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THE INVESTIGATION OF ECONOMIC EXPECTATIONS

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Fifty years ago economists rather generally allowed themselves to ignore economic expectations by concentrating attention on the "static state." Today we talk a great deal about economic expectations, but how much do we know about their formation and their behavior? How much can we say about them that might not have been said by Alfred Marshall or by John Stuart Mill, or even by Adam Smith, if they had chosen to discuss expectations?

I. *The General Problem*

There are two broad lines of possible attack on the subject of economic expectations: the individualistic and the aggregative. In its fundamentals, the problem of individual expectations may be one for psychology rather than for economics, but the economist must not hesitate to go as far as he profitably can in considering the individual.

A good deal of information has been collected from time to time on economic expectations of individuals. Now and then someone becomes interested in the opinions of economists and makes a survey of their expectations regarding prices. Surveys of the expectations of businessmen have been made frequently, by various agencies. If these surveys had seemed to produce results of much practical or scientific value, they could and should be organized more systematically and made the business, perhaps, of the federal Department of Commerce.[1] Thus far, at least, it seems that the information gathered falls more in the category of news of current interest than in that of significant economic information.

Perhaps the information has lacked scientific value because the nets of inquiry have not been cast widely enough, or in the right quarters. Or perhaps the absence of recognized scientific significance reflects inadequacy of our perception rather than absence of real significance.

I have not made a search for serious analyses of data on individual economic expectations, but I think it safe to say that there have been few of them. There is one which comes to mind, however, and the results are worth noting. Some years ago Alfred Cowles made a

[1] The annual surveys of consumer finances instituted a few years ago by the Board of Governors of the Federal Reserve System are providing some information on consumer expectations and could easily provide more if the effort should seem worth while.

critical study of the economic expectations recorded by professional stock-market forecasters. The main conclusion which emerged was that these expectations had characteristics substantially identical with those of random guesses.[2] We shall see before we finish that such an observation is not necessarily evidence of poor forecasting.

Anyone who contemplates analysis of existing data on individual economic expectations, and especially anyone who proposes large-scale collection of more data, should consider certain difficulties that must be faced in their interpretation. First is the question of validity of the expressions of expectation. Many people will state expectations with great confidence, but evince no confidence at all when faced with a challenge to act on their stated opinions. Second is the problem of classifying persons queried in relation to the economic significance of any expectations which they may have. Some expectations of some people carry great weight in economic affairs, yet most expectations of most people carry little weight.

I do not mean these comments as deprecation of the study of individual expectations, but only to suggest that any other feasible approach needs consideration also.

II. *The Aggregative Approach*

The aggregative approach to study of economic expectations takes for analysis some recorded economic variable which reflects a sort of consensus of expectations. Many economic quantities are strongly influenced by expectations and some are little influenced by anything else. Prices of industrial stocks, for example, are predominantly reflections of economic expectations. They may be regarded as expectations capitalized at going rates of interest, and in that sense they appear to reflect both expectations and existing rates of interest—or should I say that the capitalization is at expected rates of interest? The price of a commodity future is even more clearly and specifically a composite measure of economic expectations.

When we undertake study of the behavior of futures prices as a means toward knowledge of economic expectations, we must consider the objection, or qualification, that a futures price is not precisely a composite measure of expectations. British writers, including such distinguished economists as Keynes, Kaldor, Hicks, and Hawtrey, have given the question a good deal of attention since Keynes advanced his "theory of normal backwardation" in 1930.[3] They have disagreed somewhat, as economists are wont to do, but I think that they are

[2] Alfred Cowles, "Can Stock Market Forecasters Forecast?" *Econometrica*, July, 1933, pp. 309-324.

[3] J. M. Keynes, *Treatise on Money* (New York, 1930), II, p. 143.

united in the opinion that the relation of a futures price to expectations may be expressed by the equation,

$$P_f = E - r, \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where P_f is a futures price, E is some sort of combination of effects of individual expectations, and r is some sort of combination of (effects of?) individual risk premiums.[4]

This equation serves to indicate that in studying P_f we may learn facts about r rather than about E. That need not trouble us. If we find significant behavior characteristics, we shall know that they derive from corresponding characteristics in either E or r, or from some combination of both. We shall in fact find that the main characteristics derive from E expectations.

We may now profitably adopt the term "market expectation" as synonymous with "futures price." Firmly established habits of thought make it difficult to think of any price as resting primarily on expectations. By using "market expectation" as an alternative to "futures price," we may keep attention focused on the predominantly expectational quality of such a price. Because the term "market expectation" has no strongly fixed connotations, it should not be difficult to remember that, writing E_m for market expectation, its definition leads, by equation (1) to

$$E_m = E - r, \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

and therefore that market expectation is not precisely equivalent to E unless $r = 0$.

