/dk^2|_{d\lambda/dk=0} < 0$.

Using a similar approach, it is straightforward to show that $d^2\lambda/dA^2|_{d\lambda/dA=0} < 0$ and that $d^2\lambda/d\phi^2|_{d\lambda/d\phi=0} < 0$. Also, using (A1) and (A2), it can easily be shown that $\lambda \to 0$ whenever $k \to \infty$, or $A \to \infty$, or $\phi \to \infty$. Thus, λ as a function of k, A, or ϕ cannot monotonically increase. We have thus proved that λ is a unimodal function of k, A, or ϕ, or is monotonically decreasing in these variables. That λ decreases monotonically in k when $A = 0$ follows from Lemma 2 (i).

It remains to be shown that $d\lambda/dA < 0$ and that $d\lambda/d\phi < 0$ for $k = 1$. Now, from the definition of Y, $d\lambda/dA|_{k=1} = -(\partial Y/\partial A)/(\partial Y/\partial \lambda)|_{k=1} < 0$ is equivalent to

$$2\lambda\alpha\sigma_z^2 > 1,$$

or, from (A1),

$$\frac{2\alpha^2\sigma_z^2}{(1+\phi) + \alpha^2\sigma_z^2} > 1,$$

or

$$\alpha^2\sigma_z^2 > 1 + \phi.$$

From (A1), the above condition is equivalent to

$$\alpha/\lambda > 2(1+\phi).$$

From (A2), this is equivalent to

$$A\lambda\sigma_z^2(1+\phi) + A\phi/\lambda > 0,$$

which is true. Thus, $d\lambda/dA|_{k=1} < 0$. Now

$$\operatorname{sgn}\left[\left.\frac{d\lambda}{d\phi}\right|_{k=1}\right] = \operatorname{sgn}\left[\left.\frac{\partial Y/\partial \phi}{\partial Y/\partial \lambda}\right|_{k=1}\right] = -\operatorname{sgn}\left[\lambda + \alpha_\phi(2\lambda\alpha\sigma_z^2 - 1)\right].$$

Since $\alpha_\phi > 0$, the above evaluates to a negative sign. ■

Proof of Proposition 2
Note that

$$Q = \frac{k^2(1 + \phi) + \alpha^2\sigma_z^2}{k^2\phi + \alpha^2\sigma_z^2}.$$

This can be rewritten as

$$Q = \frac{\alpha}{\alpha - k\lambda}. \tag{A8}$$

Now

$$\frac{dQ}{d\sigma_z^2} = \frac{k\left[(d\lambda/d\sigma_z^2)\alpha - (d\alpha/d\sigma_z^2)\lambda\right]}{(\alpha - k\lambda)^2}.$$

Thus, $\text{sgn}\left[dQ/d\sigma_z^2\right] = -\text{sgn}\left[d(\alpha/\lambda)/d\sigma_z^2\right]$. Now

$$\frac{d(\alpha/\lambda)}{d\sigma_z^2} = A\left[\frac{d(\lambda\sigma_z^2)}{d\sigma_z^2}\right](1 + \phi) - \frac{A\phi}{\lambda^2}\left[\frac{d\lambda}{d\sigma_z^2}\right].$$

Since we have proved that $d\lambda/d\sigma_z^2 < 0$, it is sufficient to prove that $d(\lambda\sigma_z^2)/d\sigma_z^2 > 0$. Note that if $A = 0$, then $dQ/d\sigma_z^2 = 0$, proving claim (ii) of the proposition. Now, from (A3),

$$\text{sgn}\left[\frac{d(\lambda\sigma_z^2)}{d\sigma_z^2}\right] = \text{sgn}\left[\lambda\frac{\partial Y}{\partial\lambda} - \sigma_z^2\frac{\partial Y}{\partial\sigma_z^2}\right].$$

Substituting for the partial derivatives in the above expression, it can be shown that the sign of the expression in square brackets on the RHS is equal to the sign of

$$k(1 + \phi)\lambda + 2A\phi(k - \lambda\alpha\sigma_z^2).$$

Since, from (A6), $k > \lambda\alpha\sigma_z^2$, $dQ/d\sigma_z^2 < 0$. ∎

We now prove that $dQ/dA < 0$. Note from (A8) that this is equivalent to proving $d(\alpha/\lambda)/dA > 0$. Substituting for λ from (A1), we have $\alpha/\lambda = k^{-1}[\alpha^2\sigma_z^2 + k^2(1 + \phi)]$. Proving $d(\alpha/\lambda)/dA > 0$ is therefore equivalent to proving $d\alpha/dA > 0$. This can be proved as follows:

$$\frac{d\alpha}{dA} = \left[(k + 1)(1 + \phi) + 2A\lambda\sigma_z^2(1 + \phi)\right]\frac{d\lambda}{dA} + \lambda^2\sigma_z^2(1 + \phi) + \phi$$

and

$$\frac{d\lambda}{dA} = \frac{-\partial Y/\partial A}{\partial Y/\partial\lambda}.$$