III. *Bias in Market Expectations*

Expectations may be defective in either or both of two respects: they may be randomly inaccurate or they may be biased. Let us consider first the evidence under the head of bias.

At the outset, however, I want to mention, and to set aside, one kind of aberration in futures prices which might be treated as bias; namely, the effects of corners and squeezes. Corners are, or were, consequences of an excessive freedom of enterprise which seems largely a thing of the past in American futures markets. The British grain trade has never permitted either corners or significant squeezes in its futures markets. Squeezes continue to occur in American futures markets, though they can and should be eliminated. Like corners, they involve gaining and exploiting a temporary monopolistic position. The monopolistic element produces effects of quite a different nature

[4] I give r a negative sign in equation (1) because we shall usually want to use that equation in interpreting statistical evidence arising under conditions where the effect of r, if r is not equal to 0, is to give $P_f < E$. On purely algebraic grounds it would be better to use a positive sign, recognizing that r may take either a positive or a negative value.

from the others we have to consider, and I choose not to try to deal with them here.

Bias itself may be of several sorts. I shall use a classification involving four categories of bias which will be defined as we proceed.

1. The kind of bias in market expectations which has received most attention in theoretical discussion and in statistical studies is what may be called "general bias." There has been a prevalent view in agricultural circles that futures prices of a crop tend to be depressed in the period of heavy marketing soon after harvest, and thereafter to advance, perhaps tending to be highest shortly before the next harvest. Theoretical analysis, moreover, has led to the opinion that some such behavior should be expected as a reflection of risk premium. Such was the argument of J. M. Keynes's "theory of normal backwardation," and similar conclusions were reached by Kaldor and other British writers who extended and improved on Keynes's theoretical analysis.[5]

Keynes himself held that "the statistics of organized markets show that 10 per cent per annum is a modest estimate of the amount of this backwardation in the case of seasonal crops. . . ."[6] That statement is significant mainly as evidence of Keynes's opinion that an indication of the effective risk premium is obtainable from statistical calculations of general bias in futures prices. His acquaintance with the pertinent statistical studies was evidently superficial, for it is only data for short and unrepresentative periods which appear to support so high an estimate of general bias.

For Chicago wheat futures, the average general bias or "normal backwardation" cannot be estimated properly at over 2 or 3 per cent per annum. For Chicago corn futures, it may approach 5 per cent.[7]

[5] Nicholas Kaldor, "Speculation and Economic Stability," *Review of Economic Studies*, 1939-40, pp. 1-27; J. C. R. Dow, "A Theoretical Account of Futures Markets," *ibid.*, 1939-40, pp. 185-195; Nicholas Kaldor, J. C. R. Dow, and R. G. Hawtrey, "A Symposium on the Theory of the Futures Market," *ibid.*, 1939-40, pp. 196-205; and Gerda Blau, "Some Aspects of the Theory of the Futures Market," *ibid.*, 1944-45, pp. 1-30.

[6] *Treatise on Money* (New York, 1930), p. 143.

[7] Holbrook Working, "Theory of the Inverse Carrying Charge in Futures Markets," *Journal of Farm Economics*, February, 1948, pp. 8-12. (The citation in footnote 18 there should read "p. 214.")

Detailed inquiry can bring out some curious minor characteristics of general bias. For example, the reference above examines an apparent general tendency for corn futures prices to rise slightly from May to August and to decline similarly from August to October. This, however, is probably a spurious indication of general bias, arising from the fact that exaggerative bias (to be considered below) operates mainly in the upward direction and occurs most often in July and August. Chicago wheat futures prices show an apparently true general tendency to decline slightly from the latter part of February to late March. ("Price Relations between May and New-Crop Wheat Futures at Chicago since 1885," *Wheat Studies of the Food Research Institute*, February, 1934, pp. 214-216.) There is also a quite prevalent tendency for the price of any future to rise slightly (perhaps about 1 per cent) during two or three weeks before the beginning of a delivery month, when many hedges are being transferred from that future to more distant ones. This

The small size of general bias indicated by a proper interpretation of the statistics raises an interesting theoretical question: Is the effective risk premium really so small; or is there actually a positive bias in expectations proper (the E of the equation $E_m = E - r$) which partially offsets the effects of risk premium? A good argument can be made for either interpretation. On the one hand, it is clear that speculators tend to be people who are not particularly averse to taking risks; many of them, like many people who enjoy gambling, may willingly accept even *positive* risk premiums. On the other hand, prevalence of the mistaken impression that futures prices tend to be strongly depressed in the period shortly after harvest undoubtedly produces a real positive bias in the price expectations of many individual speculators. Their bias could form the basis for a small positive bias in the consensus of expectations, E.

In any case, it is clear that any errors in market expectations which arise from general bias must be exceedingly small in comparison with errors from other sources.

2. We now turn to the sorts of bias which may result in large errors of market expectation. One of these may be called "conservative bias," and defined as a tendency for market expectations to delay in adjusting fully to new substantive information. When I was studying potato prices a good many years ago, I thought that I perceived such a bias, involving a tendency for prices to be too low early in the season following a short harvest and too high in the early part of a season following a large harvest, these errors being corrected more or less gradually as the season progressed. Those observations were based, of course, on spot prices of a commodity in which there was then no futures trading.

So far as I know, there has been no demonstration that such a tendency to conservative bias exists in prices of any commodity which has a prominent futures market, with one rather obscure exception. That exception is in wheat and is associated with what I called a "long cycle."[8] Market expectations in wheat seem to have tended

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tendency is chiefly noteworthy as a source of mistaken conclusions; it permits a combination of data for different futures in such a way as to give a much exaggerated indication of general bias over a period of a year.

[8] In applying the term (in "Cycles in Wheat Prices," *Wheat Studies,* November, 1931, pp. 1-66), I intended no implication of regularity of repetition, or periodicity.

People with a good deal of mathematical training accustomed to associate "cycles" with the perfect regularity of a sine curve, have often misinterpreted economists' discussions of cycles, I think. Economists have commonly used the term in the broad sense in which it may designate only the effects of a tendency toward oscillation following a disturbance, including oscillation so highly damped that one disturbance produces only a single perceptible "wave." It is hard to find a good alternative term to avoid possible misinterpretation by the mathematically-minded. "Fluctuations" will not serve because it is too broad, lacking the connotation of progress from one phase to another.

usually to respond very tardily to emergence of a large world wheat surplus, and sometimes also to respond tardily to shortage of supplies when low stocks at the beginning of a crop year have aggravated the seriousness of a poor crop.

On this interpretation, the "long cycle" in wheat prices is, or was, a reflection of conservatism in response to the stocks position. Possibly wheat market expectations will behave differently in the future, with the benefit of better information on actual wheat stocks. If so, the phenomena of the long cycle cannot continue to be taken as evidence of true conservatism of market expectations. They will have to be regarded as effects of bias related to inadequacy of information, to which we now turn.

3. Bias in market expectations arising from inadequacy of information is probably the main source of reliable profits for speculators. Many traders in futures markets give a great part of their attention to acquiring information which has not become generally available, and thus has not been reflected in market expectations. These traders, or the commission houses which serve them, gather crop information ahead of its reporting by public agencies; they study the weather reports and seek to predict effects of the weather on the crops, thus trying actually to base expectations on crop developments which have not yet occurred; and they have even employed a long-range weather forecaster to predict the weather several weeks ahead.

The tendency in futures markets, therefore, is for most classes of pertinent information to be brought to bear on market expectations with extraordinary promptness. Errors arising from failure to obtain or to use available information of known pertinence are small and very short-lived. There are, nevertheless, certain recognizable tendencies toward bias arising from failure to give due attention to available information. The bias associated with level of wheat stocks, mentioned above, may be an example. A clearer example, again from the wheat market, appears in the tendency for market expectations in the United States to give too little attenion to much news from overseas.[9]

Critical studies of price behavior in other futures markets are needed before confident interpretation can be given the tendency of American wheat markets to ignore pertinent foreign news. Possibly there is a general tendency for any market to ignore events that are distant; but this conclusion should not be drawn too hastily on superficial evidence. There is much evidence to the contrary. It may

[9] Perhaps the best evidence on this point is in Robert D. Calkins, "Price Leadership and Interaction among Major Wheat Futures Markets," *Wheat Studies,* November, 1933, pp. 35-70.

be that the wheat market is a somewhat exceptional case, and that its tendency to underestimate the importance of news from overseas arises largely from misapprehension, fostered by the fact that such news is usually unimportant for other grain markets in the United States, and persisting because of inadequacy of basic economic studies and education regarding the major price factors in wheat. When painstaking statistical research uncovers bias in market expectations, the results may deserve to be taken mainly as evidence that the research was needed to lay a basis for better functioning of the markets.

4. The main objectionable bias commonly charged against speculative markets is exaggerative bias—a supposed tendency for market expectations to respond excessively to day-to-day news and rumors and to generate unwarranted price fluctuations. As with so much of economic thought today, the current prevalence of this idea must be attributed in considerable part to the gifts for vigorous and picturesque expression of John Maynard Keynes. His statement, applied specifically to the stock market, that "we have reached the third degree where we devote our intelligences to anticipating what average opinion expects average opinion to be,"[10] is a gem.

It takes no great amount of conversation with traders, when one can get on a basis of frank expression of ideas, to learn that many of them are mainly interested in anticipating the swings of opinion of others. This, however, is not necessarily objectionable; it may represent merely an extension over somewhat longer time intervals of the clearly useful function of the professional "scalper," whose activities certainly tend to diminish price fluctuation and make a better market.[11] To pass judgment, we need to know the effects on price behavior.