Since $\partial Y/\partial \lambda > 0$,

$$\operatorname{sgn}\left[\frac{d\alpha}{dA}\right] = \operatorname{sgn}\left[\left[(k+1)(1+\phi) + 2A\lambda\sigma_z^2(1+\phi)\right]\left(-\frac{\partial Y}{\partial A}\right) + \left[\lambda^2\sigma_z^2(1+\phi) + \phi\right]\frac{\partial Y}{\partial \lambda}\right].$$

Substituting for the derivatives in the RHS of the above expression, it is straightforward to show that it simplifies to

$$[\lambda^2\sigma_z^2(1+\phi) + \phi][k^2(1+\phi) + \alpha^2\sigma_z^2],$$

which is necessarily positive, proving that $d(\alpha/\lambda)/dA > 0$ and thus that $dQ/dA < 0$. ■

Proof of Lemma 3

We will first prove a general result on the properties of normal random variables.

Let $x_1, x_2 \sim N(0, V)$. Then, denoting the joint density of x_1 and x_2 by f, we have

$$f(x_1, x_2) = \frac{1}{2\pi|V|^{1/2}} \exp\left[-\frac{1}{2}(x_1\, x_2)(V^{-1})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right].$$

We are interested in computing $E[\exp(-x_1 x_2)]$. Thus,

$$\begin{aligned} &E\left[\exp(-x_1 x_2)\right] \\ &= \frac{1}{2\pi|V|^{1/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}(x_1\, x_2)(V^{-1})\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - x_1 x_2\right] \\ &= \frac{1}{2\pi|V|^{1/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}(x_1\, x_2)\left(V^{-1} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right] \\ &= \left(|V|^{1/2}\left|\left[V^{-1} + \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}\right]\right|^{1/2}\right)^{-1} \\ &= \left(\left|I + V\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right|^{1/2}\right)^{-1}. \end{aligned}$$

The expected utility from trading on the signal $\delta + u$ is given by $-E[\exp(-x_1 x_2)]$, where

$$x_1 = A\beta(\delta + u)$$

and

$$x_2 = F - P = \delta - \lambda k\beta(\delta + u) - \lambda z.$$

Substituting for x_1 and x_2 in the above expression for $E[\exp(-x_1x_2)]$ completes the proof. ■

Proof of Proposition 5

Though we prove the proposition for $\phi = 0$ to conform to the assumption in the text, it can also be proved for $\phi > 0$. Details of the proof for the case of $\phi > 0$ are available upon request from the author. Using notation from earlier proofs, we have

$$\lambda = \frac{k\alpha + (A_m/2)\alpha^2\sigma_z^2}{k^2 + \alpha^2\sigma_z^2}.$$

Let $Z \equiv [k^2 + \alpha^2\sigma_z^2]\lambda - k\alpha - (A_m/2)[k^2 + \alpha^2\sigma_z^2]$. Then

$$\frac{d\lambda}{d\sigma_z^2} = \frac{-\partial Z/\partial\sigma_z^2}{\partial Z/\partial\lambda}.$$

Now, it can be shown that

$$\frac{\partial Z}{\partial\lambda} = \frac{A_m}{2}\alpha^2\sigma_z^2 + \lambda\sigma_z^2\left[A\lambda^2\sigma_z^2\alpha\left(2 - \frac{A_m}{\lambda}\right) + 2\alpha^2 - A\lambda k - \frac{A_m}{\lambda}\alpha^2\right]. \tag{A9}$$

Also,

$$\alpha^2\sigma_z^2 = A\lambda k\sigma_z^2 + k + \frac{A_m}{2}\frac{\alpha^2\sigma_z^2}{\lambda}. \tag{A10}$$

It is evident from the above expression that

$$\alpha^2 > A\lambda k + \frac{A_m}{2\lambda}\alpha^2. \tag{A11}$$

From (A10), it can also be seen that, in order to have a positive α, the condition

$$\lambda > \frac{A_m}{2} \tag{A12}$$

must be satisfied. Thus, from (A11) and (A12), the RHS of (A9) must be positive, so $\partial Z/\partial\lambda > 0$. We now prove that $\partial Z/d\sigma_z^2 > 0$. It is straightforward to show that

$$\frac{\partial Z}{\partial\sigma_z^2} = \lambda\left[\alpha^2 - \frac{A_m}{2}\frac{\alpha^2}{\lambda} - kA\lambda + 2\alpha\alpha_{\sigma_z^2}\lambda\sigma_z^2\left(1 - \frac{A_m}{2\lambda}\right)\right].$$

From (A11) and (A12), noting that $\alpha_{\sigma_z^2} > 0$, we have $\partial Z/\partial\sigma_z^2 > 0$ and therefore $d\lambda/d\sigma_z^2 < 0$. ■

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