One consequence to be expected from an excess of activity toward "anticipating what average opinion expects average opinion to be" is that price movements should develop certain recognizable patterns. Apparent confirmation that this occurs is readily found among traders. A good many of them are firm believers in the significance of "head-and-shoulder formations," "resistance levels," and the like. Before accepting their opinions as valid evidence, however, one may wish to reflect on the fact that these traders rarely wear an air of prosperity and that their ideas are generally scorned by more substantial participators in the markets.

[10] *General Theory of Employment, Interest, and Money* (New York, 1936), p. 156.

[11] It came to me as a surprise, in discussion following presentation of this paper, that Professor Norton took his stand with those who think that the activities of scalpers have a bad rather than a good effect on price behavior. I welcome his remarks as evidence of the need for objective determination of price effects of speculative trading, including scalping. When opinions of competent observers differ so much as they do regarding the effects of scalping, it becomes clear that the issue is one which "informed judgment" is incompetent to settle. A fairly clear objective answer should be furnished by the "error-time test" proposed near the end of this paper.

If we turn to a study of actual price behavior for evidence of unwarranted fluctuations, we easily find many cases in which prices fell quickly after a rise or reacted promptly after a decline. This evidence of unwarranted fluctuations must also be regarded with suspicion. It is valid evidence only if accompanied by some proof that the fluctuations were not in fact warranted. Such proof has seemed very hard to come by; at least I know of very little published evidence on the subject which bears up under critical examination.

This inadequate sketch must serve for the present introduction of a large and difficult problem. I pass now to some specific evidence of exaggerative bias in market expectations.

The best possible evidence of unwarranted price fluctuation in a futures market is that frequently occurring price movements *associated with simple objectively specified conditions* have invariably been followed promptly by reverse movements.[12] For Chicago wheat futures prices, a specification involving only magnitude and rate of price change, with two theoretically reasonable qualifications regarding time of occurrence, serves to identify a class of price movements which have regularly been followed by approximately equal price reaction.[13]

There are four clear characteristics[14] of the evidence on exaggerative bias in the United States wheat market which have special theoretical interest: (1) Evidence of exaggerative bias has been found only in connection with upward price influences, not with downward influences; (2) exaggerative bias accounts in large part for most of the conspicuous advances of United States wheat prices during a period of over sixty years; (3) most of the very largest price advances show no evidence of the presence of exaggerative bias; (4) the patterns of price reaction after emergence of exaggerative bias have varied systematically in close conformity with the degree of confidence or uncertainty necessarily attending the formation of expectations at the time.

On the important question of *explanation of the occurrence* of exaggerative bias in the wheat market, the facts are obscure. The ready inference that it is a general characteristic of prices in speculative markets and the more limited inference that it arises from the par-

[12] The qualifications of frequency of occurrence and simplicity of the specifications are necessary to exclude significant possibility that the observed association is mere coincidence. Better evidence would be furnished by direct proof that the initial price movement in each instance was not accompanied by conditions which warranted it, but in view of the variety and complexity of legitimate price influences, such proof is rarely possible, at least in the present state of our knowledge.

[13] A fact which will carry special weight with statisticians is that the criteria of identification have worked as well in current use during the eighteen years since they were discovered as in application to the historical record. The confirmation has included evidence of validity of the qualifications regarding timing.

[14] Details, which must be omitted here, may be found in Holbrook Working, "Cycles in Wheat Prices," *Wheat Studies,* November, 1931, pp. 18-24.

ticular class of public participation which occurs in the wheat market, must apparently be rejected. I have looked for similar evidence of exaggerative bias in corn and oats prices and have failed to find it.[15]

The best hypothesis which I can offer to account for the main facts now known is that exaggerative bias in wheat prices is related to another bias previously discussed: that it occurs largely because significant overseas news tends to be ignored or inadequately weighted in the American wheat market. Pertinent in this connection are the facts that exaggerative bias in wheat market expectations has occurred most commonly in connection with crop scares, and that the crop damage involved has always been in North America and almost always in the United States.

There is one feature common to all the kinds of bias that we have considered, except the first, which deserves to be noticed before we pass to the next section. General bias, being so small, probably should be regarded as evidence of a risk premium rather than of any true tendency to error in market expectations. The other kinds of bias all reflect errors in market expectation, even though bias arising from inadequacy of pertinent information may warrant no criticism of the market itself. These kinds of errors of expectation, moreover, all have the characteristic that a given error persists over a period of time and tends to diminish progressively after reaching its maximum.

This characteristic of the errors of bias is very clearly observed in the exaggerative errors of crop-scare cycles in wheat prices. In them, prices rise rapidly and excessively and then decline more or less gradually. In a case of conservative bias, the price movement is quite different from that in a case of exaggerative bias, but if one imagines the course which the price would have taken in the absence of bias, it becomes apparent that the error due to bias must usually develop rapidly and then progressively diminish. Bias owing to inadequacy of information is like conservative bias in all respects except cause.

IV. *Accuracy of Market Expectations*

Concentration of attention on bias in market expectations involves emphasis on the imperfections of expectation without due consideration of their relative magnitude. Excessive attention to bias, if I may say so, risks creating a biased impression. We should seek, therefore, to get a balanced view of the inaccuracies of market expectations.

The most perfect expectations possible in economic affairs must be subject to substantial error because the outcome depends on un-

[15] It should be recalled, nevertheless, that in earlier mention of a peculiarity in evidence of apparent "general bias" in corn prices, I inclined to attribute it to existence of some exaggerative bias.

predictable future events. Market expectations, therefore, have a certain *necessary inaccuracy*. By necessary inaccuracy in market expectations I mean that irreducible minimum of inaccuracy which must result from response of prices to unpredictable changes in supply and in consumption demand schedules. An excess of inaccuracy over this minimum may be called *objectionable inaccuracy*.[16]

It may be helpful to consider an analogy. Imagine a rifleman shooting at a distant target equipped with an electrically operated mechanism by which the target is moved unpredictably just as the gun is fired. The inaccuracy of fire caused by movement of the target would be necessary inaccuracy in the sense in which we are speaking. Inaccuracy beyond that caused by movement of the target we call objectionable inaccuracy. The objectionable inaccuracy itself might be divided into two parts, one part arising from error by the marksman and the other attributable to inherent inaccuracy of the rifle, but we may ignore, for the present, any distinctions between kinds of objectionable inaccuracy and degrees of objectionableness.

The main concept involved in this distinction is an old one. It must have entered the mind of everyone who has given thought to the apparent imperfections of price behavior in "free" markets. The concept has naturally suggested to many minds the desirability of having determinations made of the respective relative magnitudes of what I call necessary inaccuracy and objectionable inaccuracy in specific markets. The methods which have been proposed for such determinations, so far as I am aware, have all involved, in principle, the determination of what ideal expectations should have been in individual instances.

Heavy difficulties must confront any attempt to get a valid measurement of necessary inaccuracy by direct estimation in individual instances. Such attempts should not be discouraged, for they might produce results of great value, if properly conceived and implemented. Nevertheless, it is highly desirable to find some simpler method of measuring approximately the relative magnitudes of necessary and of objectionable inaccuracy in market expectations, if there be any adequate simpler method. The remainder of this section will be devoted mainly to outlining a clearly feasible and fairly economical approach to the problem, which I think promises reliable conclusions.

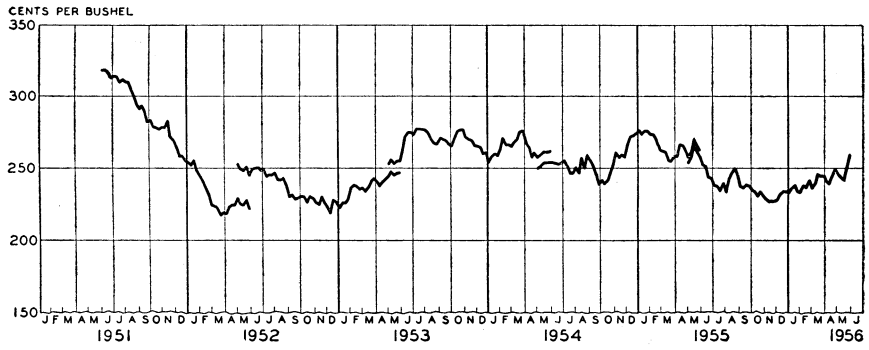
Figure 1 presents a sample of what a record of futures prices might look like, under slightly simplified conditions, if the market expecta-

[16] Professor Waite, in discussion of this paper, suggested that this is not a good term, since all inaccuracy may be considered objectionable. I grant his point and shall welcome suggestions for a better terminology. Meanwhile, I hope that my meaning has been made clear and that readers will not find it too difficult to bear in mind that by "objectionable" inaccuracy I mean inaccuracy which reflects more or less discredit on the functioning of the market.

tions were subject only to necessary inaccuracy. In other words, Figure 1 purports to show a sample of what should be considered ideal behavior of a futures price. The sample is perhaps not the most representative one which could have been chosen. I had nine such samples conveniently available from which to choose and I chose to exhibit the second sample rather than the first one, because it serves better to illustrate that ideal price behavior may easily be mistaken for quite objectionable price behavior.

The essential basis for presenting Figure 1 as a sample of ideal price behavior in a futures market is this: if the futures prices (market ex-

FIG. 1.—PRICES OF "MAY WHEAT FUTURE," IF AFFECTED BY NECESSARY ERROR ONLY



pectations) are subject only to necessary inaccuracy, the price changes will be completely unpredictable. This proposition is readily proved from a consideration of the alternative condition in which price changes are predictable. If it is possible under any given combination of circumstances to predict future price changes and have the predictions fulfilled, it follows that the market expectations must have been defective; ideal market expectations would have taken full account of the information which permitted successful prediction of the price change. Since any predictability of price change is evidence of objectionable error in market expectations, a necessary condition for absence of objectionable error is that the changes be completely unpredictable.

An interesting consequence of this proposition is that, given an ideal futures market in which market expectations exhibited only necessary error, it would be impossible for any professional forecaster to predict price changes successfully. Apparent imperfection of professional forecasting, therefore, may be evidence of perfection of the market. The failures of stock market forecasters, to which we referred earlier, reflect credit on the market.

Changes which are completely unpredictable are, by definition, random changes, in the strict statistical sense of randomness. Statisticians

have had a good deal of occasion to deal with random numbers and have found that it is not so easy as might be supposed to get a series of numbers which is strictly random. Various devices have been tried for the purpose, including tossing of coins, throwing dice, and drawing numbers "at random" out of a hat or a bowl. Each of these methods, when carefully used, produces numbers which are near enough to pure randomness to satisfy many purposes, but each has been shown to produce numbers which tend to have some minor systematic characteristics which render them predictable in some small degree. In an effort to satisfy the most exacting requirements for randomness, L. H. C. Tippett, some twenty years ago, tried another device which proved quite successful. The results were published for the convenience of other statisticians in Tippett's *Random Sampling Numbers*.[17]

Figure 1 has been constructed from numbers drawn from Tippett's table. Since the requirement for our purpose is that the *changes* be unpredictable, the series used for the figure is not that given by the original random numbers but a series obtained by adding each number to the sum of all the preceding ones.[18] The result is a series in which changes are purely random—what may be called a "random-difference series."

Finally, to avoid having attention diverted by purely superficial differences of appearance between the resulting series and actual records of futures prices, I have broken the series into sections representing one year each, have added a constant to all the numbers in each section, and have inserted designations such as would appear on a chart of prices of wheat futures. These final modifications are pure decoration and do not affect the essential characteristics of the series.[19]

There is only one essential respect in which the data of Figure 1 differ from what should be expected in a record of actual futures prices under conditions of ideal market expectations, if such a record were available; in Figure 1 the changes from point to point correspond to random numbers drawn from a single statistical population with a constant standard deviation. In any actual market the necessary inaccuracy of market expectations must vary from season to season and from year to year. Consequently prices representing ideal market expectations would show random changes with *variable* standard deviations. It is for that reason that Figure 1 was characterized as showing the consequences of purely *necessary* error under "slightly simplified conditions."

[17] Cambridge University Press, 1927.

[18] The frequency distribution of numbers drawn from Tippett's table is rectangular. To improve the correspondence with reality, I have transformed the distribution to an approximately "normal" one.

[19] The series may be seen in its original form in "A Random-Difference Series for Use in the Analysis of Time Series," *Journal of the American Statistical Association*, March, 1934, p. 13, Chart I, where the data here used are designated as applying to the years '06 to '10, inclusive.

Consider now the effects of *objectionable* error in market expectations. The effects may take a variety of forms, but perhaps the most reasonable assumption is that such errors would tend to produce a highly damped autoregressive series. (For present purposes, a sufficient definition of an autoregressive series is that it is a type of artificial time series in which random disturbances produce an appearance of irregular cyclical fluctuations, or oscillations.) Such a series represents the actual

FIG. 2.—PRICES OF "MAY WHEAT FUTURE," IF AFFECTED BY OBJECTIONABLE ERROR ONLY

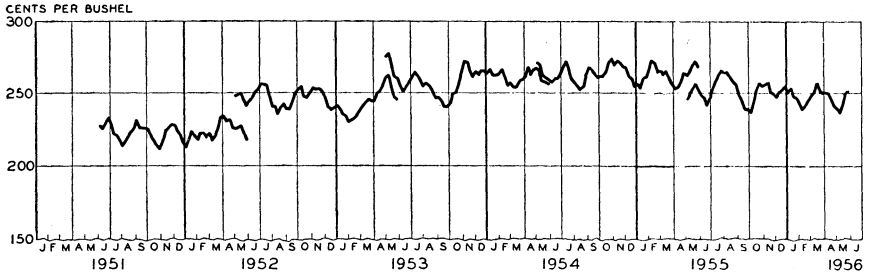
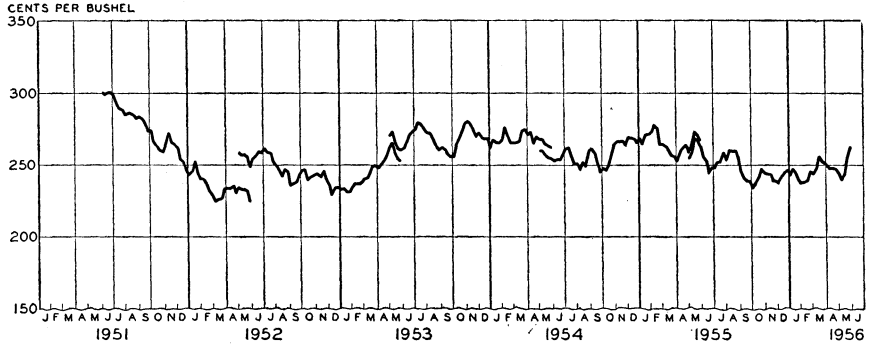


FIG. 3.—PRICES OF "MAY WHEAT FUTURE," IF AFFECTED BY BOTH NECESSARY AND OBJECTIONABLE ERROR



effects of objectionable errors of expectation better than a simple random series because the effect of any error is not confined to a single point in the series, but is spread over several adjacent points.[20] On that assumption, I take an autoregressive series which has been studied by M. G. Kendall[21] as the basis for suggesting what a record of futures

[20] A high degree of damping must also be assumed because available evidence indicates that an error of expectation produces only one clear wave in prices; it does not initiate a prominent series of oscillations such as occurs in only slightly damped autoregressive series.

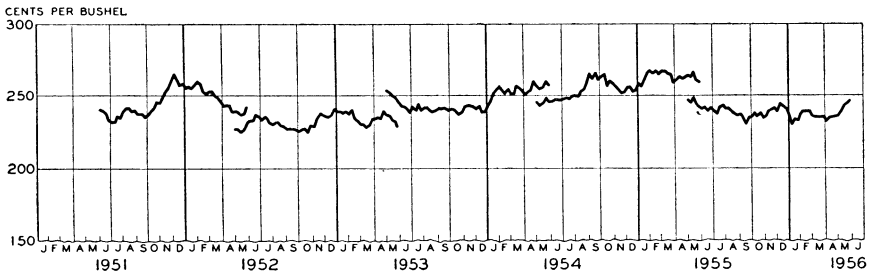
[21] "On the Analysis of Oscillatory Time-Series," *Journal of the Royal Statistical Society*, 1945, p. 111, Table 9. I have obtained 260 numbers by repeating Kendall's numbers 6-25, inclusive, at the end of his series of 240 numbers. The series happens to have characteristics which correspond extraordinarily well with known characteristics produced by exaggerative bias in wheat prices.

prices might look like if the inaccuracy of market expectations arose wholly from objectionable error. The data, with some "decorations" like those given the random-difference series to produce a superficial resemblance to actual futures prices, are reproduced in Figure 2.

The inaccuracy of actual market expectations is not either wholly necessary inaccuracy, as assumed for Figure 1, nor wholly objectionable inaccuracy, as assumed for Figure 2, but is a combination of both sorts of inaccuracy. One such combination may be obtained by simply adding each number used for Figure 2 to the corresponding number used for Figure 1. The results, after a slight modification of scale, are shown in Figure 3.

Is it possible to determine from an actual price record whether the

FIG. 4.—PRICES OF "MAY WHEAT FUTURE," CONDITION TO BE ASCERTAINED



conditions correspond to those of Figures 1, 2, or 3? There may be some persons who will feel that the distinction can be made by visual judgment. Possibly it can. For a real test of the power of visual judgment, a fourth series should be introduced for classification. Figure 4 shows a series which has exactly the same essential characteristics as one of the three preceding ones. Which of the three does it correspond to?

For the investigation of actual market expectations, we need statistical methods which will discriminate as efficiently as possible among series like those in Figures 1, 2, and 3. Any method should also provide some sort of measure of degree of approach to, or departure from, the condition of minimum inaccuracy, represented by Figure 1. We shall never expect to find that condition perfectly met, and shall be interested principally in knowing how much objectionable inaccuracy is present.

The fundamental statistical basis for discriminating between necessary and objectionable inaccuracy is that necessary inaccuracy produces price changes among which all serial correlations tend to be zero, whereas objectionable inaccuracy tends to produce price changes which have certain serial correlations that differ significantly from zero. This is not the place to discuss the relative merits of various statistical tests

which may be used to discriminate between necessary and objectionable inaccuracy, but there is one possible test which is especially interesting because it exhibits directly certain characteristics which are important for economics. I shall call this test the *error-time relation test.*

It seems obvious that inaccuracy of expectations should be expected to be greater for expectations concerning the price a week hence than for expectations concerning tomorrow's price. Similarly, expectations concerning the price a month hence are believed to be less accurate than expectations concerning next week's price. In short, we expect inaccuracy to increase as the time-span of expectation increases. The manner in which inaccuracy tends to increase with time, however, differs according to the source of inaccuracy.

For convenience, let us assume that the basic causes of inaccuracy do not change in magnitude during the period considered. That was the assumption made in deriving the curves for Figures 1 to 4. For the case of necessary inaccuracy, represented by Figure 1, the assumption means that the distribution of expected errors of expectation over a given time-span is the same in June, for example, as in January, and the same in June of one year as in June of another year. In the case of objectionable inaccuracy, represented by Figure 2, the assumption means that the probability of occurrence of a new error of given magnitude is similarly constant from time to time.

Under this assumption of constancy of the sources of error, the increase of necessary inaccuracy as the time-span of expectation increases can be expressed very simply. If inaccuracy be measured by the variance (that is, the square of the standard deviation) of errors of expectation, necessary inaccuracy tends to be exactly proportional to time-span. In other words, the variance of necessary error is directly proportional to the time-span of expectation. This remains true, in theory, however long the time-span of expectation.

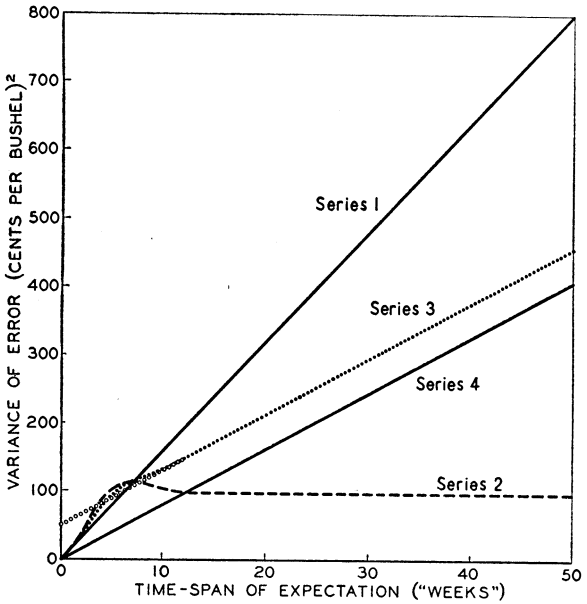
With objectionable error, the case is different. Starting from a low level for a very short time-span, the variance of objectionable error may increase, for a short distance, more rapidly than in proportion to length of time-span; but the rate of increase soon slows down and the variance of objectionable error tends to approach a maximum corresponding to a rather moderate length of time-span, and thereafter to increase not at all as the time-span increases. With objectionable errors of the sort represented in Figure 2, the inaccuracy tends actually to be somewhat smaller for time-spans greater than three months than for time-spans between two-and-a-half and three months.

The error-time relations corresponding to the data of the four preceding charts are shown, somewhat idealized, in Figure 5. If the variances plotted there were based on data for only the five "years" covered

by each of the preceding charts, the error-time curves would be quite irregular. The series shown in the preceding charts, however, are capable of indefinite extension. If they were considerably extended and the variances calculated and plotted, the resulting curves of error-time relation would still be slightly irregular, but they would conform very closely with the curves of Figure 5.

The error-time curve corresponding to the data of Figure 4, it will

FIG. 5.—ERROR-TIME RELATIONS



be noticed, is a straight line running through the origin of the chart, like that corresponding to the data of Figure 1. These are the distinguishing characteristics associated with presence of necessary inaccuracy only—the line of relationship is straight and passes through the origin of the chart. Presence of objectionable inaccuracy alone tends to give an error-time curve which flattens out after an initial rise. When both necessary and objectionable error are present, as in the data of Figure 2, an extension of the straight part of the error-time curve (the line of hollow dots in Figure 5) passes above the origin of the chart.

Some advantages might have been gained by drawing Figure 5 with both scales logarithmic. It would then have been found that presence of necessary inaccuracy alone gives a straight line with a slope of unity; presence of objectionable inaccuracy alone gives a curve which, after an initial rise, takes a slope of zero; and when necessary and objec-

tionable inaccuracy are combined, the (logarithmic) slope of the curve after its initial rapid rise is an index of the proportion of the inaccuracy which is necessary—a proportion which, in theory, may range between zero and 100 per cent in different circumstances.[22]

This discussion of the definition of necessary and objectionable inaccuracy of market expectations and of the possibility of measuring the relative proportion of observed inaccuracy which is necessary must end without any very firm conclusions regarding the state of affairs in actual markets. A good deal remains to be done in testing the reliability of measurements, and in devising means to cope with some difficulties which I have not discussed, before definite conclusions can be reached. It may be worth noting, nevertheless, that such preliminary studies as I have made suggest that the inaccuracy of actual market expectations is composed mainly of necessary inaccuracy.

[22] I did not use logarithmic scales for Figure 5 because I feared the advantages would be more than offset for present purposes by excessive emphasis on characteristics of the curves over the very short time-spans.